General Physics I

Mechanics, optics, thermal dynamics, and other basic fundamental things.

TsungChe Liu

Lecture VII : measurement

Gay-Lussac's law

The pressure of a gas of fixed mass and fixed volume is directly proportional to the gas's absolute temperature.

$$\frac{T}{p} = C \qquad \frac{T_{triple}}{p_{triple}} = C$$

1778-1850

Triple point of water

$$\frac{273.16K}{p_{triple}} = C$$

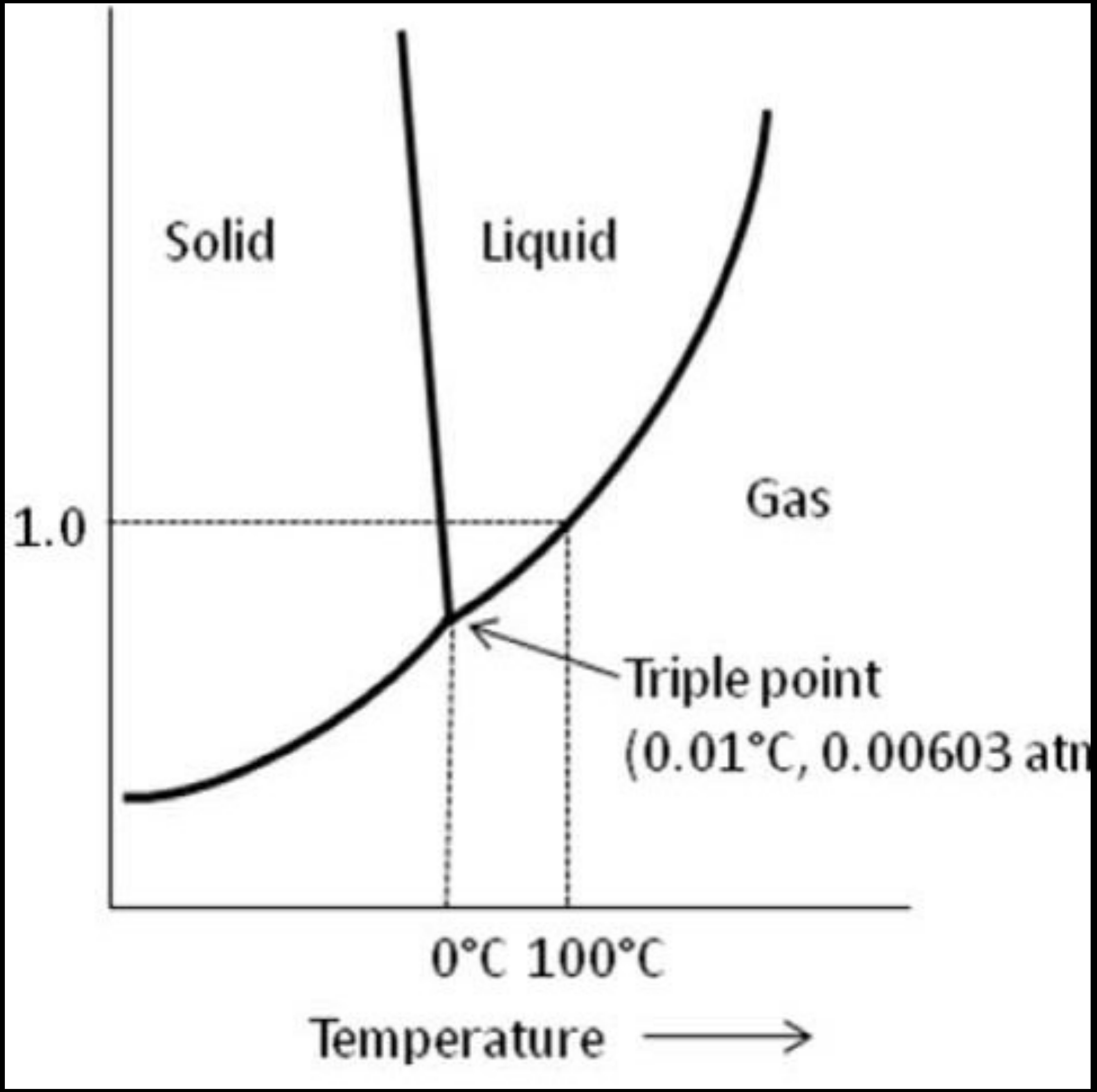
Lecture VII : Absolute zero

Limit to the "degree of cold"

In the SI system, temperature is measured on the Kelvin scale, which is based on the triple point of water (273.16 K).

$$\frac{T}{p} = C = \frac{273.16K}{p_{triple}}$$

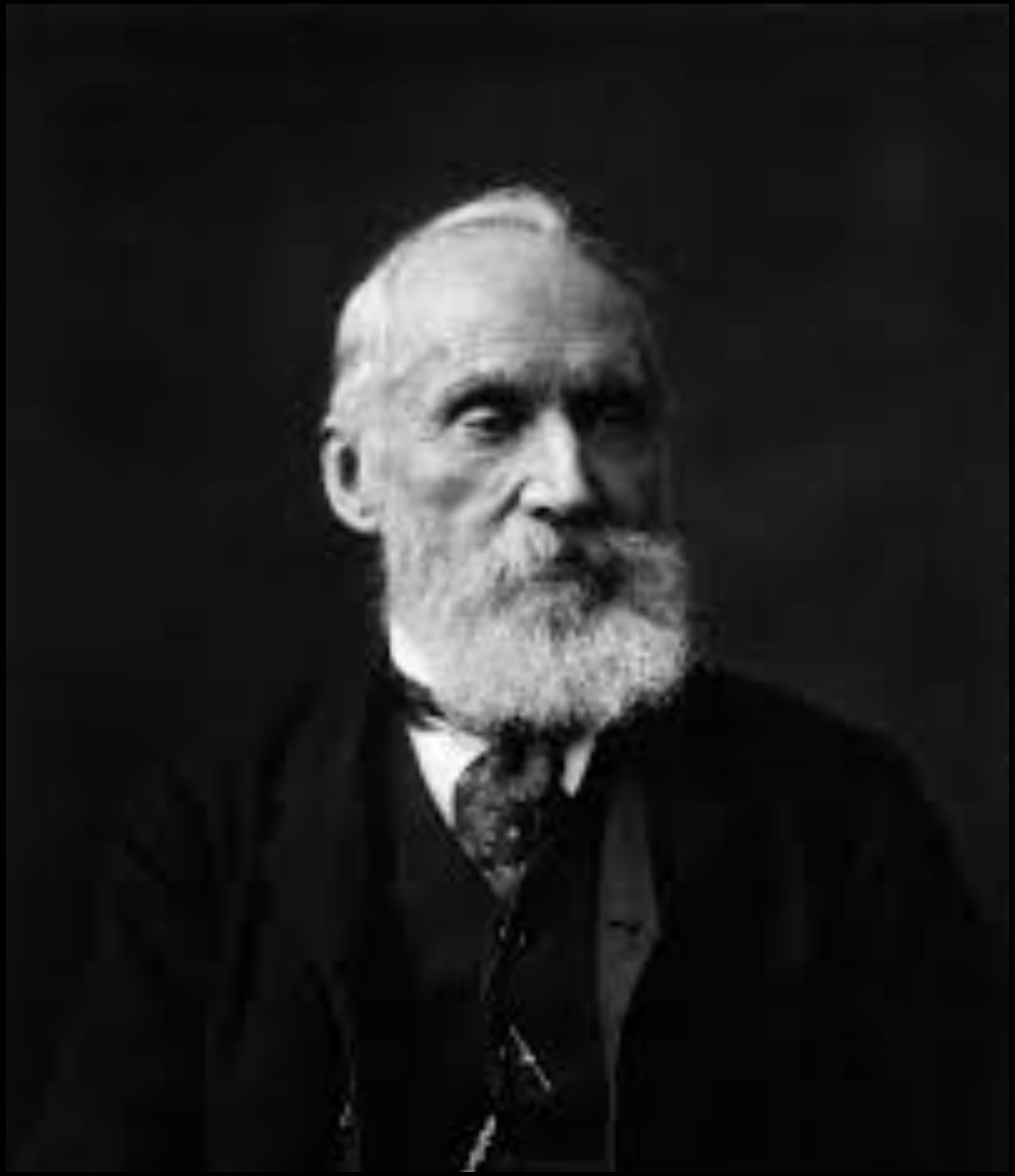
$$T = 273.16K\left(\frac{P_M}{P_{triple}}\right)$$



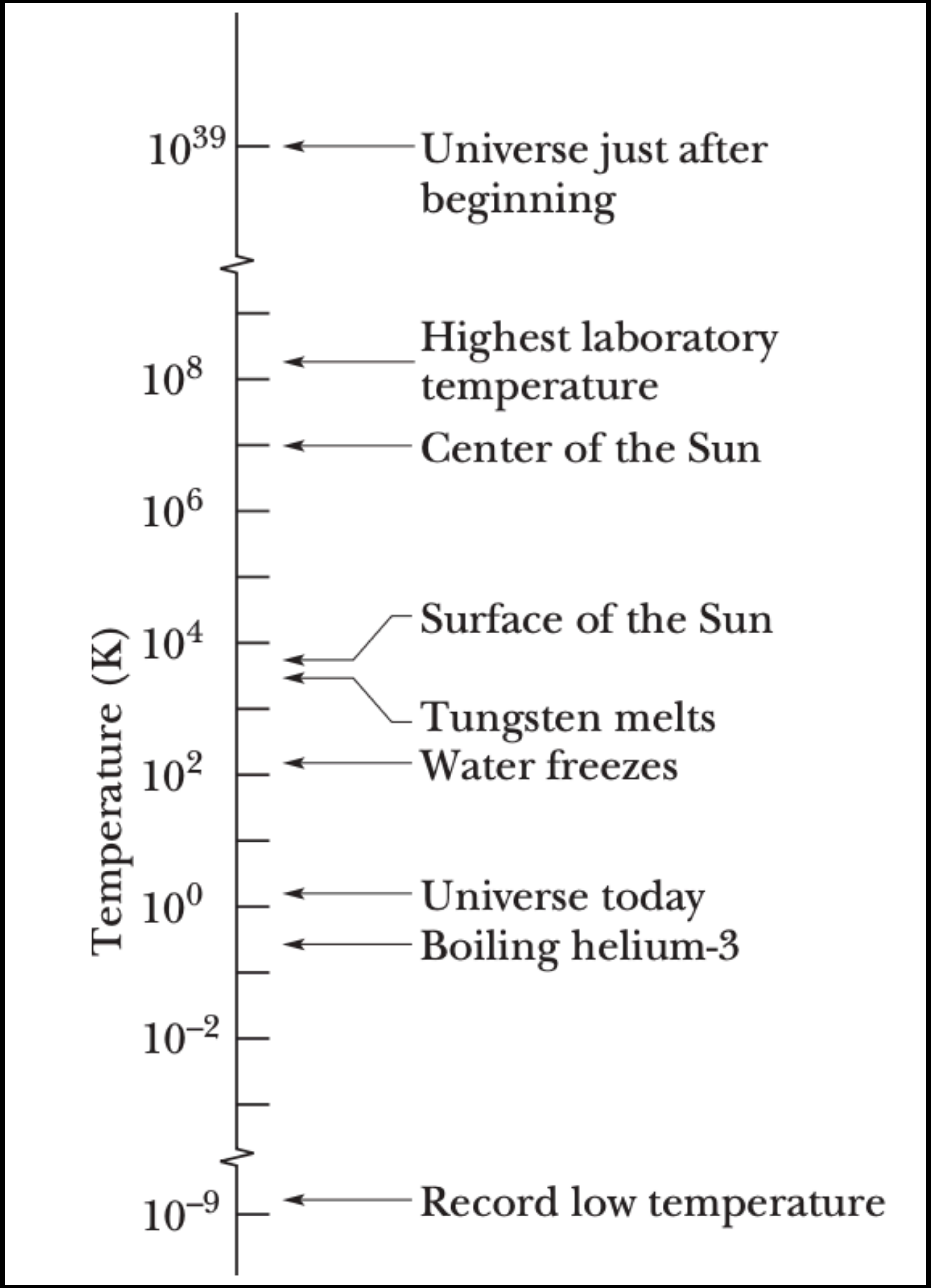
Lecture VII : Kelvin scale

Unit: kelvins

~~^{o}K~~ K



1824-1907

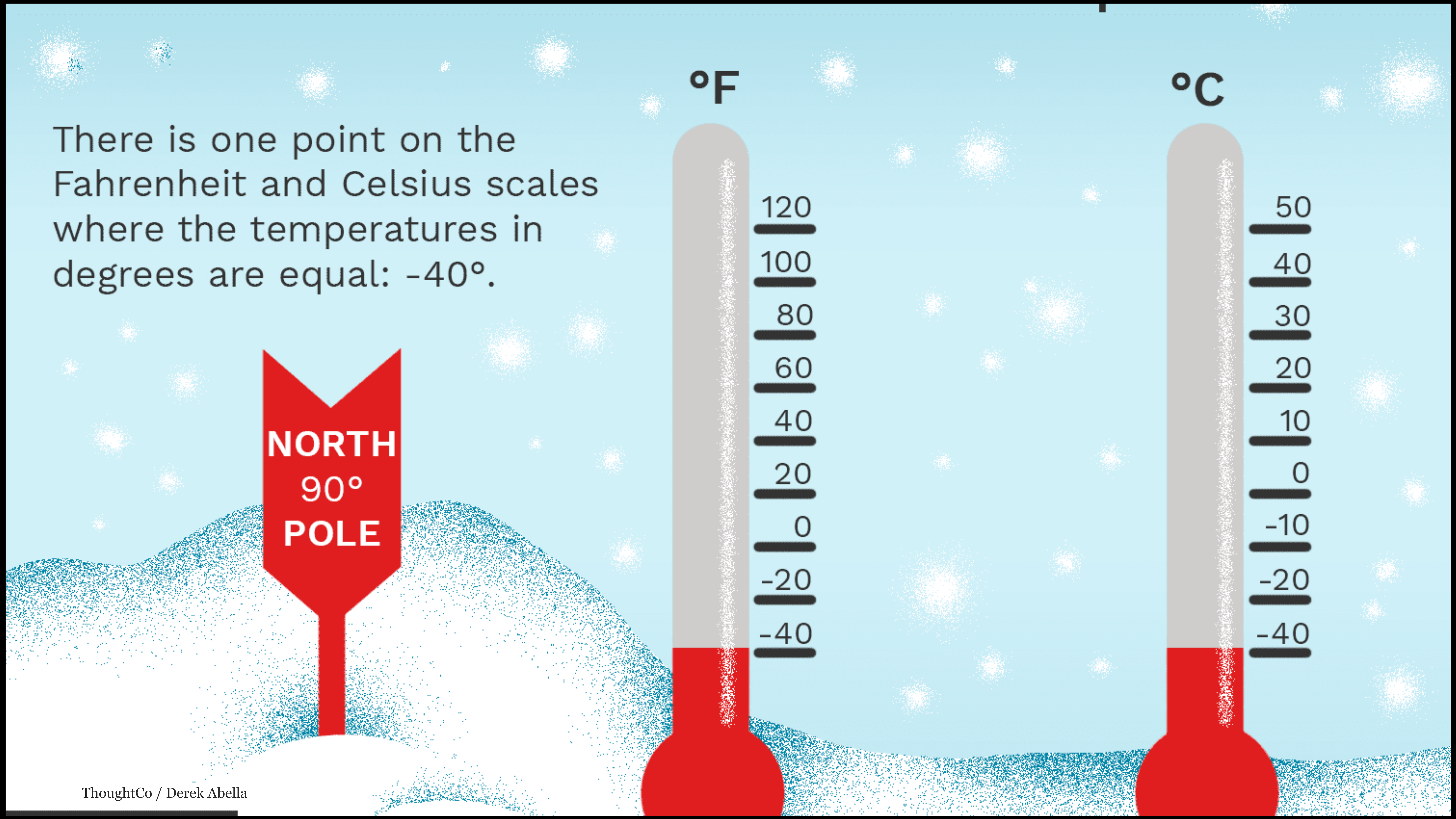


Lecture VII : scale

Temperature is a physical property of matter that quantitatively expresses hot and cold

Celsius & Fahrenheit

$$C^o = \frac{9}{5}F^o + 32$$



There is one point on the Fahrenheit and Celsius scales where the temperatures in degrees are equal: -40°.

ThoughtCo / Derek Abella

Lecture VII : scale

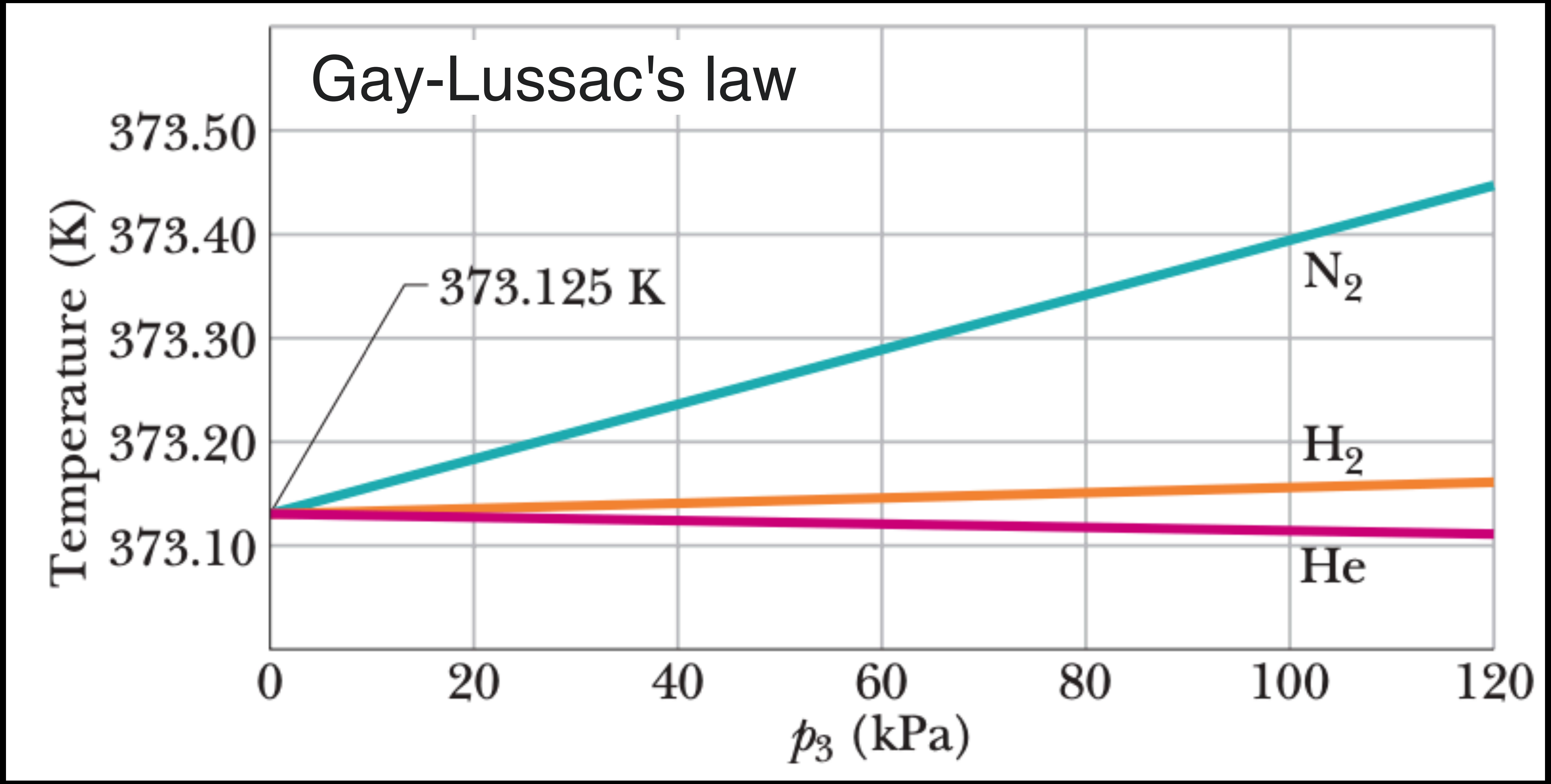
Temperature	^oC	^oF
Boiling point od water	100	212
Human body	37	98.6
Freezing point of water	0	32
Scales coincide	-40	-40

