HW. 2

Deadline: July/28th 10:00
notice:

1. Please write down your name, affiliation, & student ID.
2. Please use A4 paper.

1. What is Physical quantity?
2. What are vector and scalar?
3. What is energy, Show some examples of energy?
4. What is field, Show some examples of field?
5. What are conservative farce and non-conservation force?

6. Find the velocity $\dot{x}$ and position x as functions of the time for a particle of mass m, which starts from rest x =0 and t =0 subject to the following force function:
   1. $F_x = F_o + Ct$
   2. $F_x = F_0 \sin ct$
   3. $F_x = F_0 e^{ct}$, where F_0 and C are positive constant

   example: $F_x = F_o + Ct^2 = ma = m\ddot{x};$

   $$F_o t + \frac{Ct^3}{3} + v_0 = m\dot{x};$$

   $$\frac{F_0 t^2}{2} + \frac{Ct^4}{12} + v_0 t + x_0 = mx;$$

   Since the particle start from x= 0 and t =0;

   $$\frac{F_0 t^2}{2} + \frac{Ct^4}{12} + v_0 t + x_0 = mx \rightarrow \frac{F_0 t^2}{2} + \frac{Ct^4}{12} + v_0 t + 0 = 0\text{, it lead } x_0 = 0$$

   Since the particle start from rest

   $$F_o t + \frac{Ct^3}{3} + v_0 = m\dot{x} \rightarrow F_o t + \frac{Ct^3}{3} + 0 = m \times 0\text{, it lead } v_0 = 0$$

   ANSWAR: $F_o t + \frac{Ct^3}{3} = m\dot{x}; \frac{F_0 t^2}{2} + \frac{Ct^4}{12} = mx$

7. Find the potential energy function V(x) for following forces ?
   1. $F_x = F_o + Cx$
   2. $F_x = F_0 \cos cx$
   3. $F_x = F_0 e^{-cx}$, where F_0 and C are positive constant

   ***hint 1**. What is potential energy function:

   According the mechanical energy conservation $\int_{x_0}^{x} F(x)dx = \Delta E_k = T - T_0$

   $\int_{x_0}^{x} F(x)dx$ is the work done on the particle by the impressed force F(x), thus work is equal to the change in the kinetic energy of particle. Hence we can define a function V(x) such that $-\frac{dV}{x} = F(x)$, then $\int_{x_0}^{x} F(x)dx = -\int_{x_0}^{x} dV = T - T_0 = -V(x) + V(x_0)$ and find the function V(x) is the potential himself. Hence we can get the potential(potential energy function) from the force function. $-\frac{dV}{dx} = F(x)$

8. [Line integral in plane] I will introduce line integral on next Tuesday. ***hint 2**

considering a body that is pushed with force $\vec{F}(\vec{r}(t)) = -10\hat{j}$ alone the path $\vec{r}(t) = t\hat{i} + t\hat{j}$ (a). Draw the path from t=0 to t=1 sec. (b)calculate the work done by the force $\vec{F}(\vec{r}(t))$ from t = 0 sec to 1 sec

***hint 2** $\int_c \vec{F}(\vec{r})d\vec{r} = \int_a^b (F_x dx + F_y dy + F_z dz) = \int_a^b (F_x x' + F_y y' + F_z z')dt$

C is contour of the integral path from initial point a to point b, F_i is the component of $\vec{F}$, $\vec{r}$ is position vector.

Example: $\vec{F}(t) = x\hat{i} - 10\hat{j}, \vec{r}(t) = t\hat{i} + e^t\hat{j}$, calculate the work from t=0 to t= 1:

$$\vec{F}(\vec{r}(t)) = x(t)\hat{i} - 10\hat{j} = t\hat{i} - 10\hat{j}, \vec{r}'(t) = r_x'\hat{i} + r_y'\hat{j} = \hat{i} + e^t\hat{j}$$

$$\int_c \vec{F}(\vec{r})d\vec{r} = \int_0^1 (t\hat{i} - 10\hat{j}) \cdot (\hat{i} + e^t\hat{j})dt = \int_0^1 (t - 10e^t)dt = \frac{1}{2} - 10e$$

9. considering a body that is pushed with force $\vec{F}(\vec{r}(t)) = 1\hat{i} + 1\hat{j}$ alone the path $\vec{r}(t) = t\hat{i} + t\hat{j}$ (a)calculate the work done by the force $\vec{F}(\vec{r}(t))$ from t = 0 sec to 1 sec

10. considering a body that is pushed with force $\vec{F}(\vec{r}(t)) = t\hat{i} + t\hat{j}$ alone the path $\vec{r}(t) = t\hat{i} + t\hat{j}$ (a)calculate the work done by the force $\vec{F}(\vec{r}(t))$ from t = 0 sec to 1 sec

11. considering a body that is pushed with force $\vec{F}(\vec{r}(t)) = t\hat{i} + t\hat{j}$ alone x axis from t = 0 sec to 1 sec and change direction to y-axis from 1 to 2 second.

    $\vec{r}(t) = t\hat{i}$; for t = 0-1 second

    $= \hat{i} + (t-1)\hat{j}$; for t = 1-2 second

    calculate the work done by the force $\vec{F}(\vec{r}(t))$ alone the path from t = 0 sec to 2 sec

12. According to the work result of problem 10 & 1, The force $\vec{F}(\vec{r}(t)) = t\hat{i} + t\hat{j}$ is conservative force or non-conservative force, Why?

13. A ball (mass =1 kg) is dropped from the rest from the top of Taipei 101(508m). Calculate **(a)** the initial potential energy of the baseball, **(b)** its finial kinetic energy(ignore air resistance)

14. A ball (mass =m) is dropped from the rest from the top of Taipei 101. Show the velocity v is $\frac{mg}{c} - e^{-\frac{ct}{m} + \frac{cc'}{m}}$, with air resistance $F_g = c\vec{v}$, c' is some constant

    ***hint 4:** start from $m\frac{dv}{dt} = mg - cv$

Advance:

15. A ball (mass =m) is dropped from the rest from the top of Taipei 101(set top is 0 m). Show the velocity v is $a \tanh \frac{act}{m}$, with air resistance $F_g = c\vec{v}^2$ and $a = \sqrt{\frac{mg}{c}}$.

***hint 5:** start from $m\frac{dv}{dt} = mg - cv^2$, and use the integral table

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{a}\tanh^{-1}\frac{x}{a}$$

16. Show the falling distance y is $\frac{m}{c}\ln\cosh\frac{act}{m}$, other condition is same as problem 15.

***hint 6 :** $\int \tanh u\, du = \ln\cosh u$