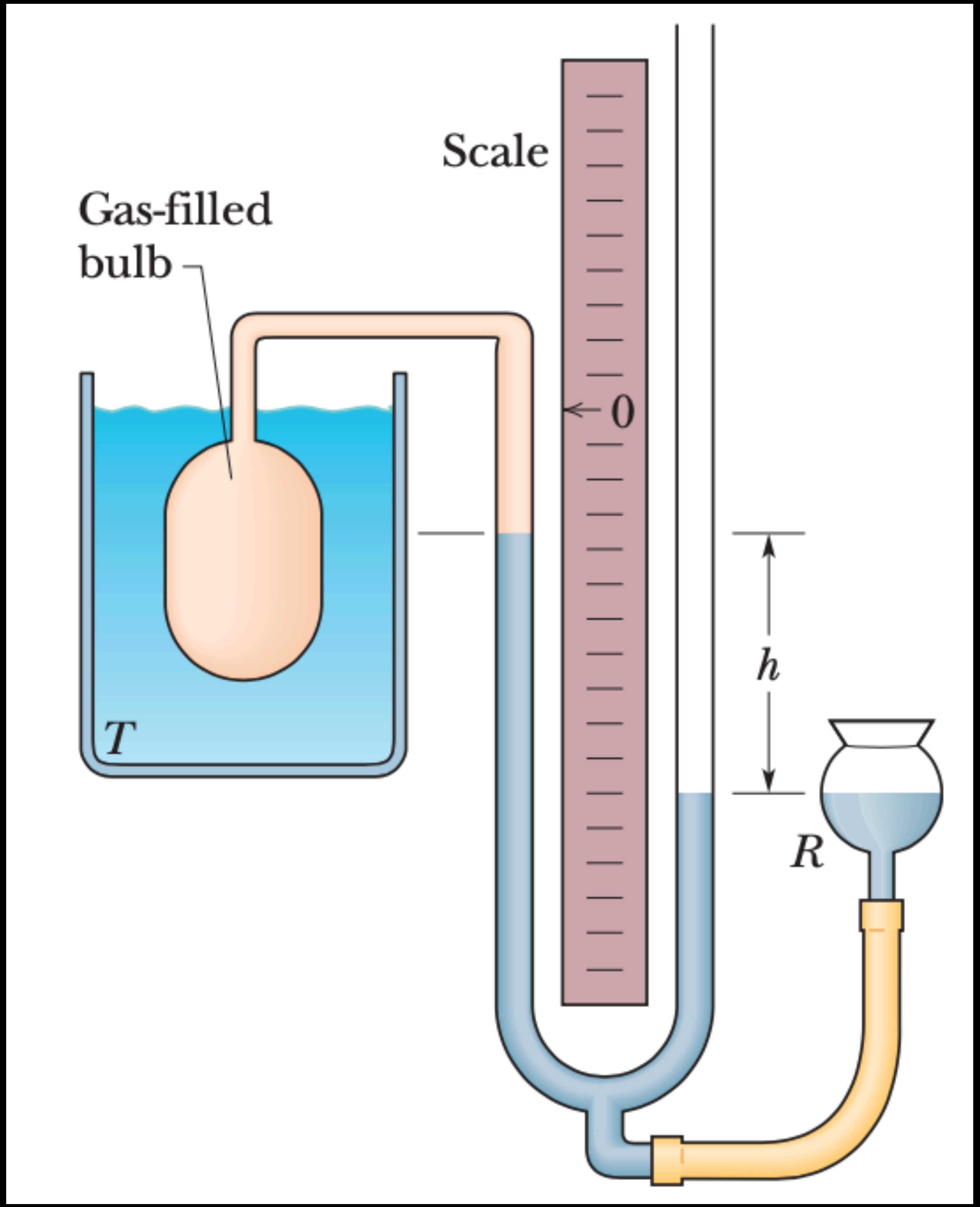
Lecture VII : measurement

The Constant-Volume Gas Thermometer

The temperature of any body in thermal contact with the bulb is then defined to be

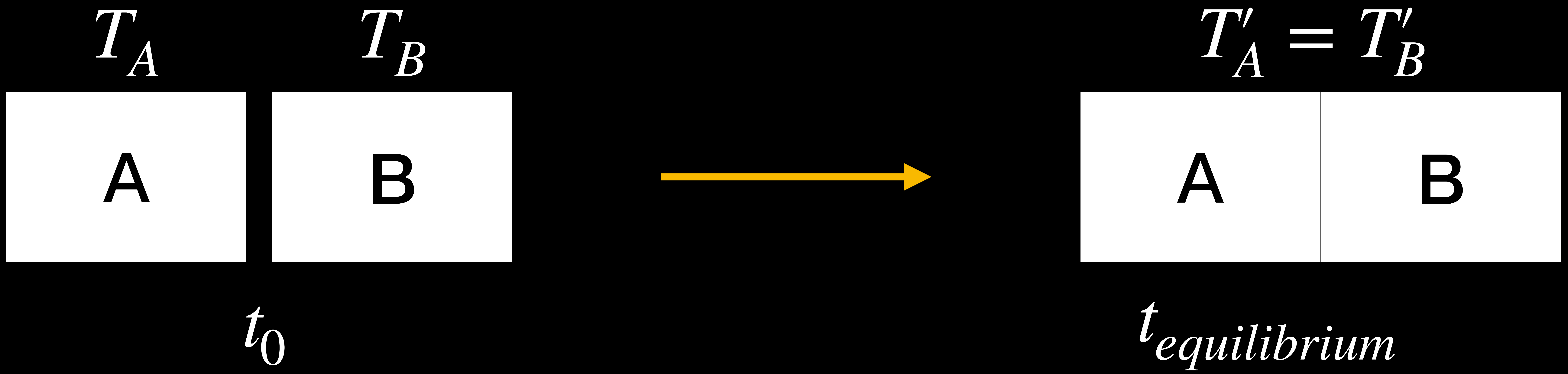
$$T = Cp \qquad p = p_0 - \rho g h$$





Lecture VII : Zeroth law of thermodynamics

If bodies A and B are each in thermal equilibrium with a temperature T, then A and B are in thermal equilibrium with each other.



T_A T_B

A B

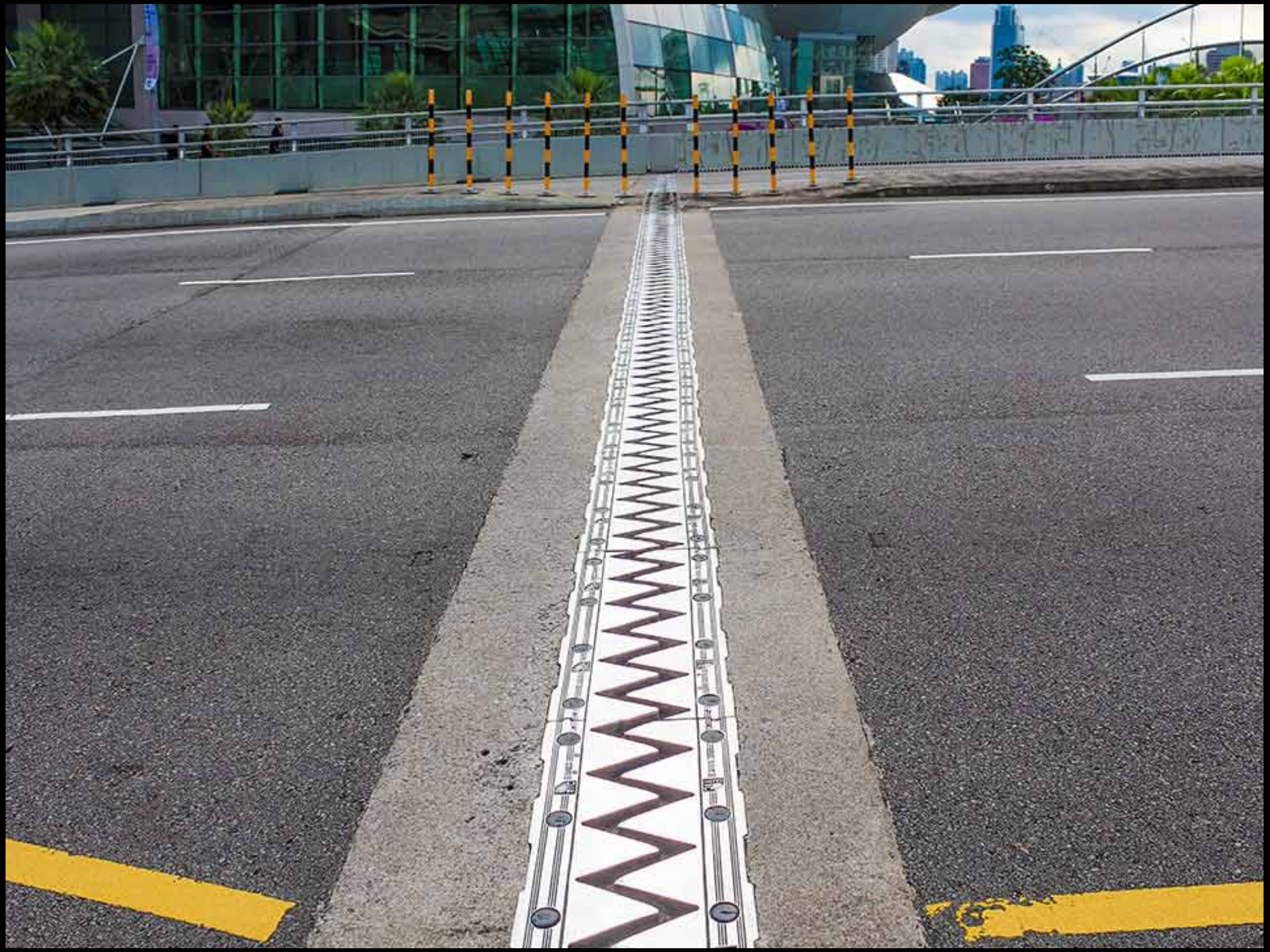
t_0

$T'_A = T'_B$

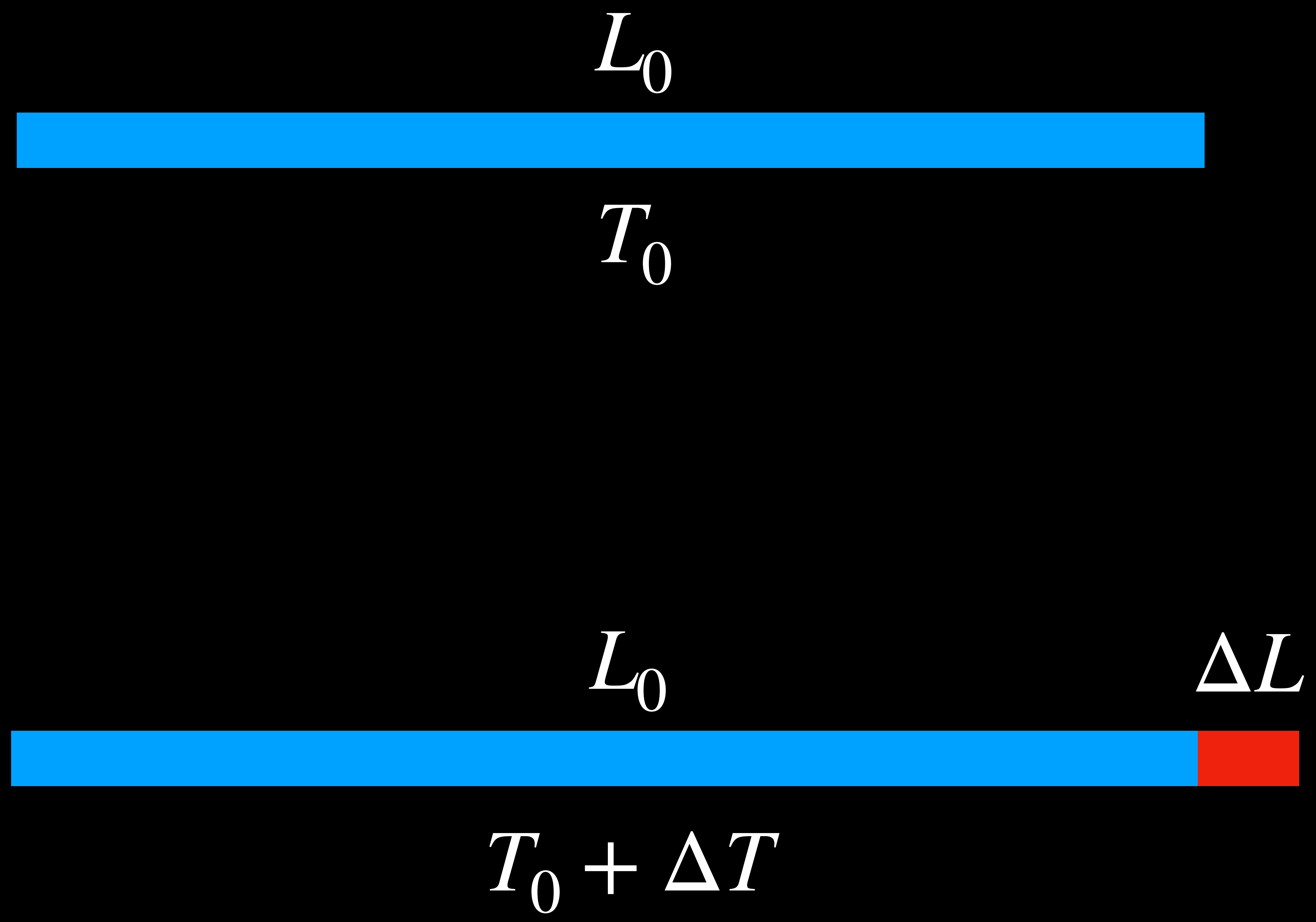
A B

$t_{equilibrium}$

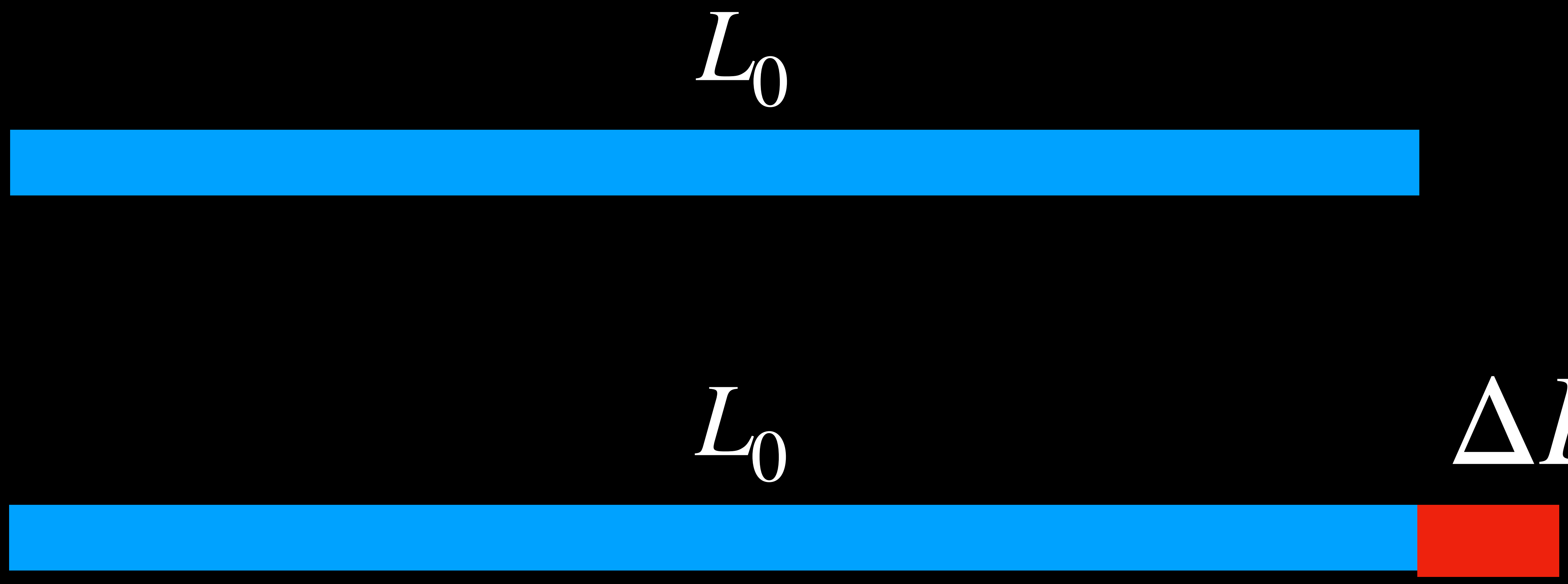
Lecture VII :



Lecture VII : Thermal expansion

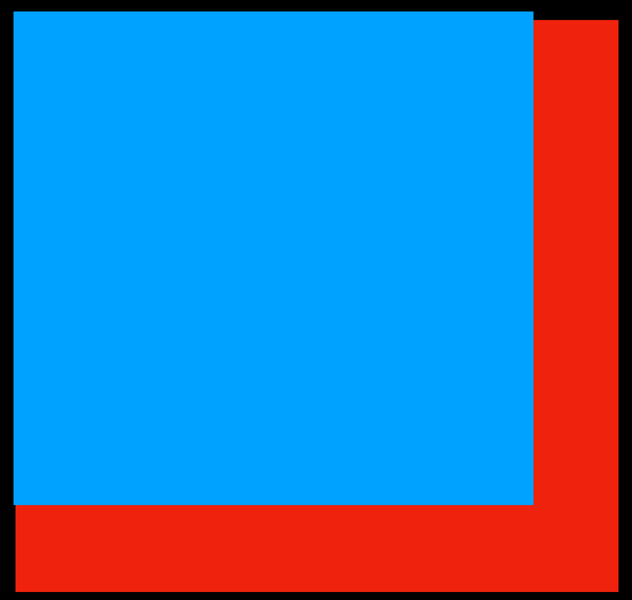


Lecture VII : Thermal expansion



$$\Delta L = L_0 \alpha \Delta T$$

α coefficient of linear expansion



$$A = (L_0 + L_0 \alpha \Delta T)^2$$

$$A = L_0^2 (1 + \alpha \Delta T)^2 \sim A(1 + 2\alpha \Delta T)$$

$$\beta \equiv 2\alpha$$

coefficient of area expansion

Lecture VII : Thermal expansion

coefficient of volume expansion

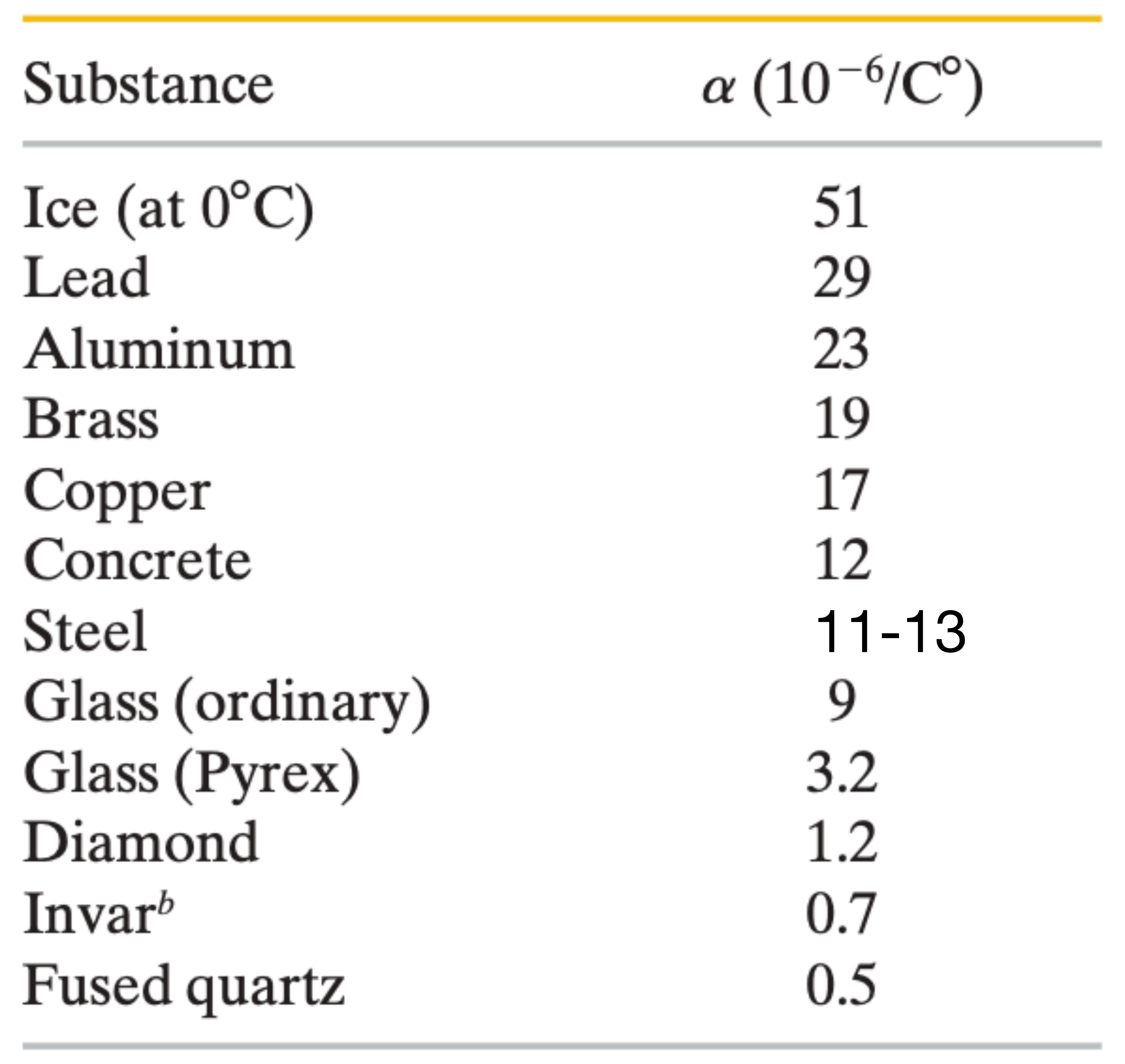
$$\Delta V = (L_0 + L_0 \alpha \Delta T)^3$$

$$\Delta V = L_0^3 (1 + \alpha \Delta T)^3 \sim V_0 (1 + 3\alpha \Delta T)$$

$$\gamma \equiv 3\alpha$$

coefficient of volume expansion

Lecture VII : Thermal expansion



Substance	α (10^{-6}/C°)
Ice (at 0°C)	51
Lead	29
Aluminum	23
Brass	19
Copper	17
Concrete	12
Steel	11-13
Glass (ordinary)	9
Glass (Pyrex)	3.2
Diamond	1.2
Invar[b]	0.7
Fused quartz	0.5

Lecture VII : Example

I have a 10 cm^3 steel box that is measured by a steel ruler at room temperature (40 ^oC) in Taiwan.

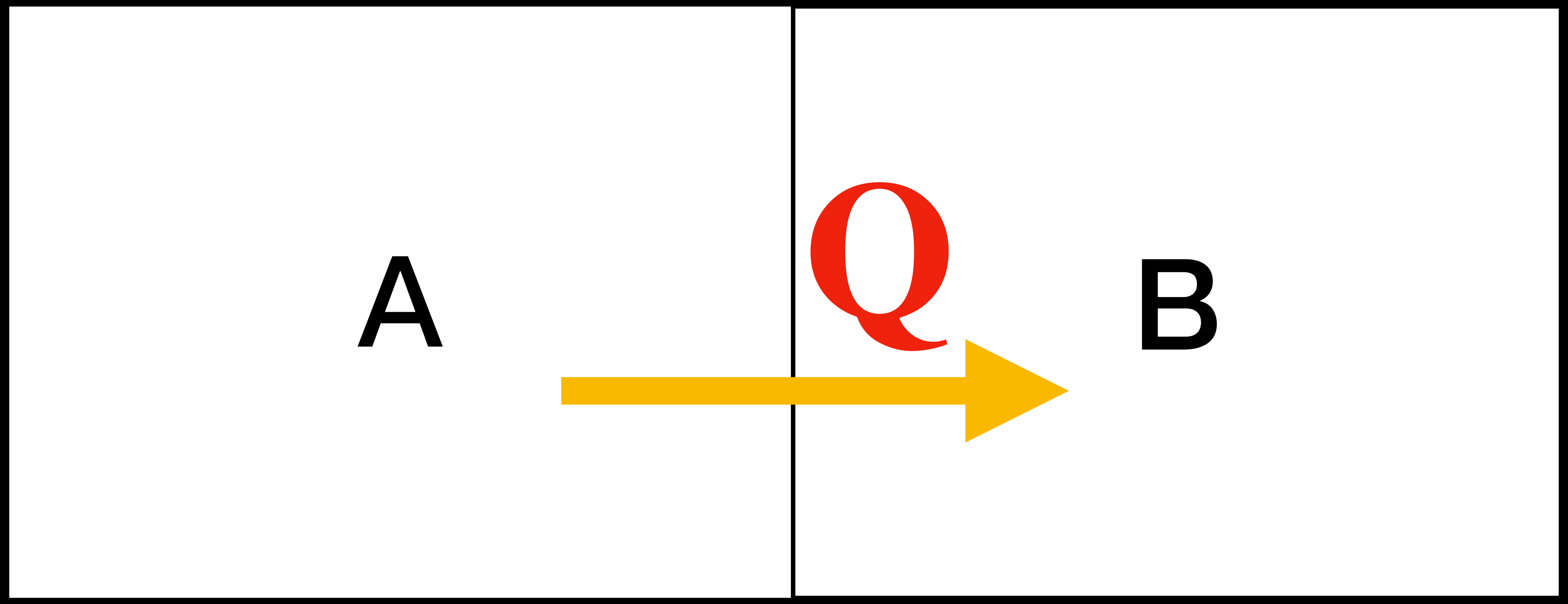
1. What is the volume of the same box that measured by the same ruler in Antarctica (233.16 K)?

2. What is volume change when the box moves to Antarctica from Taiwan?

Lecture VII : Temperature & heat (Q)

Heat is the energy transferred between two systems because of a temperature difference that exists between them.

$$T_A > T_B$$



The transferred energy is called heat and is symbolized Q

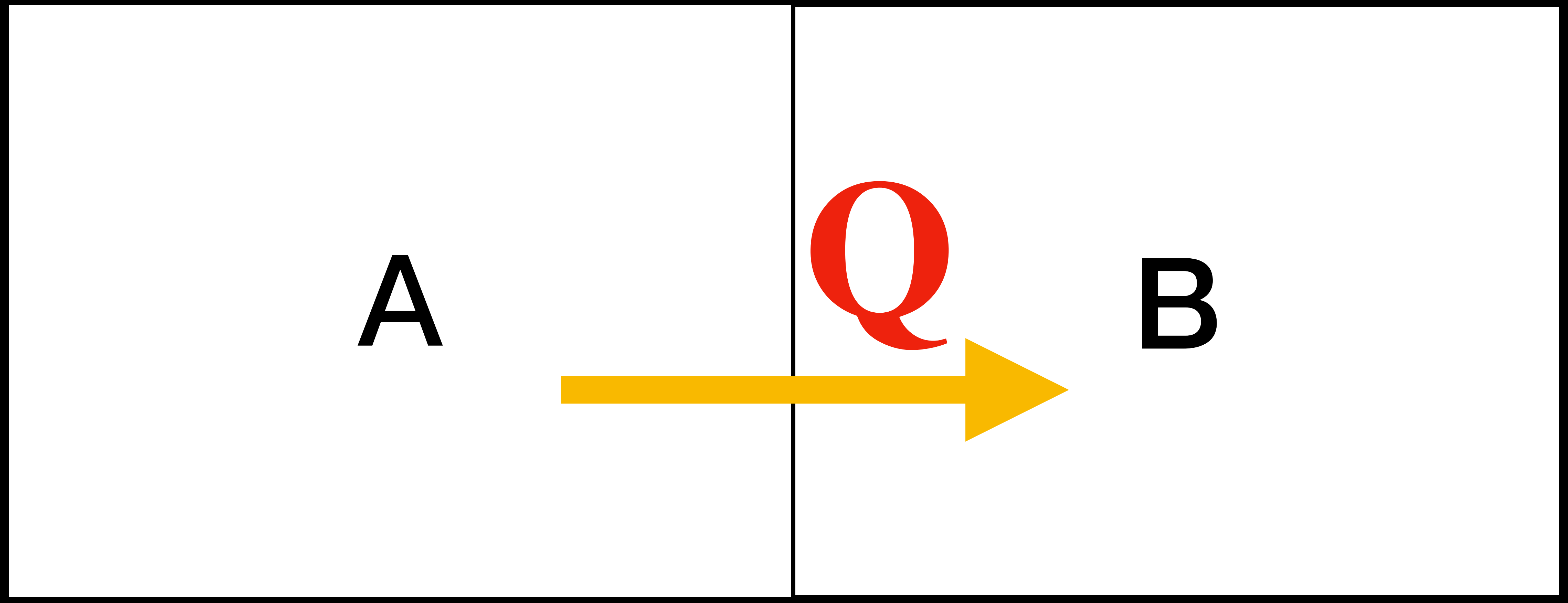
Lecture VII : Units of heat (Q)

Before scientists realized that heat is transferred energy, heat was measured in terms of its ability to raise the temperature of water. Thus, the **calorie (cal)** was defined as the amount of heat that would raise the temperature of 1 g of water from 14.5C to 15.5C. In the British system, the corresponding unit of heat was the British thermal unit (Btu), defined as the amount of heat that would raise the temperature of 1 lb of water from 63F to 64F. In 1948, the scientific community decided that since heat (like work) is transferred energy, the SI unit for heat should be the one we use for energy— namely, the **joule**. The calorie is now defined to be 4.1868 J (exactly), with no refer- ence to the heating of water. (The “calorie” used in nutrition, sometimes called the Calorie (Cal), is really a kilocalorie.) The relations among the various heat units are

$$1\ cal = 10^{-3}\ Cal = 3.968 \times 10^{-}3\ Btu = 4.178\ J$$

Lecture VII : Specific heat (c)

$$T_A > T_B$$



The transferred energy is called heat and is symbolized Q

$$Q = c\,m\Delta T = C\Delta T$$

c : “heat capacity per unit mass” or specific heat

C : heat capacity

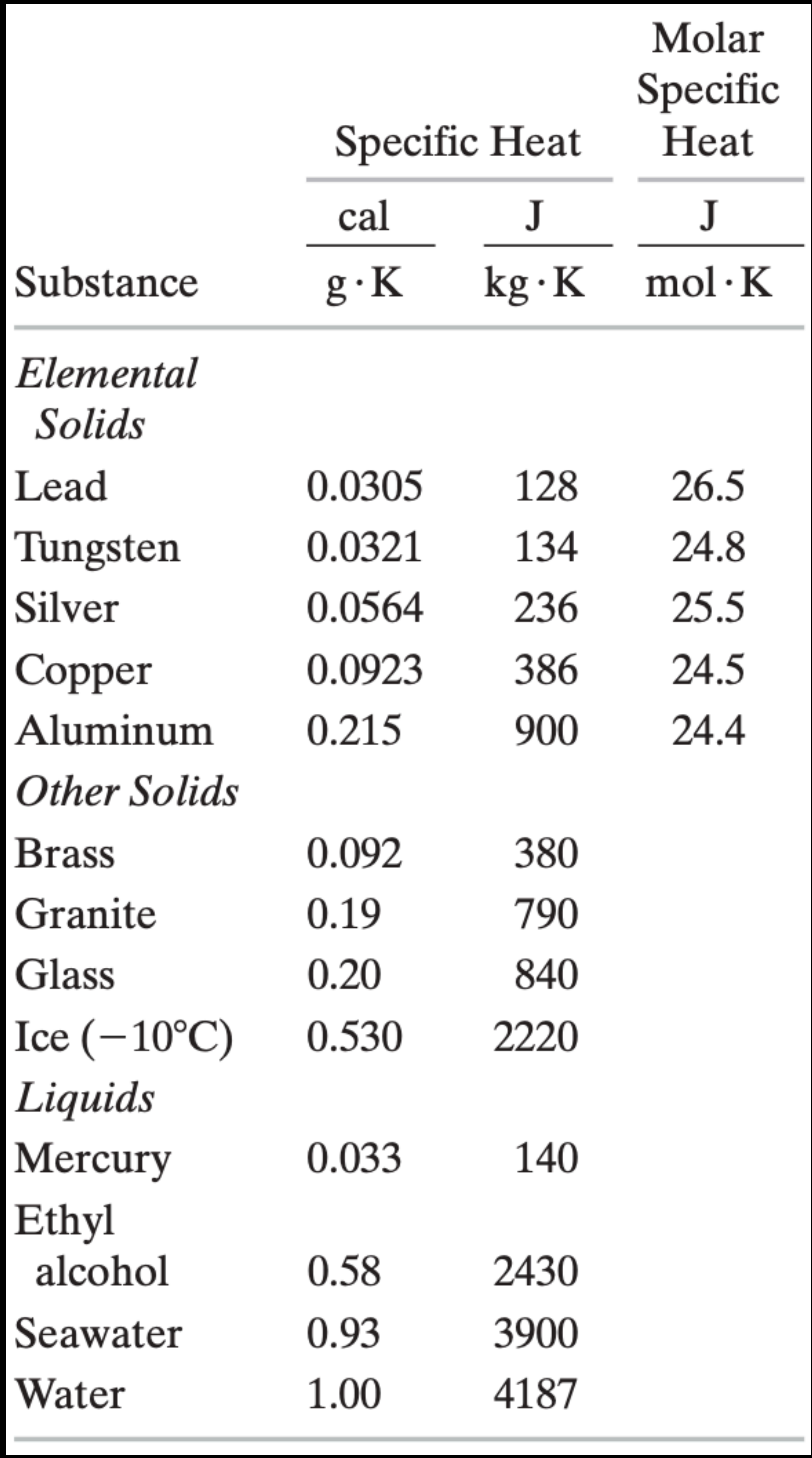
Lecture VII : Molar specific heat

$$1\ mol = 6.02 \times 10^{23}\ elementary\ unit$$

“heat capacity of per molar material” or Molar specific heat

Avogadro’s Number: $N_A = 6.02 * 10^{23}\ mol^{-1}$

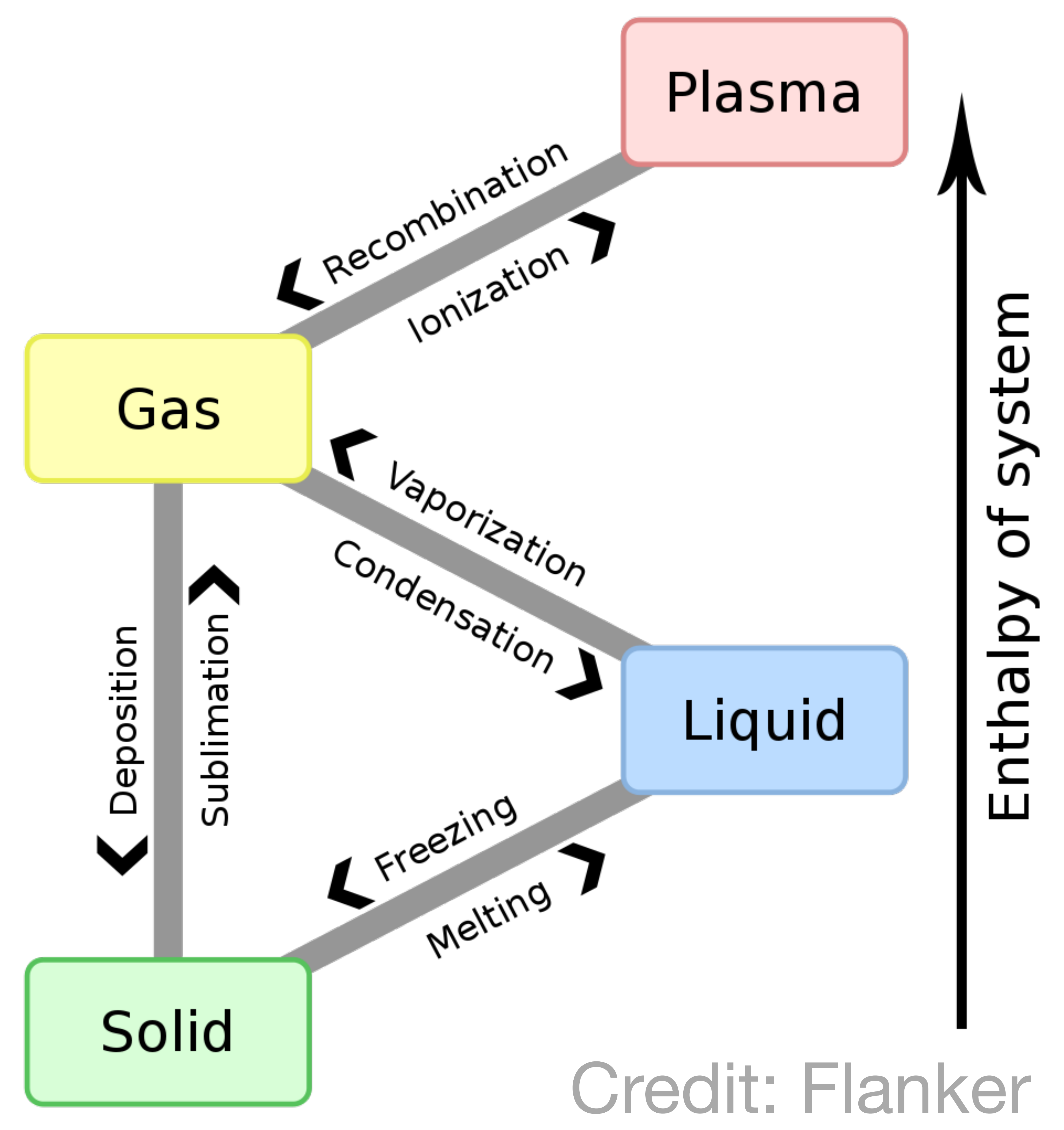
One mole is the number of atoms in a 12 g sample of carbon-12.



Substance	Specific Heat $\frac{cal}{g \cdot K}$	Specific Heat $\frac{J}{kg \cdot K}$	Molar Specific Heat $\frac{J}{mol \cdot K}$
Elemental Solids			
Lead	0.0305	128	26.5
Tungsten	0.0321	134	24.8
Silver	0.0564	236	25.5
Copper	0.0923	386	24.5
Aluminum	0.215	900	24.4
Other Solids			
Brass	0.092	380	
Granite	0.19	790	
Glass	0.20	840	
Ice (−10°C)	0.530	2220	
Liquids			
Mercury	0.033	140	
Ethyl alcohol	0.58	2430	
Seawater	0.93	3900	
Water	1.00	4187	

Lecture VII : Phase Transition

	Solid	Liquid	Gas	Plasma
Solid		Melting		
Liquid	Freezing		Vaporization	
Gas	Deposition	Condensation		Ionization
Plasma			Recombination	



Credit: Flanker

Lecture VII : Phase Transformation

The amount of energy per unit mass that must be transferred as heat when a sample completely undergoes a phase change is called the heat of transformation L.

$$Q = Lm$$

When the phase change is from liquid to gas (then the sample must absorb heat) or from gas to liquid (then the sample must release heat), the heat of transformation is called the heat of vaporization.

$$L_V = 539\ cal/g = 40.7\ kJ/mol$$

When the phase change is from solid to liquid (then the sample must absorb heat) or from liquid to solid (then the sample must release heat), the heat of transformation is called the heat of fusion.

$$L_F = 79.5\ cal/g = 6.01\ kJ/mol$$

Lecture VII : The First Law of Thermodynamics

The internal energy E_{int} of a system tends to increase if energy is added as heat Q and tends to decrease if energy is lost as work W done by the system.

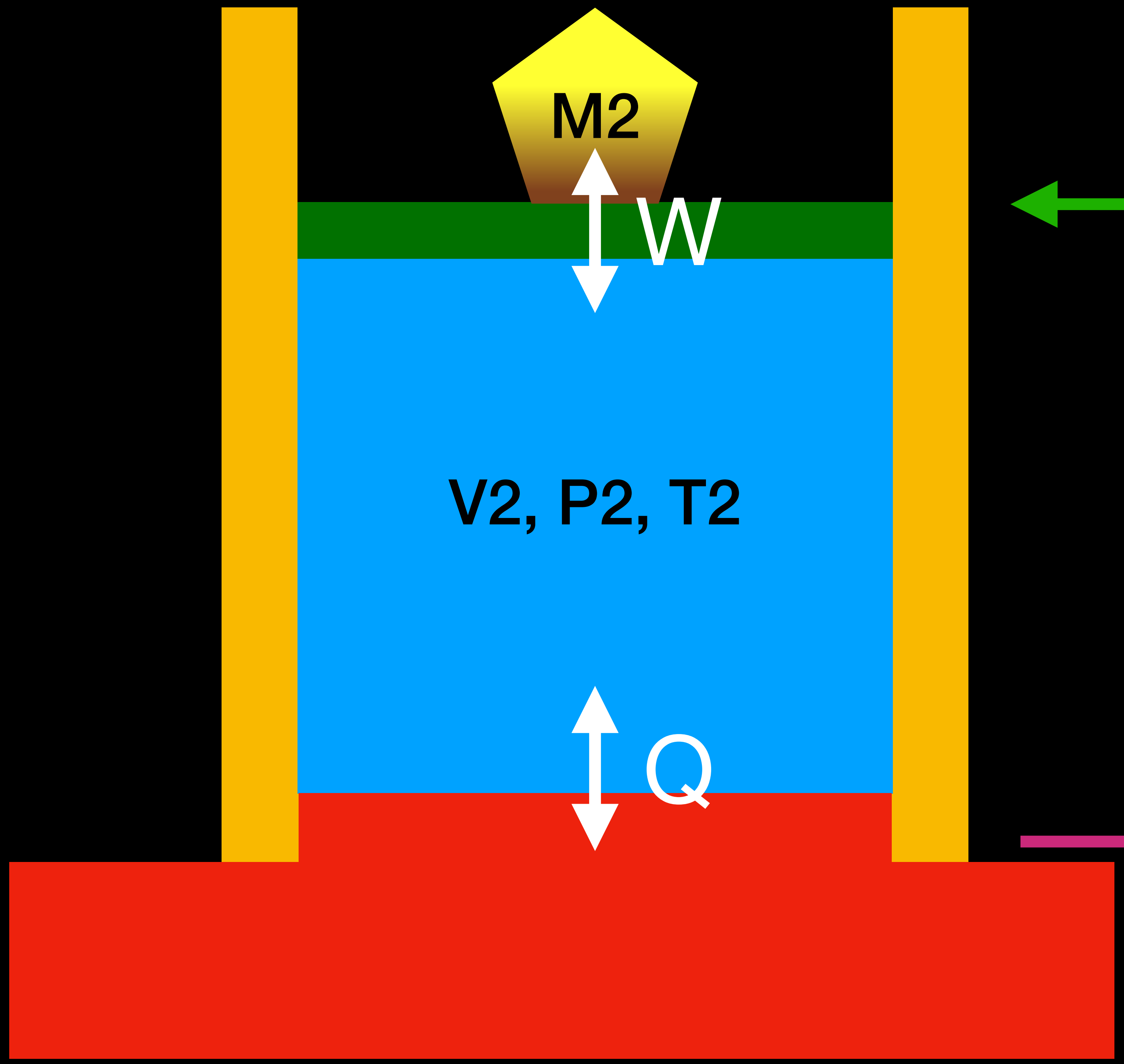
$$dE_{int} = dQ - dW$$

The work done by system

change of the total energy

The heat flow into system

Lecture VII : Start from a Gas System



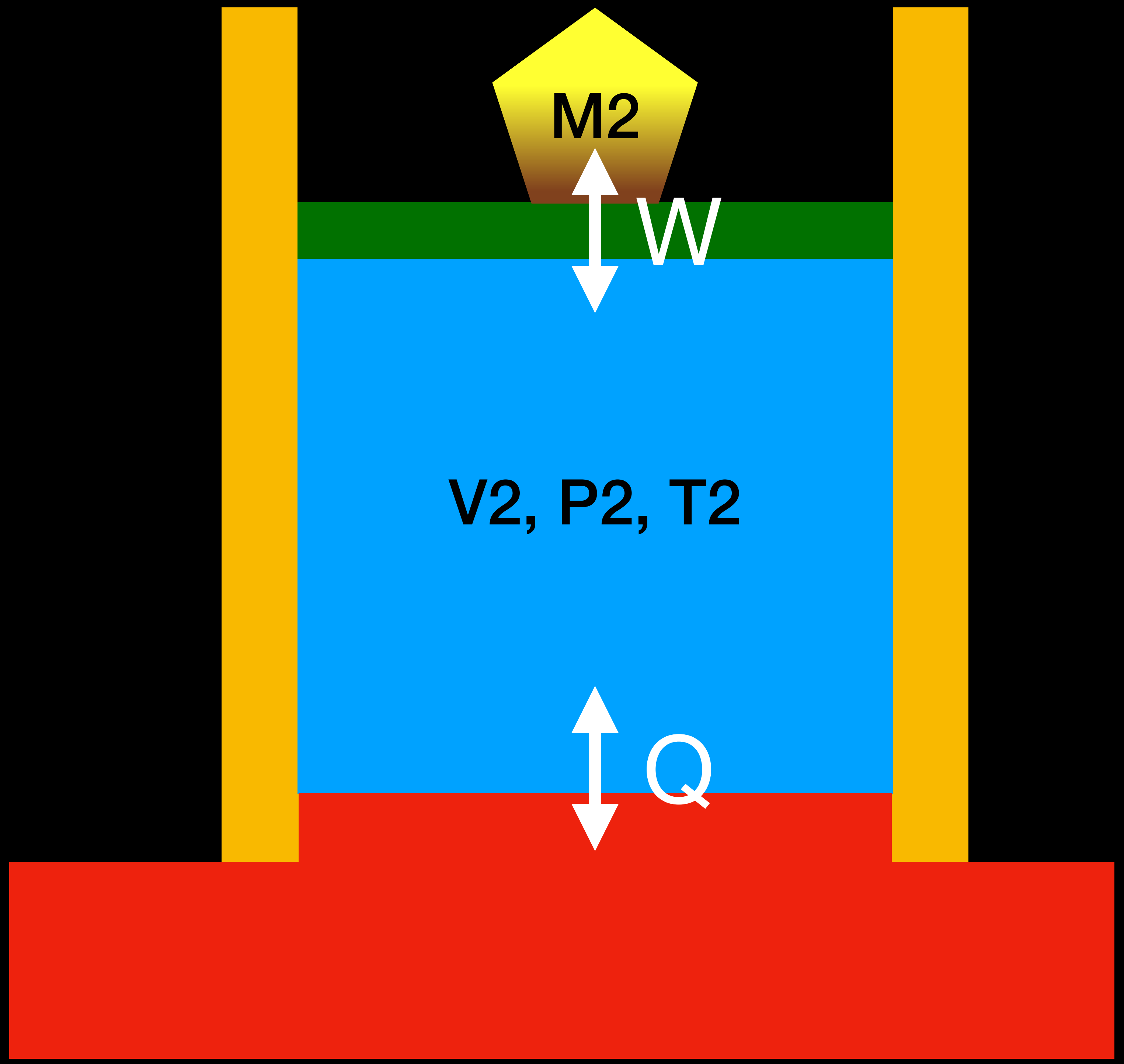
The work done by system

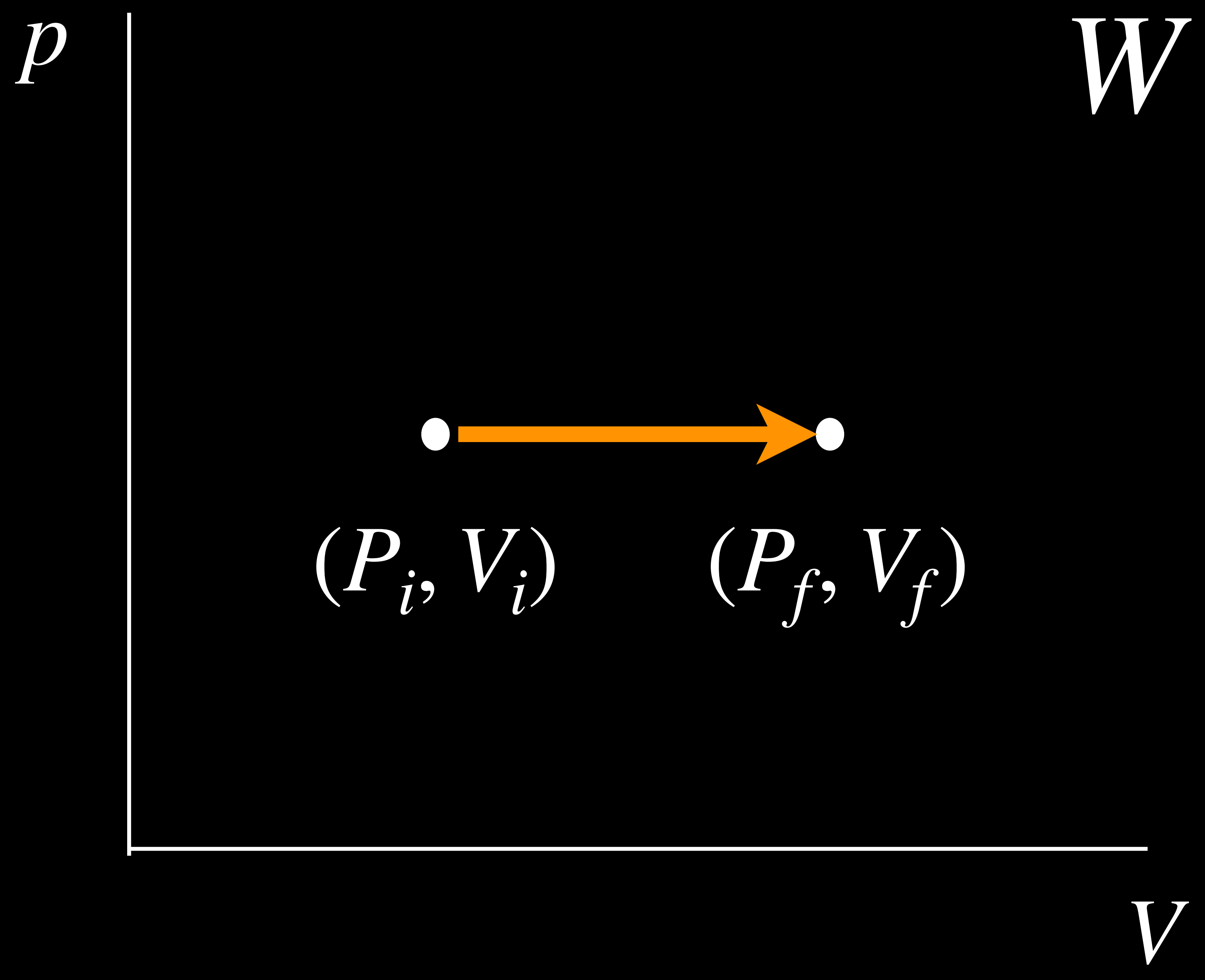
$$dE_{int} = dQ - dW$$

The heat flow into system

Lecture VII : Start from a Gas System

$$dE_{int} = dQ - dW$$





$$W = \int_{V_i}^{V_f} p dV$$

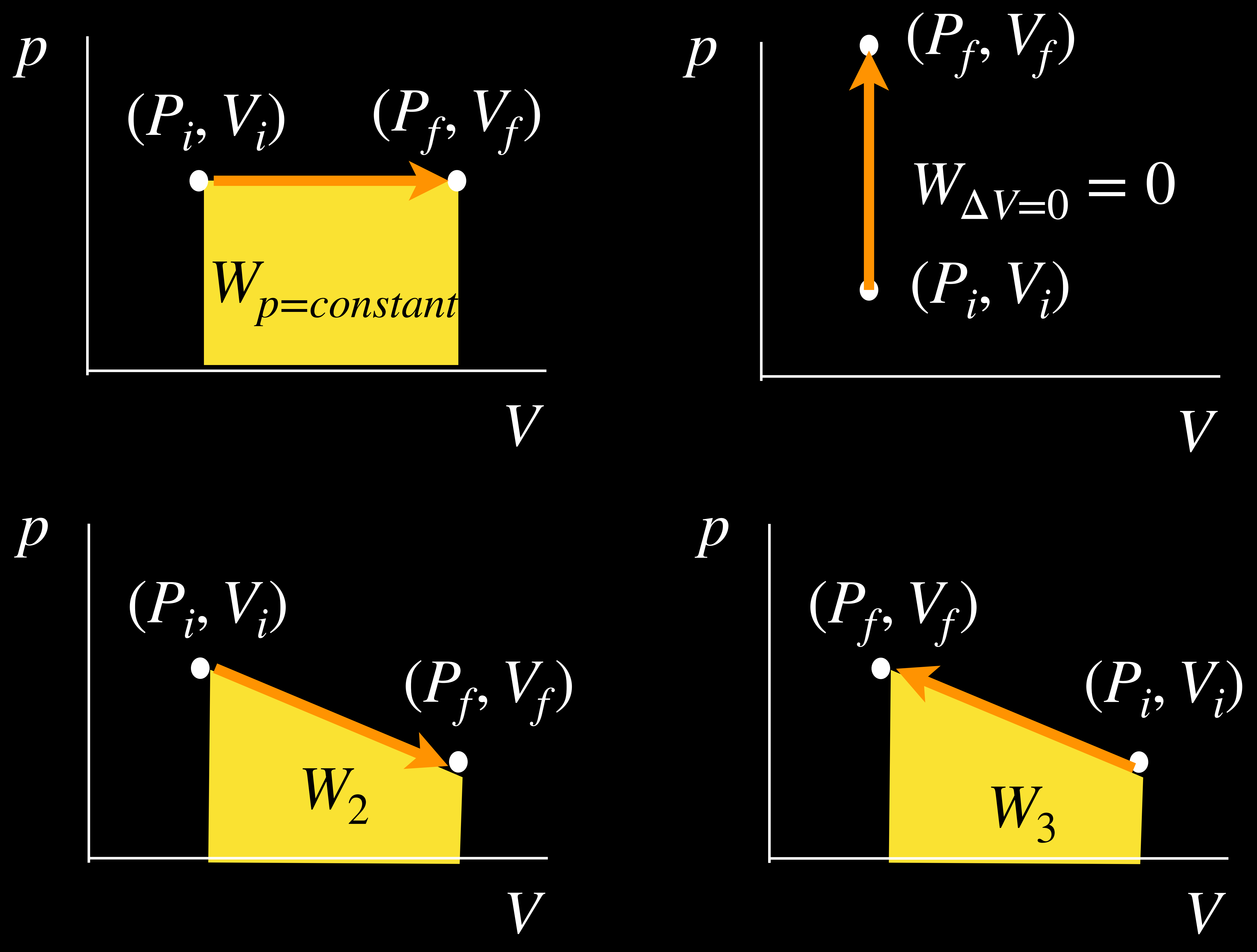
Lecture VII : Start from a Gas System

$$W = \int_{V_i}^{V_f} p dV$$

$$W_{p=constant} > W_2 > 0$$

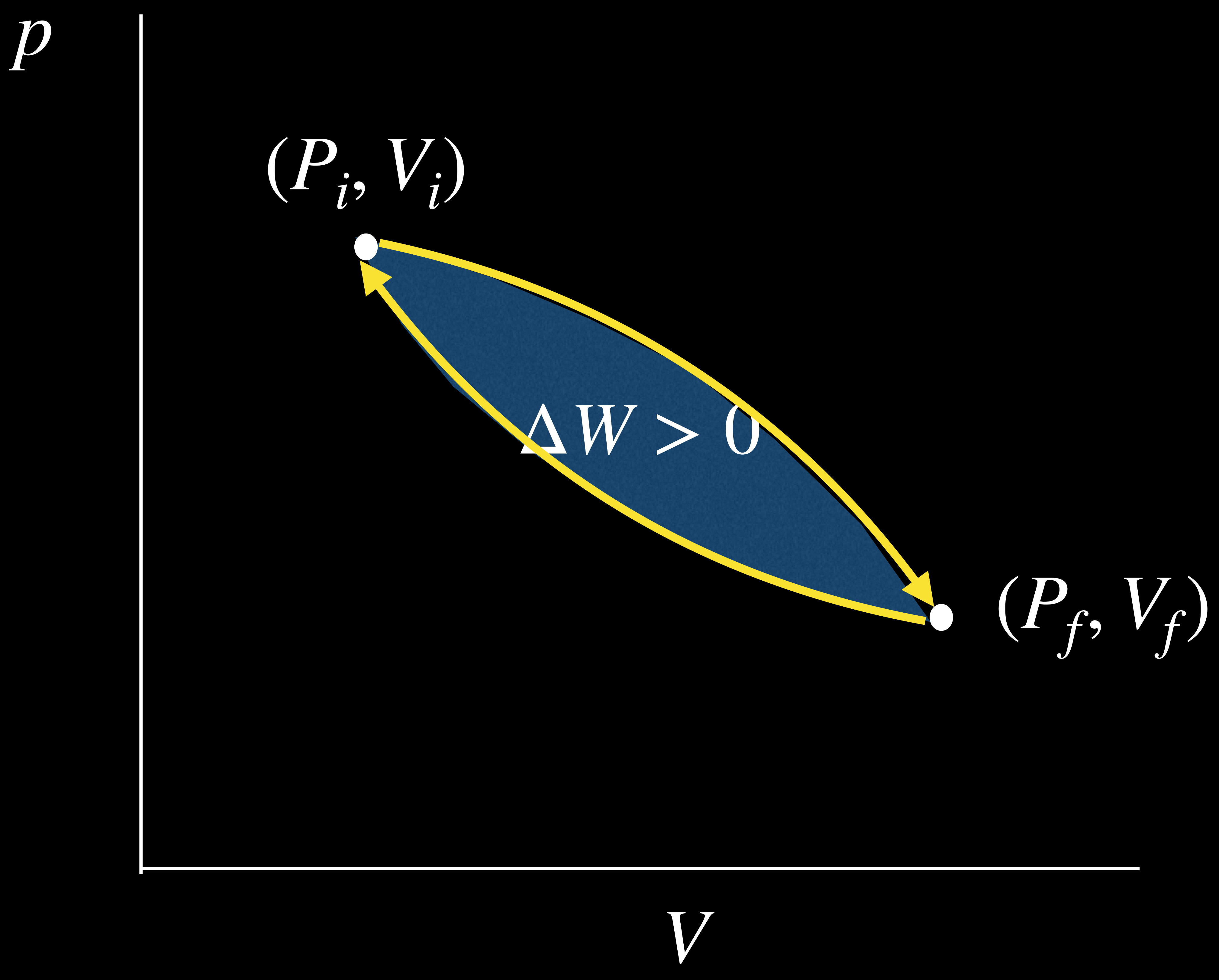
$$W_{\Delta V=0} = 0$$

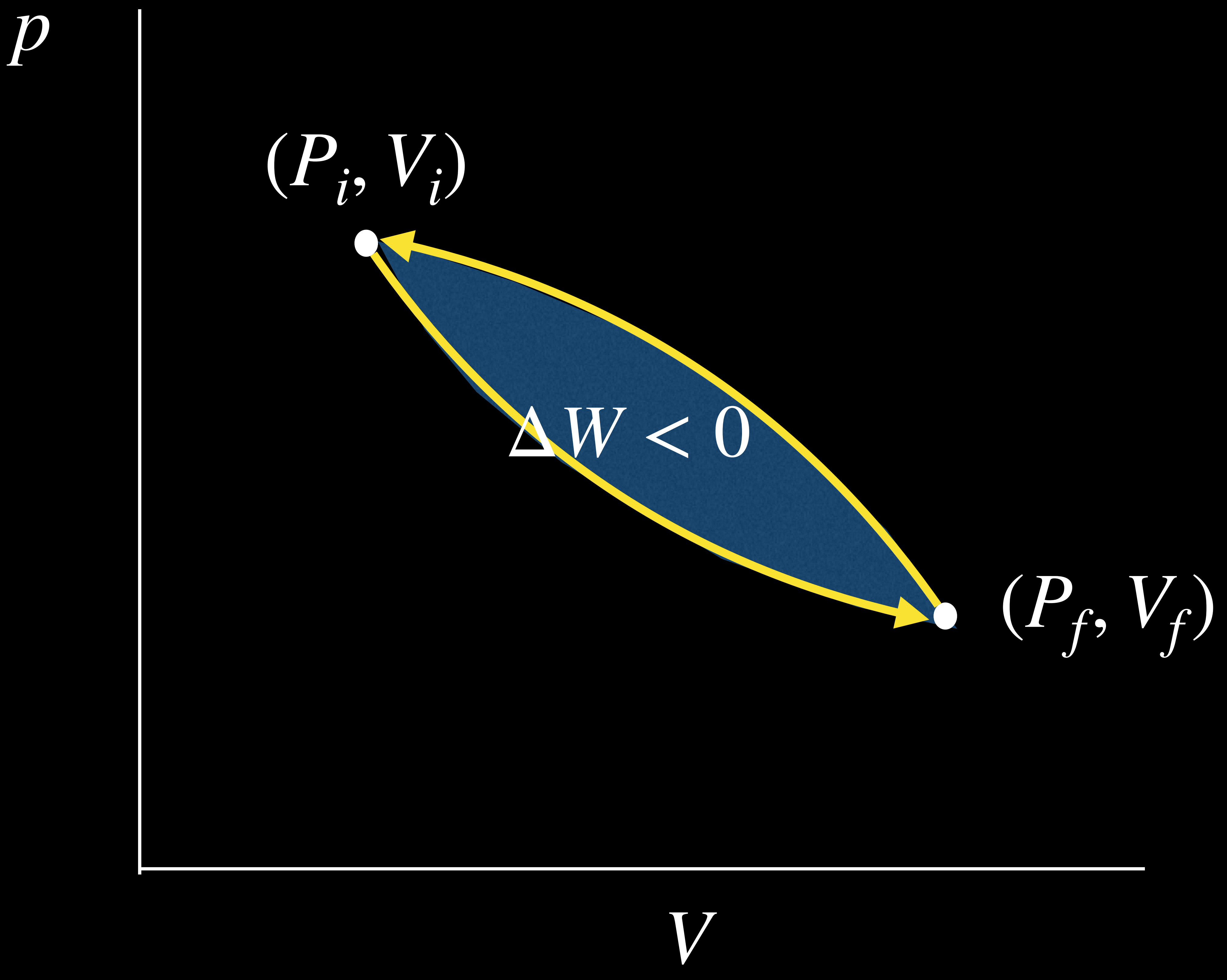
$$W_3 < 0$$



Lecture VII : Cycle

$$W = \int_{V_i}^{V_f} p dV$$





Lecture VII : The First Law of Thermodynamics

Special process: Adiabatic processes dQ=0 絕熱

$$\Delta E_{int} = Q - W$$

The work done by system

The heat flow into system =0

change of the total energy

$$\Delta E_{int} = -W$$

Lecture VII : The First Law of Thermodynamics

Special process: constant-volume processes dV=0 定容

$$\Delta E_{int} = Q - W$$

The work done by system p*dV =p*0

The heat flow into system

change of the total energy

$$\Delta E_{int} = Q$$

Lecture VII : The First Law of Thermodynamics

Cyclical process: dE=0

$$\Delta E_{int} = Q - W$$

The work done by system

The heat flow into system

change of the total energy=0

$$Q = W$$

There are processes in which, after certain interchanges of heat and work, the system is restored to its initial state. In that case, no intrinsic property of the system—including its internal energy—can possibly change.

Lecture VII : The First Law of Thermodynamics

Free expansion: Q=0, W=0

$$\Delta E_{int} = Q - W$$

The work done by system

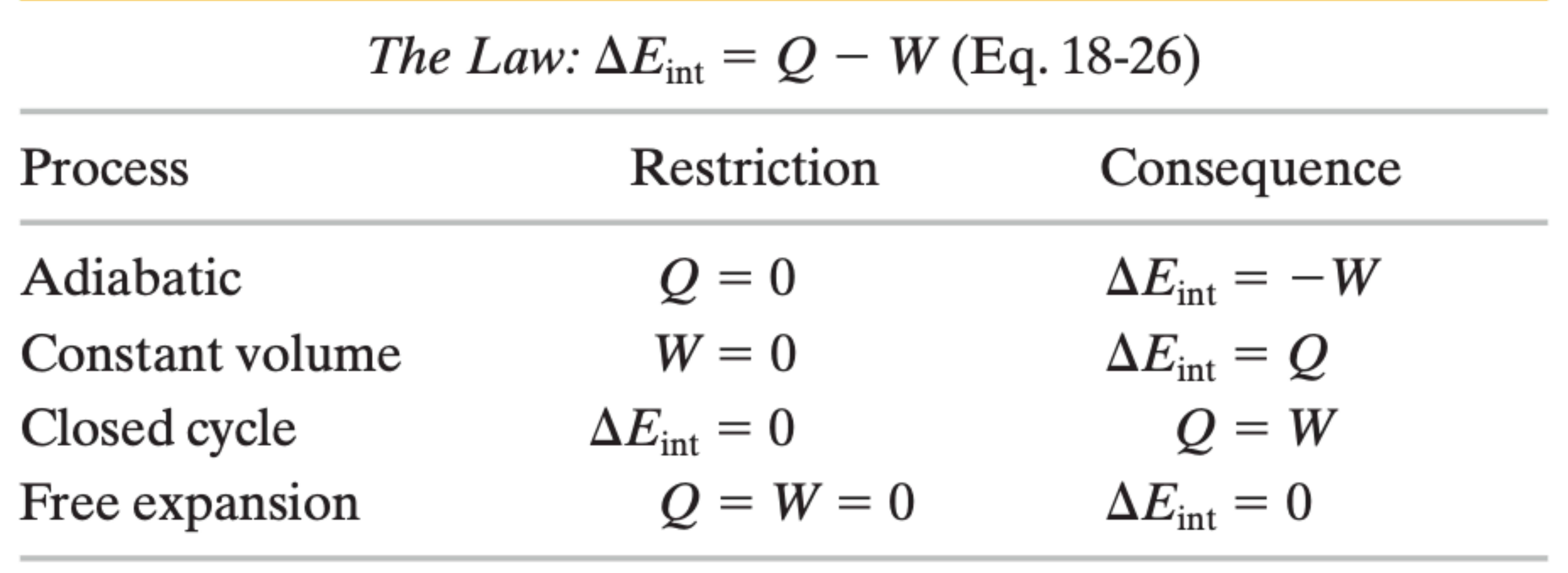
The heat flow into system =0

change of the total energy=0

$$\Delta E_{int} = 0$$

These are adiabatic processes in which no transfer of heat occurs between the system and its environment and no work is done on or by the system. Thus, Q =W=0

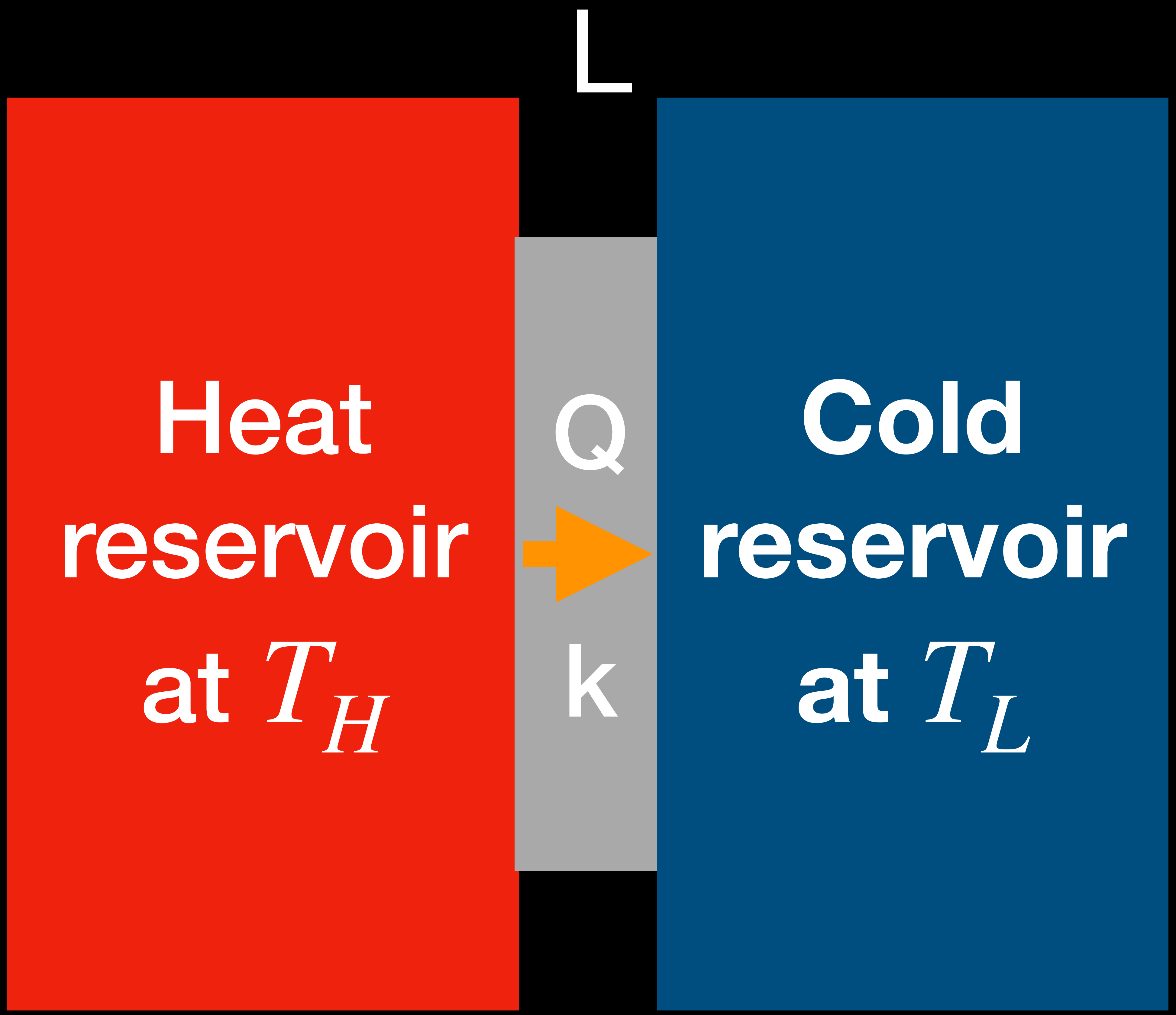
Lecture VII : Four Special Processes



The Law: $\Delta E_{\text{int}} = Q - W$ (Eq. 18-26)

Process	Restriction	Consequence
Adiabatic	$Q = 0$	$\Delta E_{\text{int}} = -W$
Constant volume	$W = 0$	$\Delta E_{\text{int}} = Q$
Closed cycle	$\Delta E_{\text{int}} = 0$	$Q = W$
Free expansion	$Q = W = 0$	$\Delta E_{\text{int}} = 0$

Lecture VII : Heat transfer



$$Q = k\frac{(T_H - T_C)A}{L} \times t$$

$$Conduction\ rate\ P_{cond} = \frac{Q}{t}$$

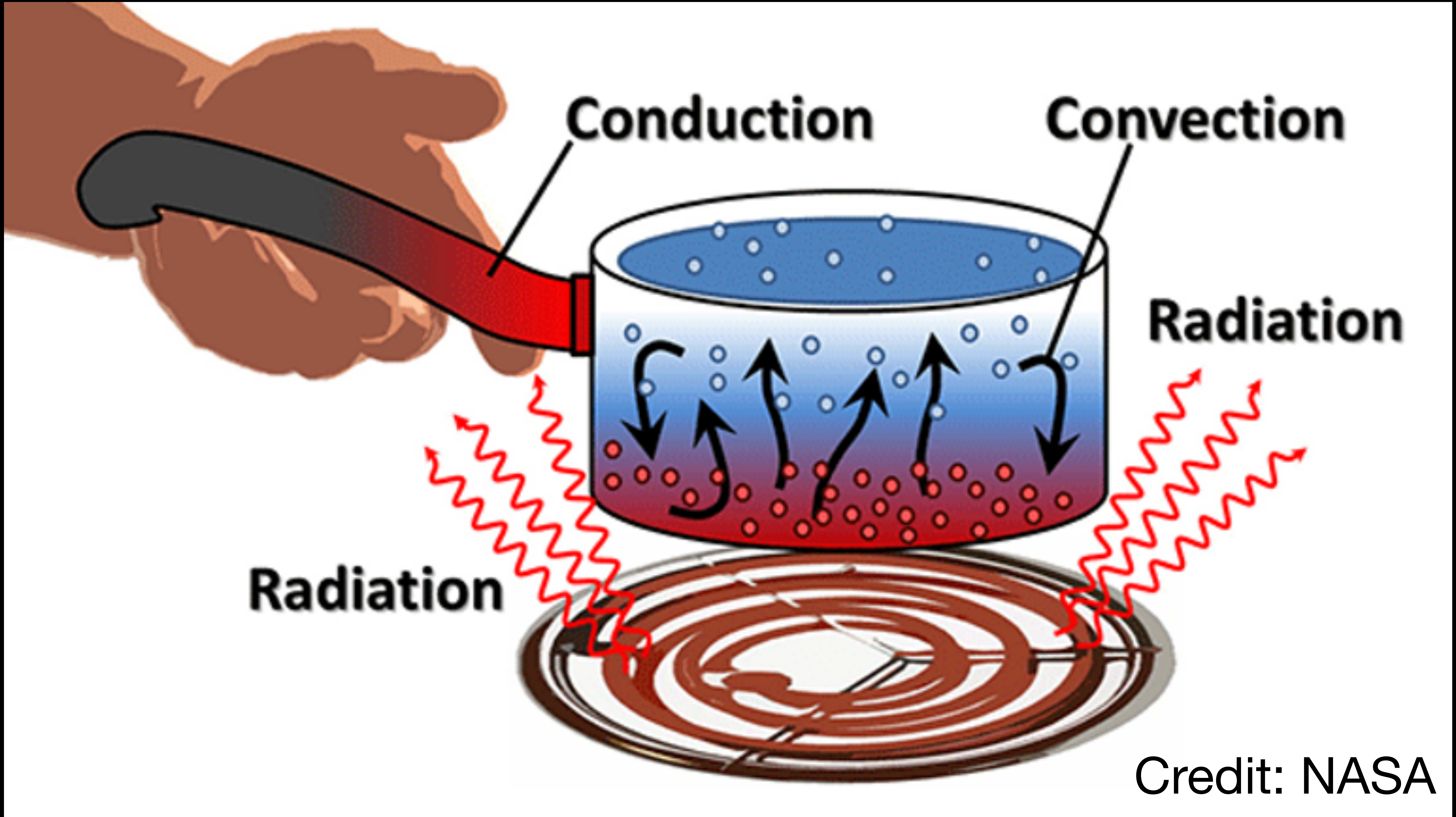
$thermal\ conductivity\ k$

Lecture VII : Heat transfer

Conduction

Convetion

Radiation



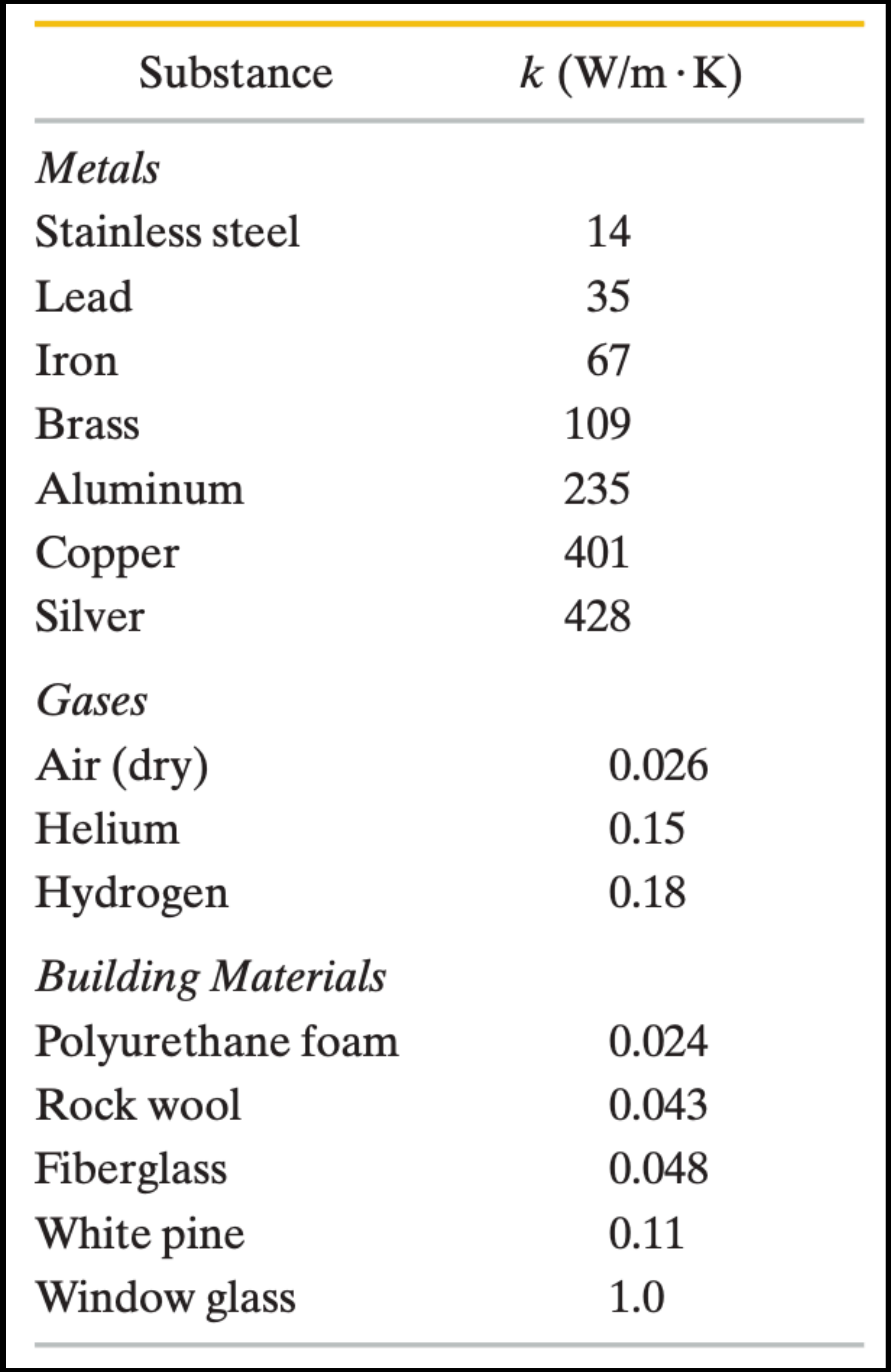
Credit: NASA

Lecture VII : Conduction

$$Q = k\frac{(T_H - T_C)A}{L} \times t$$

$$Conduction\ rate\ P_{cond} = \frac{Q}{t}$$

$$thermal\ conductivity\ k$$

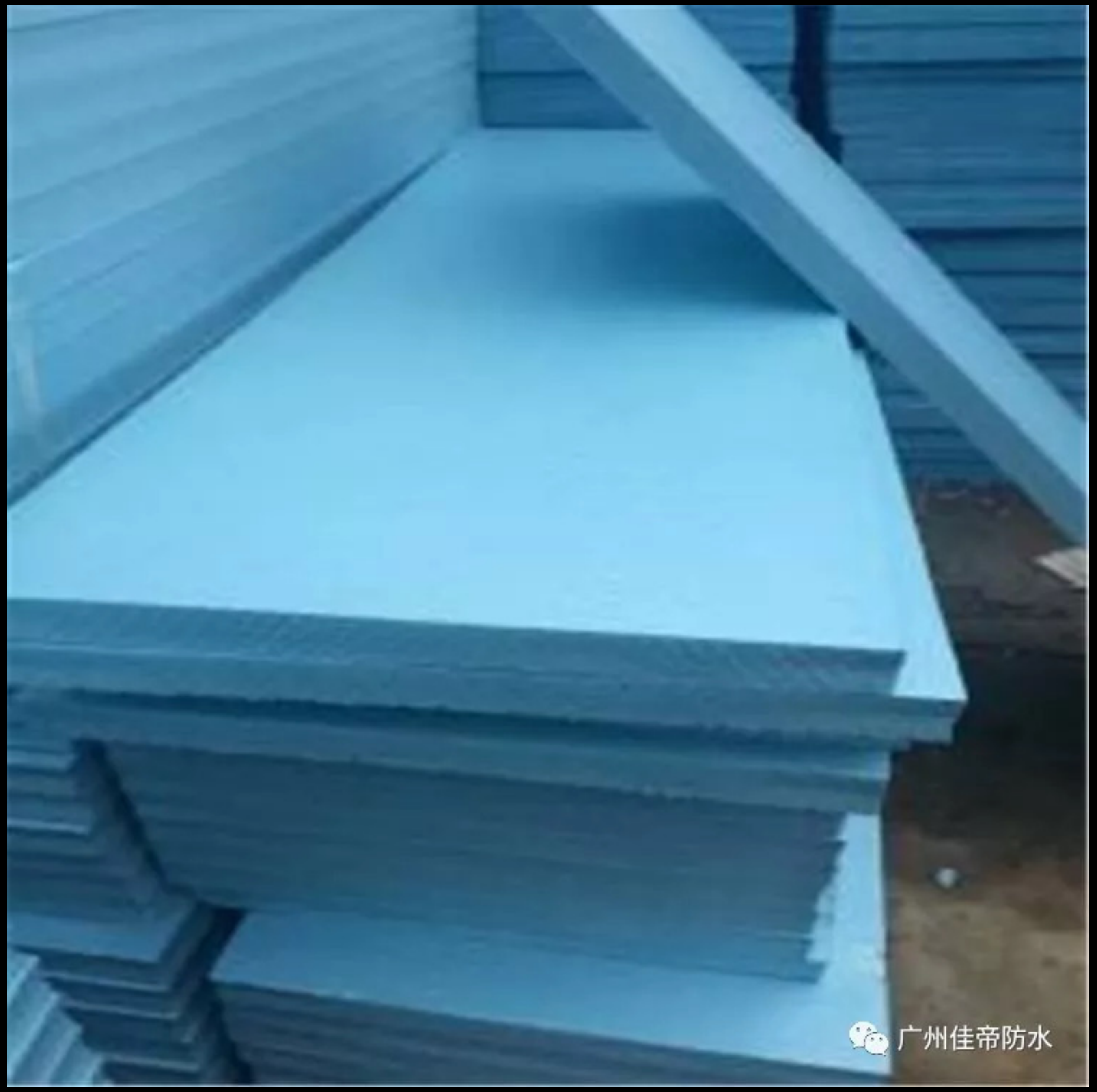


Substance	k (W/m·K)
Metals	
Stainless steel	14
Lead	35
Iron	67
Brass	109
Aluminum	235
Copper	401
Silver	428
Gases	
Air (dry)	0.026
Helium	0.15
Hydrogen	0.18
Building Materials	
Polyurethane foam	0.024
Rock wool	0.043
Fiberglass	0.048
White pine	0.11
Window glass	1.0

Lecture VII : Thermal Resistance to Conduction (R-Value)

$$Conduction\ rate\ P_{cond} = \frac{Q}{t} = \frac{k}{L} A \Delta T$$

$$R \equiv \frac{L}{k}$$



Something that has a high R-value is a poor thermal conductor and thus a good thermal insulator.

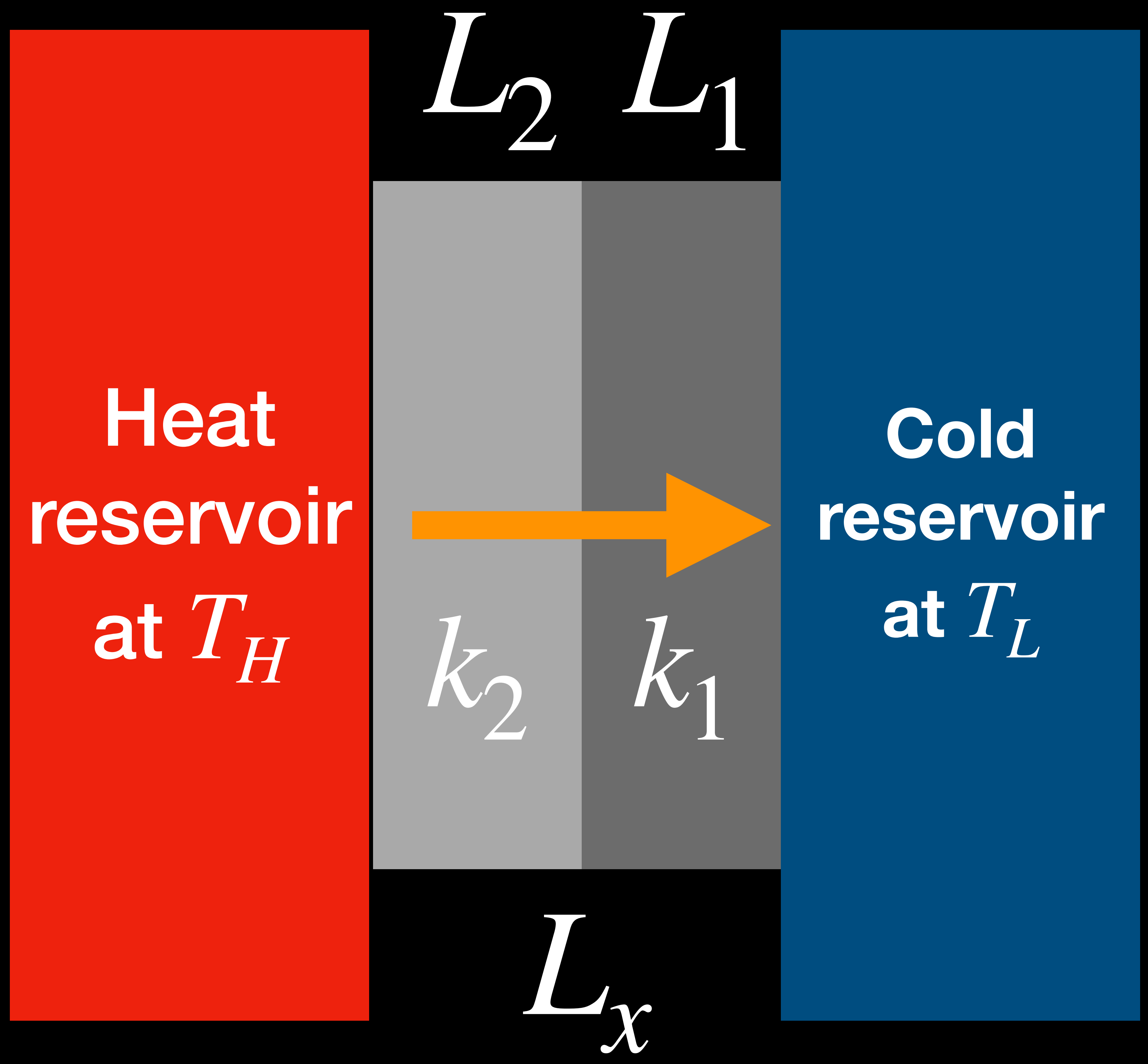
Lecture VII : Conduction Through a Composite Slab

$$P_{cond} = \frac{Q}{t} = \frac{k}{L} A \Delta T$$

$$P = \frac{k_2}{L_2} A(T_H - T_X) = \frac{k_1}{L_1} A(T_X - T_C)$$

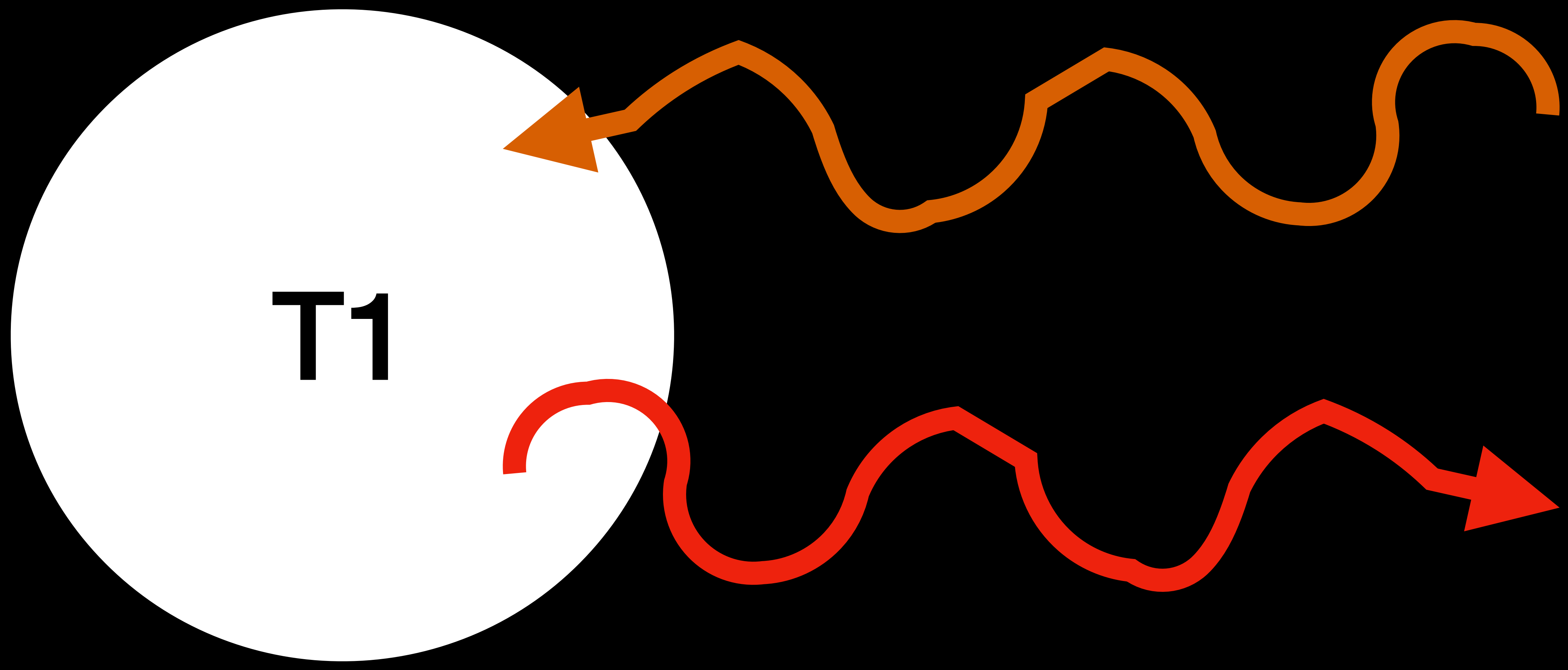
$$T_X = \frac{k_2 L_1 T_H + k_1 L_2 T_c}{(k_1 L_2 + k_2 L_1)}$$

$$P = \frac{A(T_H - T_c)}{(L_1/k_1 + L_2/k_2)}$$



Lecture VII : Radiation

$$P_{rad} = \sigma \epsilon A T^4$$



$$P_{abs} = \sigma \epsilon A T_{env}^4$$

$$P_{rad} = \sigma \epsilon A T_1^4$$

$$P_{net} = \sigma \epsilon A (T_1^4 - T_{env}^4)$$

$\sigma = 5.6704 \times 10^8 W/m^2K^4$ is called the *Stefan–Boltzmann constant*

ϵ represents the emissivity of the object's surface, which has a value between 0 and 1, depending on the composition of the surface.

Lecture VII : Kinetic theory of gases

Mole

One mole is the number of atoms in a 12 g sample of carbon-12.

The number of moles n, $n = \dfrac{N}{N_A} = \dfrac{M_{sum}}{M} = \dfrac{M_{sum}}{mN_A}$

molar mass M (the mass of 1 mol)

molecular mass m (the mass of one molecule):

Lecture VII : Ideal gas

Our goal in this chapter is to explain the macroscopic properties of a gas — such as its pressure and its temperature

$$pV = RnT$$

$$R = 8.31\ J/(mol \cdot K)$$

R gas constant

n numbers of moles of gas

$n = N/N_A$

$$pV = RnT = \frac{R}{N_A}NT = kNT$$

Lecture VII : Ideal gas

$$pV = RnT = \frac{R}{N_A}RNT = kNT$$

High T

Courtesy www.doctorslime.com

Low T

Courtesy www.doctorslime.com

Lecture VII : Kinetic theory of gases (pressure)

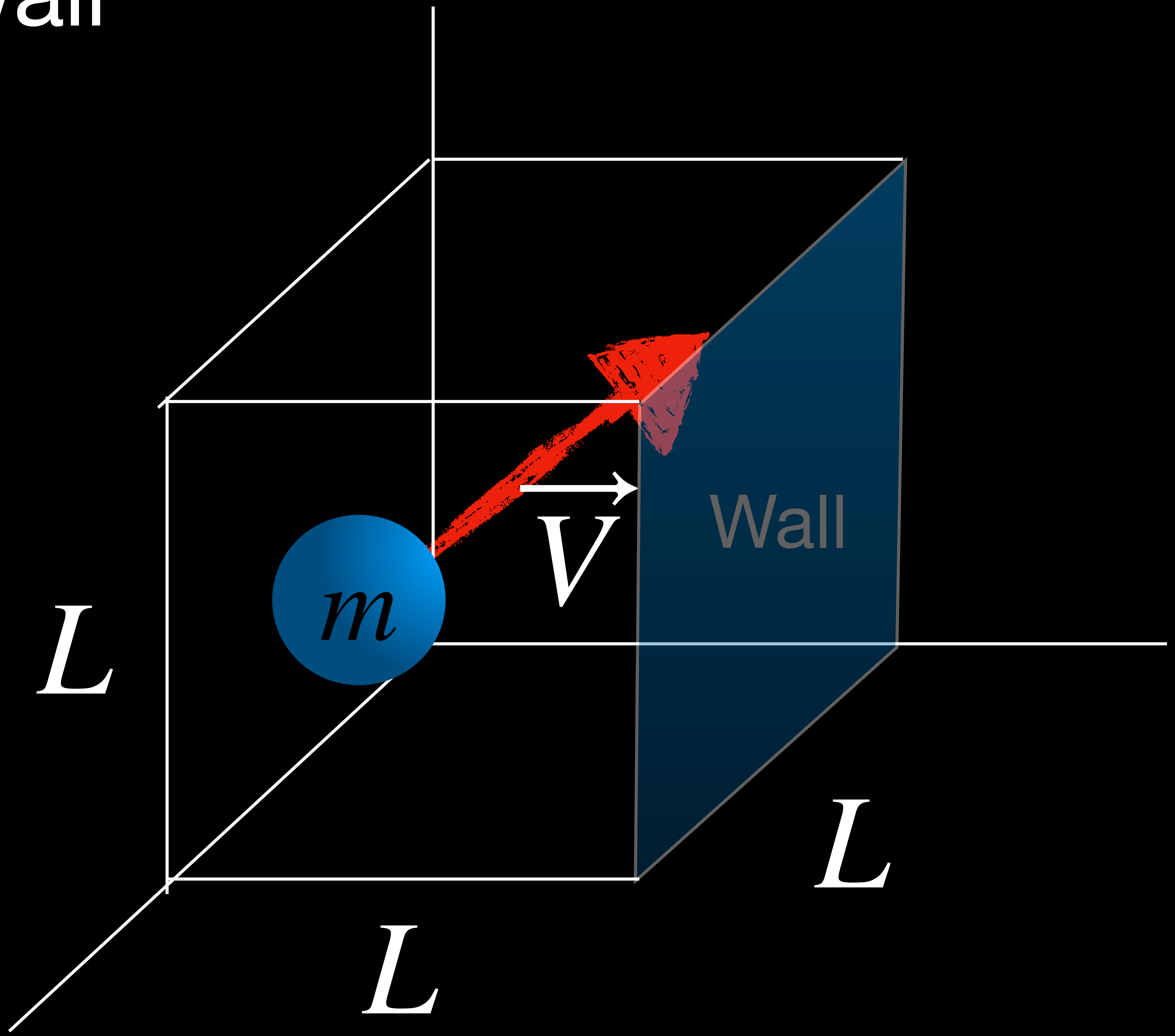
Considering the single particle hits a wall

$$\Delta P_x = mV_x - (mV_x) = 2mV_x$$

Finial Initial

$$\frac{\Delta p_x}{\Delta t} = \frac{2mV_x}{2L/V_x} = \frac{mV_x^2}{L}$$

$$Presure\ p = \frac{F}{L^2} = \frac{mV_x^2}{L^3}$$



Lecture VII : Kinetic theory of gases (pressure)

Considering N particle system $N = nN_A$

$$Presure\ p = \sum_{i=1}^{N} \frac{F_i}{L^2} = \sum_{i=1}^{N} \frac{\Delta p_{x,i}}{\Delta t L^2} = \sum \frac{mV_{x,i}}{L^3/V_x} = \sum \frac{mV_{x,i}^2}{L^3}$$

$$\sum_{i=1}^{N} V_{x,i}^2 = N < V_x^2 >$$

$$Presure\ p = \frac{nMN_A < V_x^2 >}{L^3} = \frac{nM < V_x^2 >}{L^3}$$

$$Presure\ p = \frac{nM < V^2 >}{3L^3}$$

Lecture VII : root-mean-square speed

$$\sqrt{< V_x^2 >} \equiv V_{rms}$$

$$Presure\ p = \frac{nM < V^2 >}{3V} = \frac{nMV_{rms}^2}{3V}$$

$$pV = \frac{nMV_{rms}^2}{3} = nRT$$

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

Lecture VII : Kinetic Energy

$$K_{average} = \sum_{i=1}^{N} \frac{mv_i^2}{2N} = \frac{m}{2N} \sum_{i=1}^{N} v_i^2$$

$$= \frac{m}{2} V_{rms}^2$$

$$= \frac{m}{2} (\frac{3RT}{M})$$

$$= (\frac{3kT}{2})$$

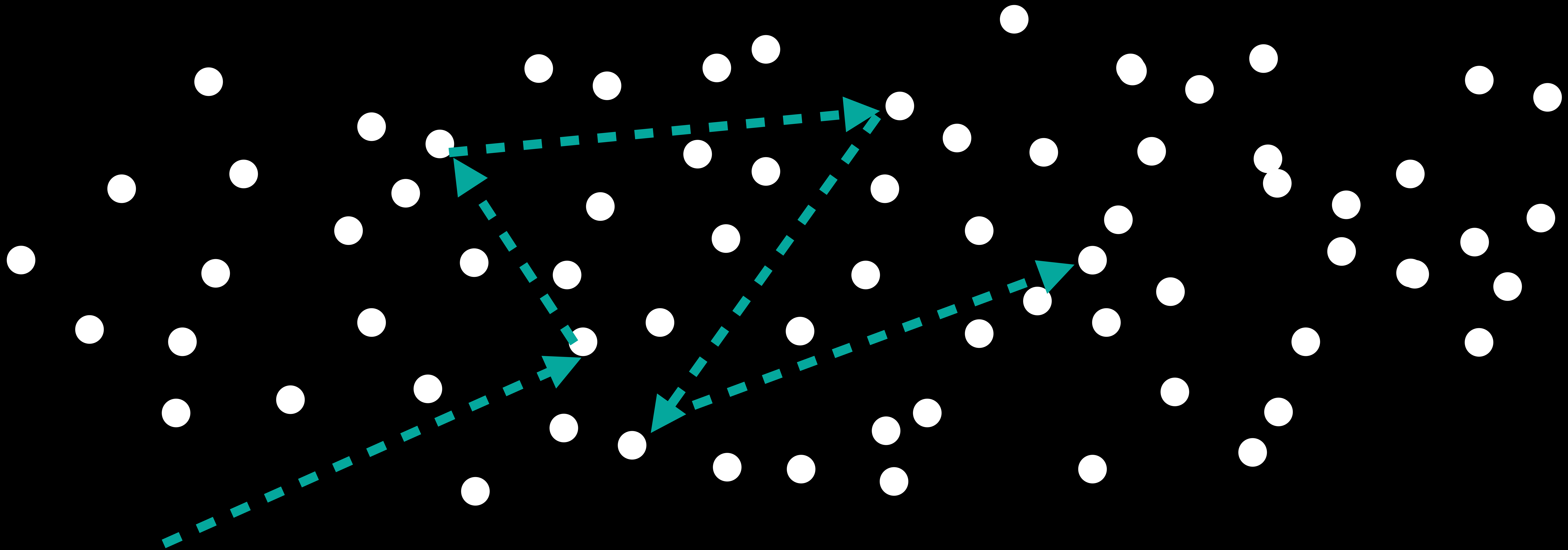
$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

$$n = \frac{N}{N_A} = \frac{M_{sum}}{M} = \frac{M_{sum}}{mN_A}$$

$$k = R/N_A$$

Lecture VII : Mean Free Path

Mean free path λ is the average distance traversed by a molecule between collisions.

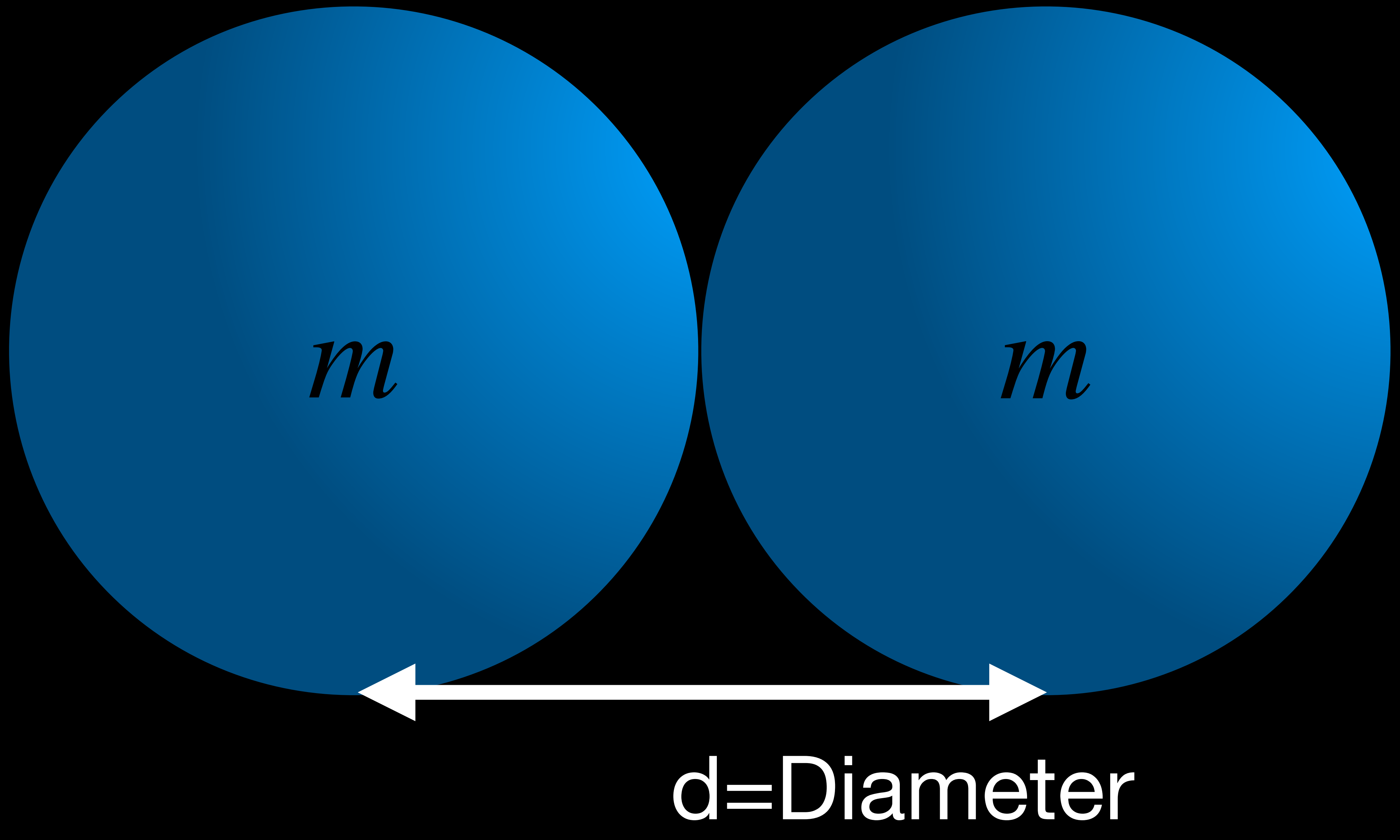


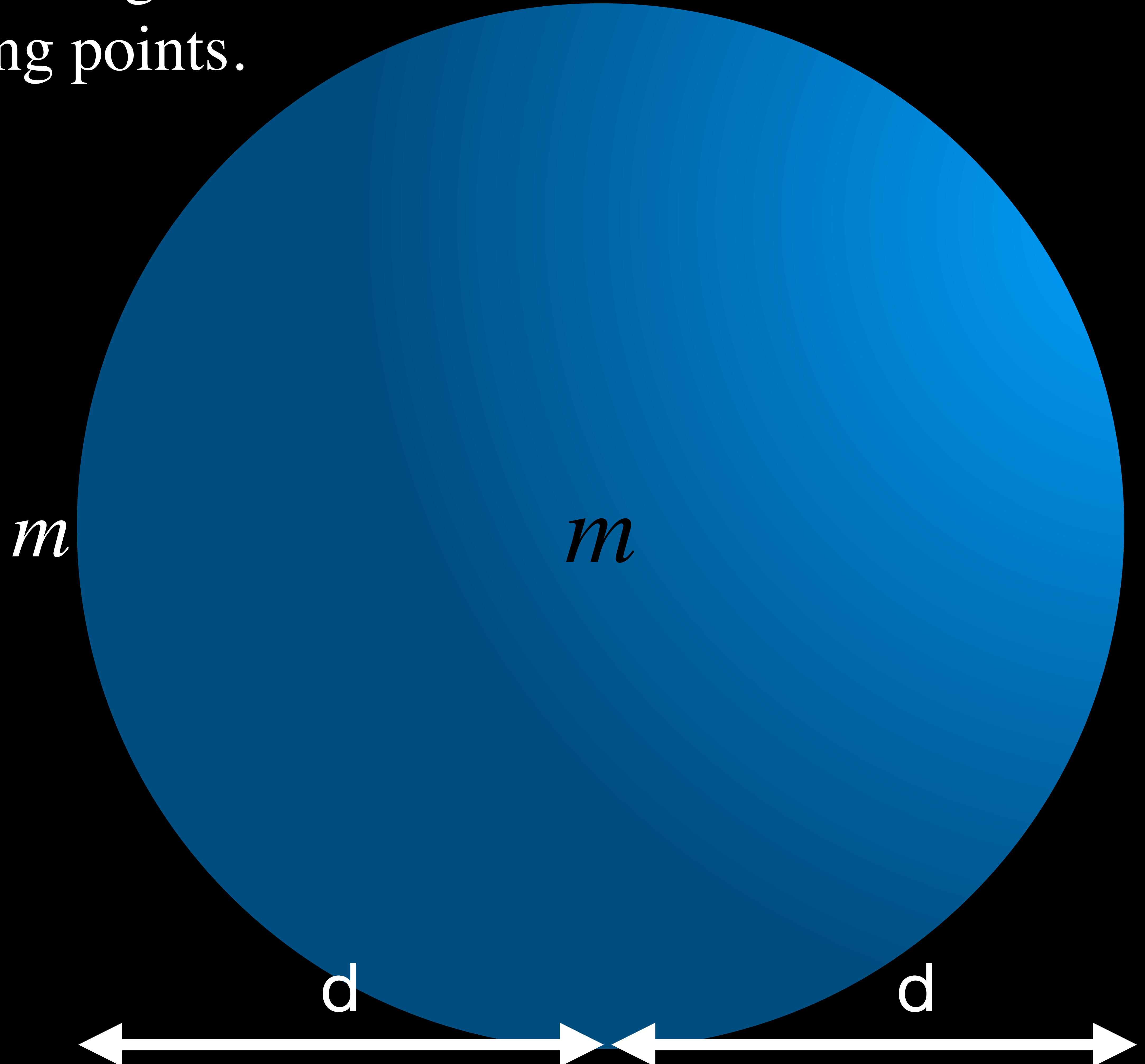
Lecture VII : Mean Free Path

Considering the molecular first.

An equivalent: to think of the moving molecule as having a *radius d* and all other molecules as being points.

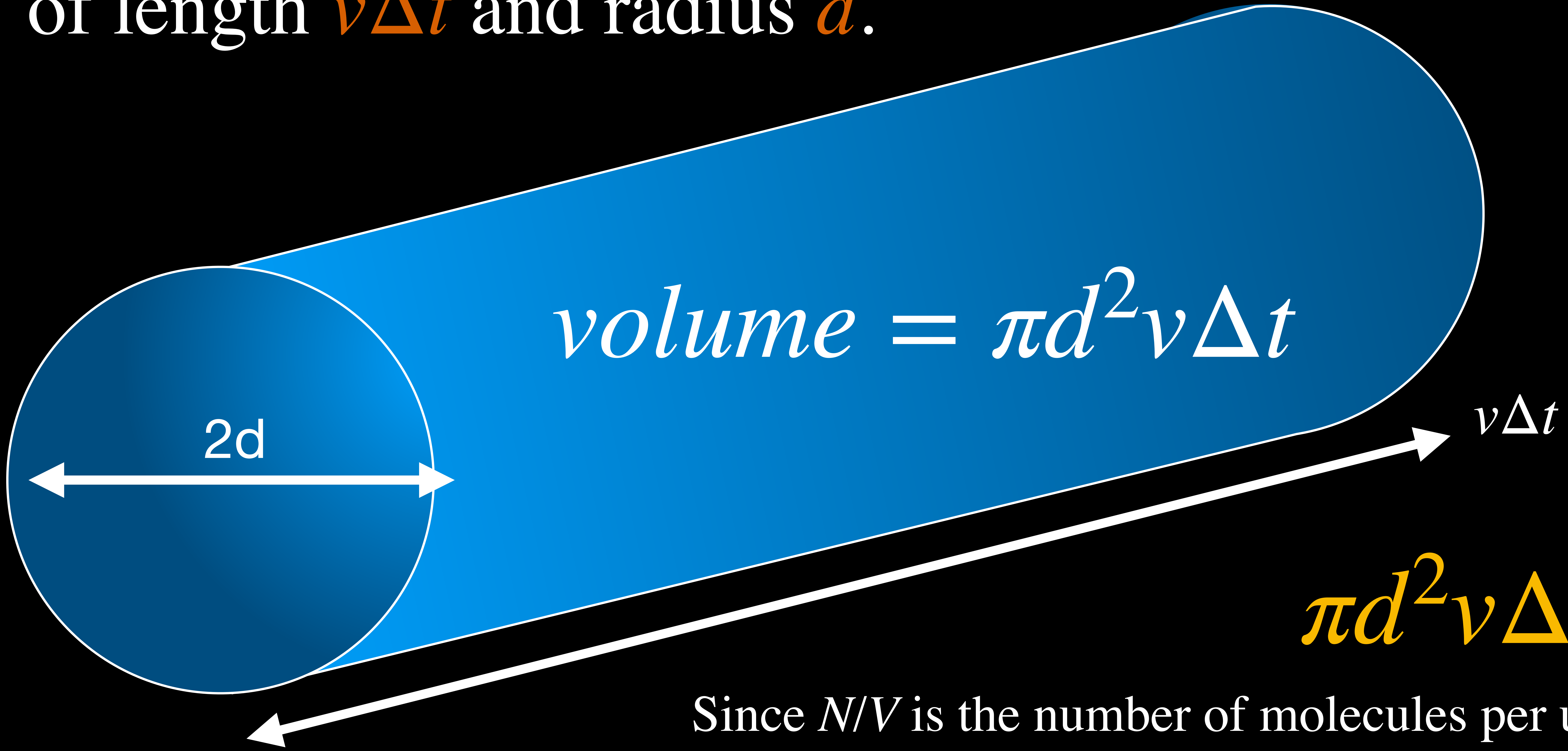
A collision occurs when the centers of two molecules come within a distance *d* of each other, *d* being the molecular diameter.





Lecture VII : Mean Free Path

In time t the moving molecule effectively sweeps out a cylinder of length $v\Delta t$ and radius d.



$$volume = \pi d^2 v\Delta t$$

$$\pi d^2 v\Delta t N/V$$

Since N/V is the number of molecules per unit volume, the number of molecules in the cylinder is N/V times the volume of the cylinder

Lecture VII : Mean Free Path

Mean free path λ is the average distance traversed by a molecule between collisions.

$$\lambda = \frac{lrngth\ of\ path\ during\ \Delta t}{number\ of\ the\ coilions\ during\ \Delta t}$$

$$\sim \frac{v\Delta t}{\pi d^2 v \Delta t N/V} = \frac{V}{\pi d^2 N}$$

All molecules are moving

Lecture VII : Maxwell’s speed distribution law

$$P(v) = 4\pi\left(\frac{M}{2\pi RT}\right)^{3/2} v^2 e^{-Mv^2/2RT}.$$

probability distribution function

$$\int_0^\infty P(\nu)d\nu \equiv 1$$

Normalize condition

$$< x > = \int_0^\infty xP(\nu)d\nu$$

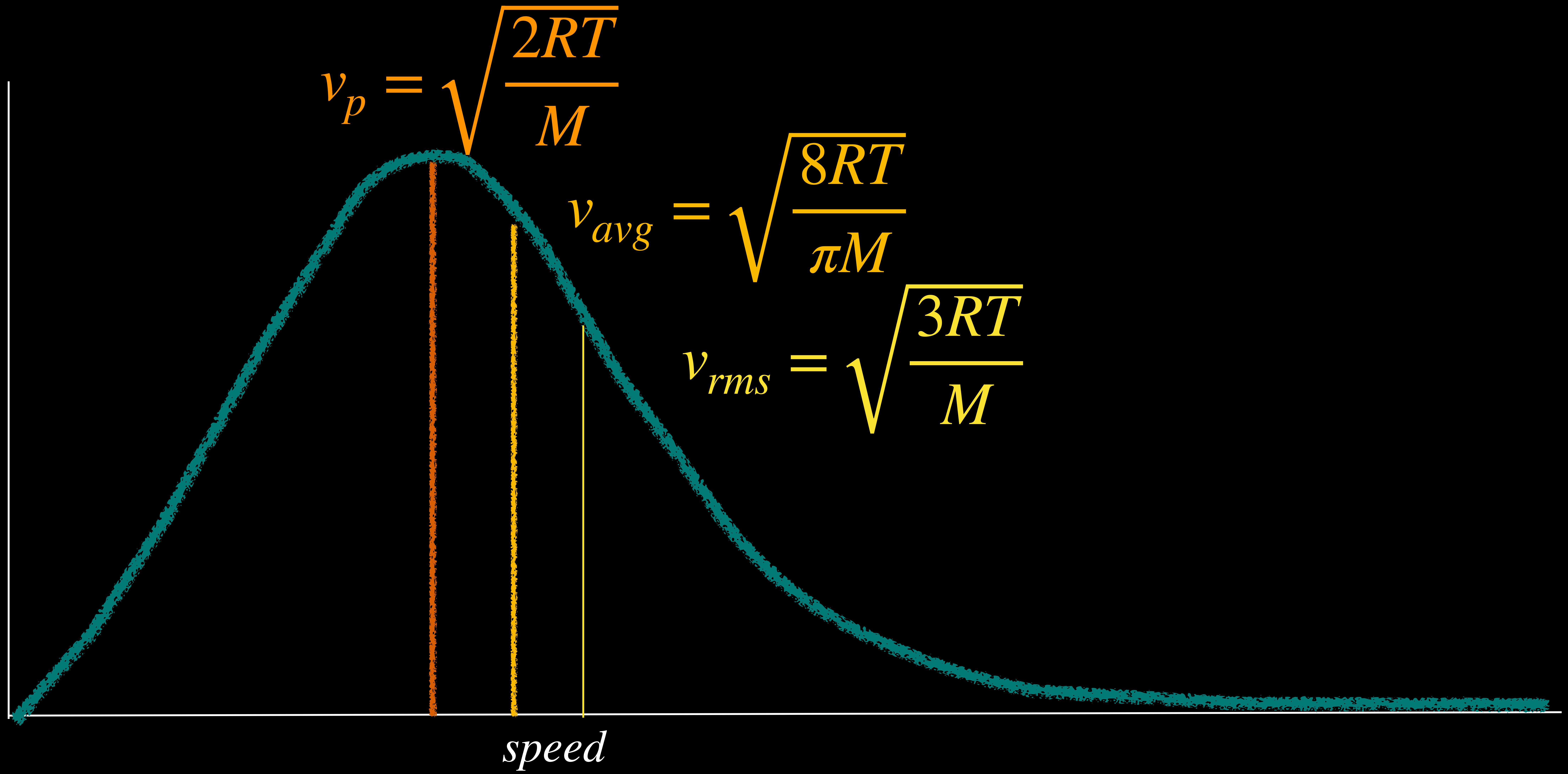
Expected value

$$< v > = v_{avg} = \int_0^\infty vP(\nu)d\nu = \sqrt{\frac{8RT}{\pi M}}$$

$$< v^2 > = \int_0^\infty v^2P(\nu)d\nu = \frac{3RT}{M} \quad v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$v_p = \sqrt{\frac{2RT}{M}}$$

Lecture VII : Maxwell’s speed distribution law

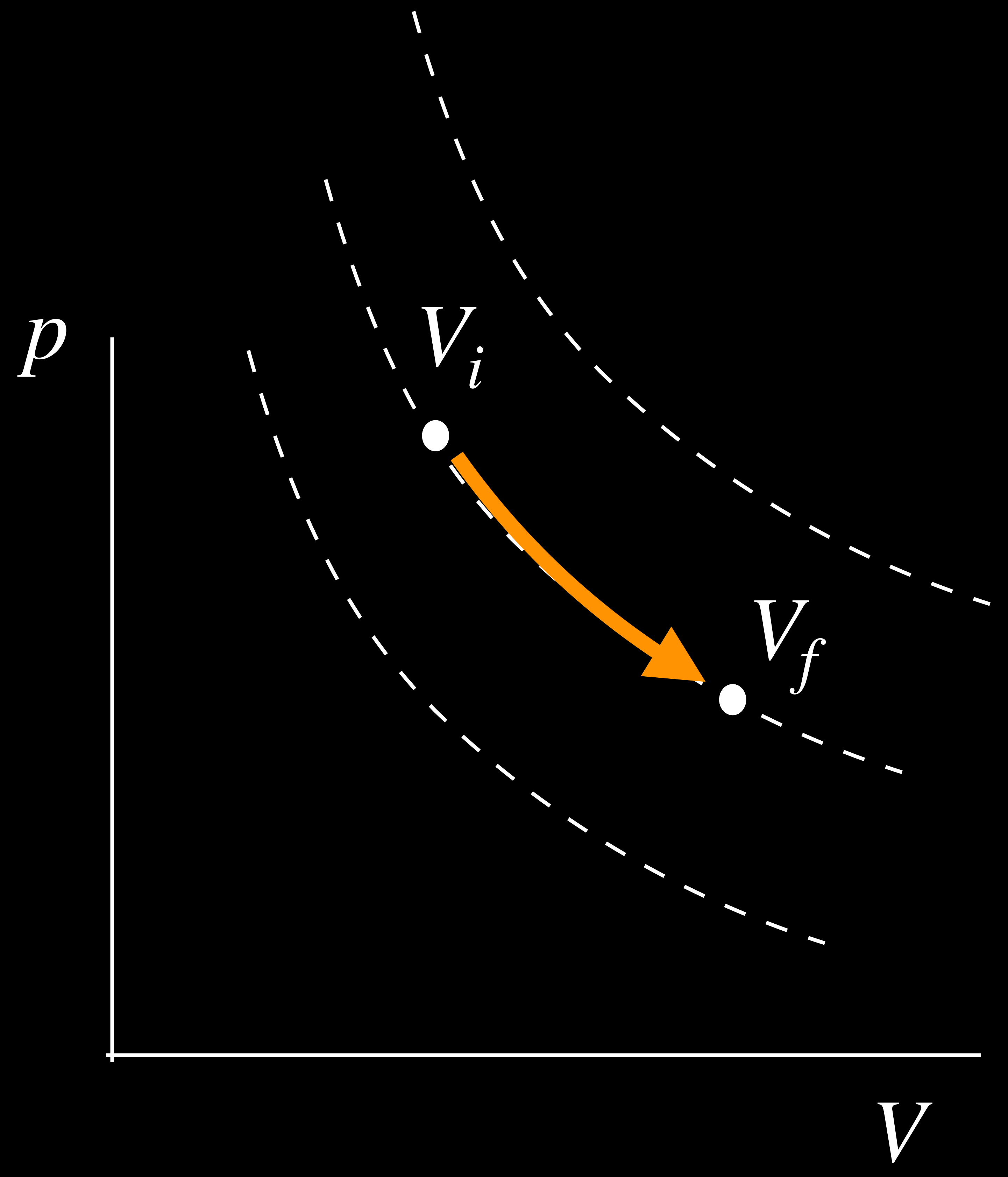


$$v_p = \sqrt{\frac{2RT}{M}}$$

$$v_{avg} = \sqrt{\frac{8RT}{\pi M}}$$

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

Lecture VII : work done by ideal gas (isothermal)



$$pV = RnT = \frac{R}{N_A}RNT = kNT$$

$$W = \int_i^f pdV = \int_i^f \frac{kNT}{V}dV$$

$$= kNT\ln(V_f - V_i) = kNT\ln(\frac{V_f}{V_i})$$

Lecture VII : work done by ideal gas

Constant V

$$W = \int_i^f p dV = 0$$

Constant P

$$W = \int_i^f p dV = p(V_f - V_i)$$

Lecture VII : Kinetic Energy

$$K_{average} = \sum_{i=1}^{N} \frac{mv_i^2}{2N} = \frac{m}{2N} \sum_{i=1}^{N} v_i^2$$

$$= \frac{m}{2} V_{rms}^2$$

$$= \frac{m}{2} (\frac{3RT}{M})$$

$$= (\frac{3kT}{2})$$

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

$$n = \frac{N}{N_A} = \frac{M_{sum}}{M} = \frac{M_{sum}}{mN_A}$$

$$k = R/N_A$$

Lecture VII : internal energy

$$K_{average} = \sum_{i=1}^{N} \frac{mv_i^2}{2N} = \frac{m}{2N} \sum_{i=1}^{N} v_i^2 = (\frac{3kT}{2})$$

The average translational kinetic energy of a single atom depends only on the gas temperature and is given by $3\ kT$. A sample of n moles of such a gas contains nNA atoms. The internal energy Eint of the sample is then

$$E_{int} = (nN_A)K_{avg} = (nN_A)(\frac{3}{2}kT)$$

$$E_{int} = \frac{3}{2}kT$$ Monoatomic ideal gas

Lecture VII : internal energy

$$E_{int} = (nN_A)K_{avg} = (nN_A)(\frac{3}{2}kT)$$

$$E_{int} = \frac{3}{2}kT$$ Monoatomic ideal gas

The internal energy Eint of an ideal gas is a function of the gas temperature only; it does not depend on any other variable

Lecture VII : Molar Specific Heat at Constant Volume

CV is a constant called the molar specific heat at constant volume.

$$Q = nC_v\Delta T$$

Constant V

$$W = \int_i^f p dV = 0$$

$$\Delta E_{int} = nC_v\Delta T - W$$

$$\Delta E_{int} = nC_v\Delta T$$

$$C_v = \frac{\Delta E_{int}}{n\Delta T}$$

Lecture VII : Molar Specific Heat at Constant Volume

CV is a constant called the molar specific heat at constant volume.

$$Q = nC_v\Delta T$$

Constant V

$$C_v = \frac{\Delta E_{int}}{n\Delta T} \qquad E_{int} = \frac{3}{2}kT$$

$$C_v = \frac{3}{2}R = 12.5 J/mol \cdot K$$

Monatomic gas

Lecture VII : Molar Specific Heat at Constant Volume

$$C_v = \frac{\Delta E_{int}}{n\Delta T}$$

$$E_{int} = C_v T$$ For any ideal gas

$$dE_{int} = nC_v dT$$ For any iprocess

A change in the internal energy Eint of a confined ideal gas depends on only the change in the temperature, not on what type of process produces the change.

Lecture VII : Molar Specific Heat at Constant pressure

Start from $Q = nC_v\Delta T$ Constant volume

$$dE_{int} = Q - W$$

$$Q = E_{int} + W$$

$$dQ = C_V dT + pdV$$

Lecture VII : Molar Specific Heat at Constant pressure

$$dQ = C_V dT + pdV$$

$$pV = nRT$$

$$C_p = C_V + R$$

$$pdV + Vdp = RdT$$

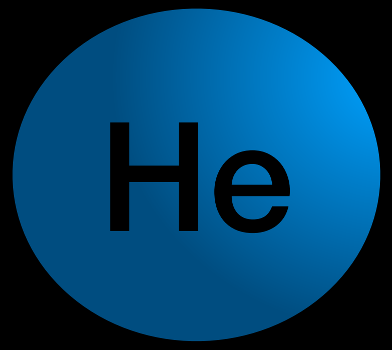
$$pdV = RdT - Vdp$$

$$dQ = C_V dT + RdT - Vdp$$

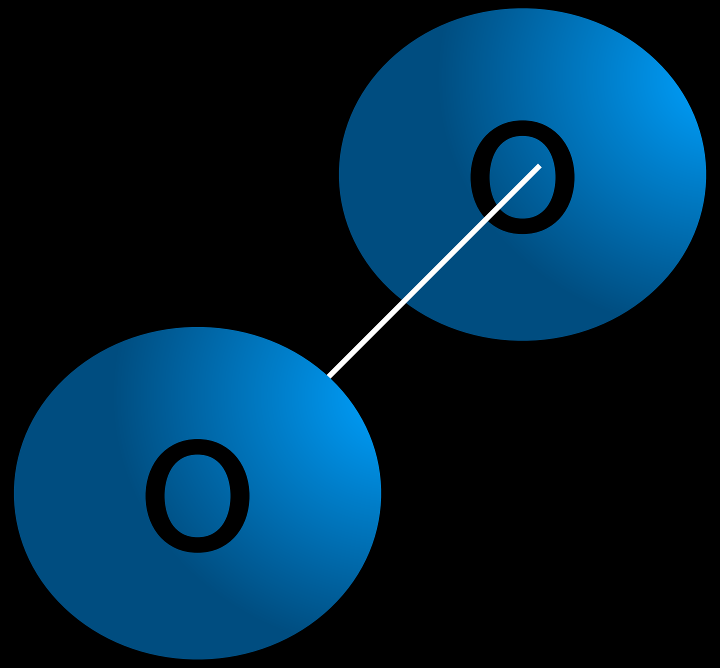
$$dQ = (C_V + R)dT - \cancel{Vdp}$$

$$dQ = (C_V + R)dT = C_p dT$$

Lecture VII : Degrees of Freedom and Molar Specific Heats



		Degrees of Freedom		
Molecule	Example	Translational	Rotational	Total (f)
Monatomic	He	3	0	3
Diatomic	O_2	3	2	5



kind of molecule has a certain number f of degrees of freedom, which are independent ways in which the molecule can store energy. Each such degree of freedom has associated with it—on average—an energy of 1/2kT per molecule (or 1/2RT per mole)

Ex $C_v = \frac{3}{2}R = \frac{f}{2}R$

Lecture VII : The Adiabatic Expansion of an Ideal Gas $pV^{\gamma} = constant$

$$dE_{int} = Q - W$$

$$dE_{int} = -W$$ Adiabatic

$$C_v dT = -W$$

$$C_v dT = -pdV$$

$$C_v dT + pdV = 0$$

$$p_i V_i^{\gamma} = p_f V_f^{\gamma}$$ Adiabatic

Lecture VII : The Adiabatic Expansion of an Ideal Gas

$$pV^{\gamma} = constant$$

$$p_iV_i^{\gamma} = p_fV_f^{\gamma}$$ Adiabatic

$$pV^{\gamma} = \frac{RT}{V}V^{\gamma},\ TV^{\gamma-1} = constant$$

$$T_iV_i^{\gamma-1} = T_fV_f^{\gamma-1}$$ Adiabatic

Lecture VII : free expansion

$$dE_{int} = dQ - dW = 0$$

$$T_i = T_f$$

$$p_i V_i = p_f V_f$$

Lecture VII : The Molar Specific Heats of an Ideal Gas

Constant V

$$W = \int_i^f p dV = 0$$

Constant P

$$W = \int_i^f p dV = p(V_f - V_i)$$

Lecture VII : Equilibrium

The equilibrium state is defined as the one macroscopic state of a closed system which is automatically attained after a sufficiently long period of time such that the macroscopic state quantities no longer change with time.

All systems which are in thermal equilibrium with a given system are also in thermal equilibrium with each other

-The zeroth law of thermodynamics

Extensive(additive) state quantities:

These quantities are proportional to the amount of matter in a system

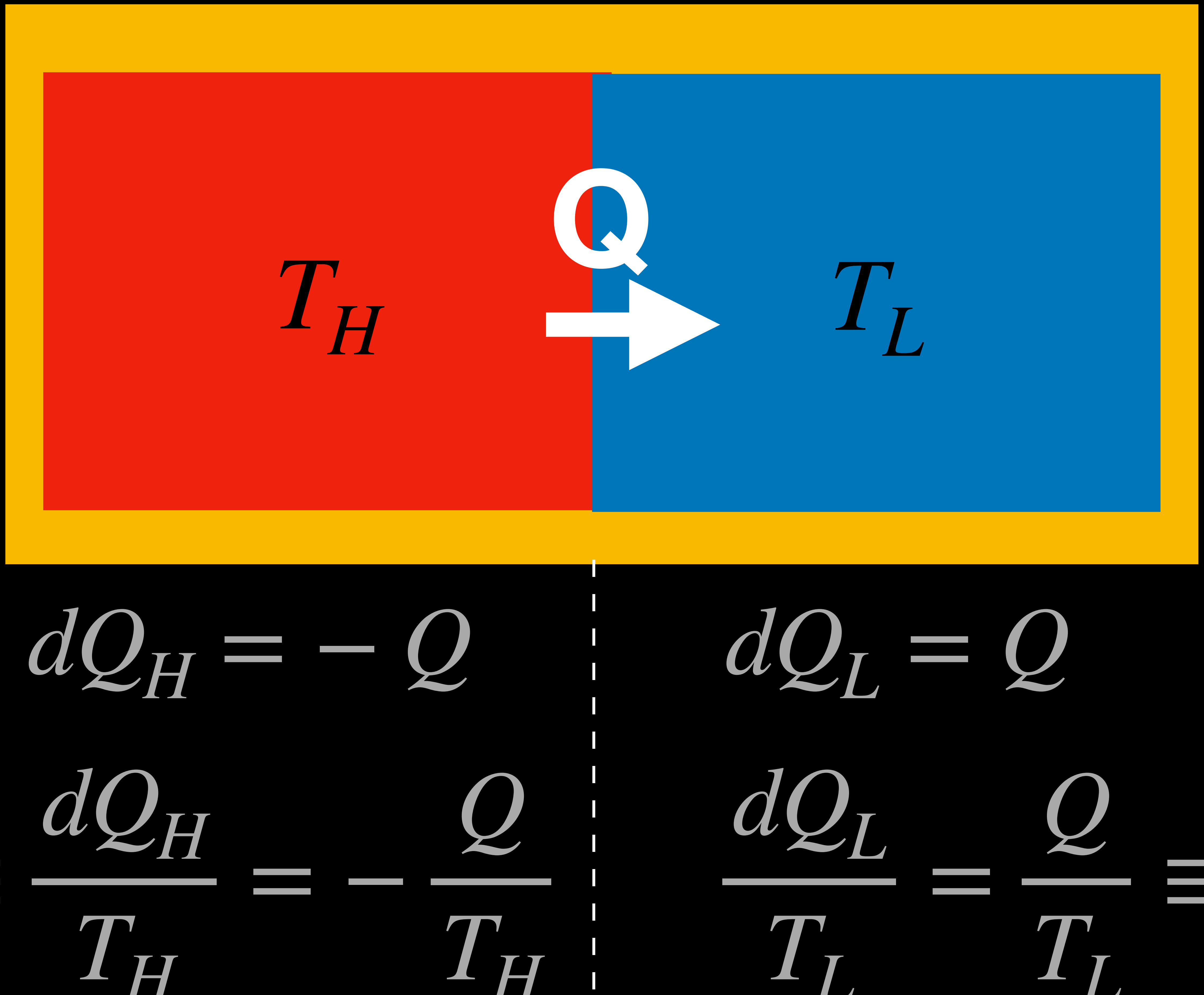
$$\mathrm{V}, \mathrm{E}, \mathrm{M}, \mathrm{S}, C_V, C_P \ldots$$

Intensive state quantities

These quantities are independent of the amount of matter in a system

$$\mathrm{T}, \mathrm{p}, \rho \ldots$$

Equalization of Temperature

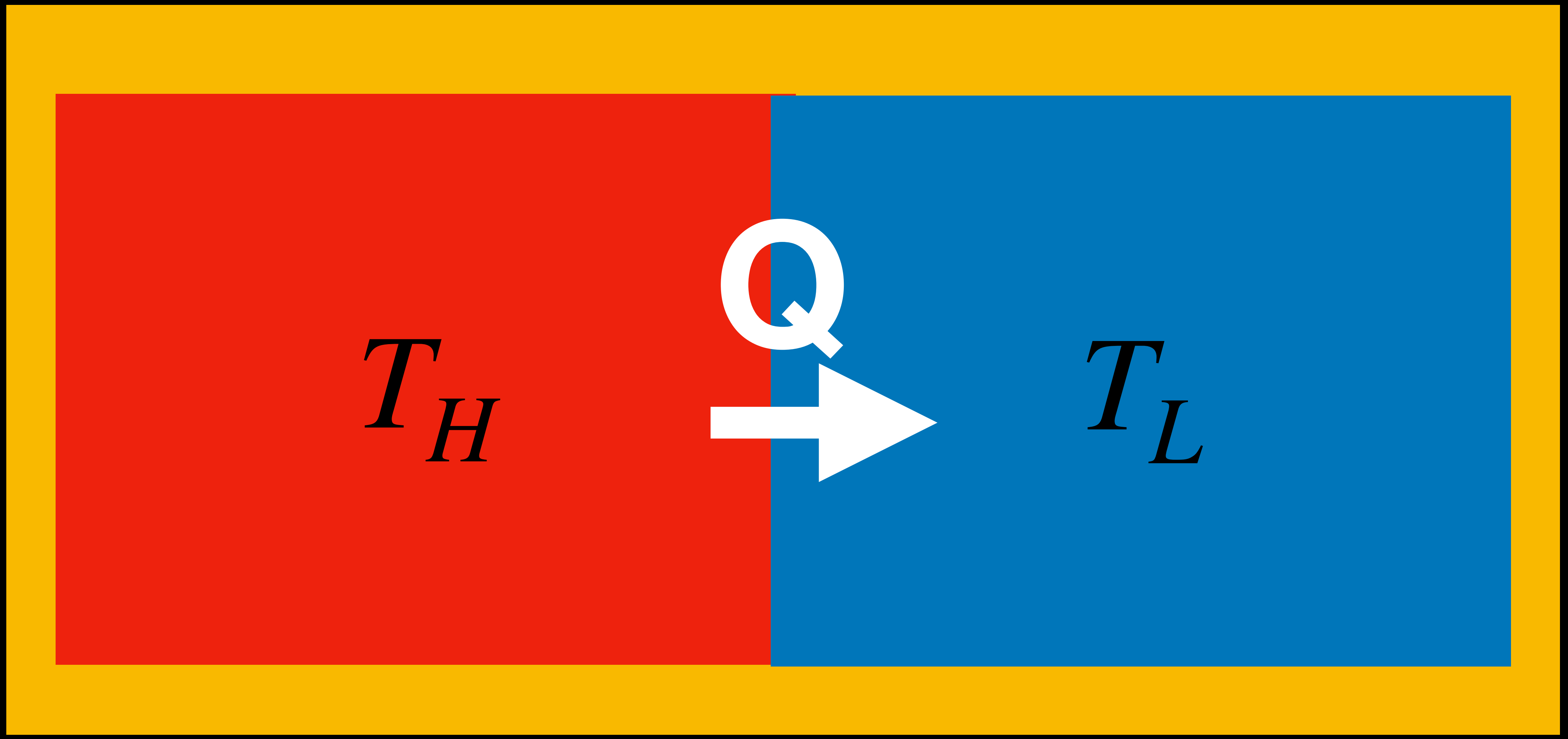


$$dQ_H = -Q \qquad dQ_L = Q$$

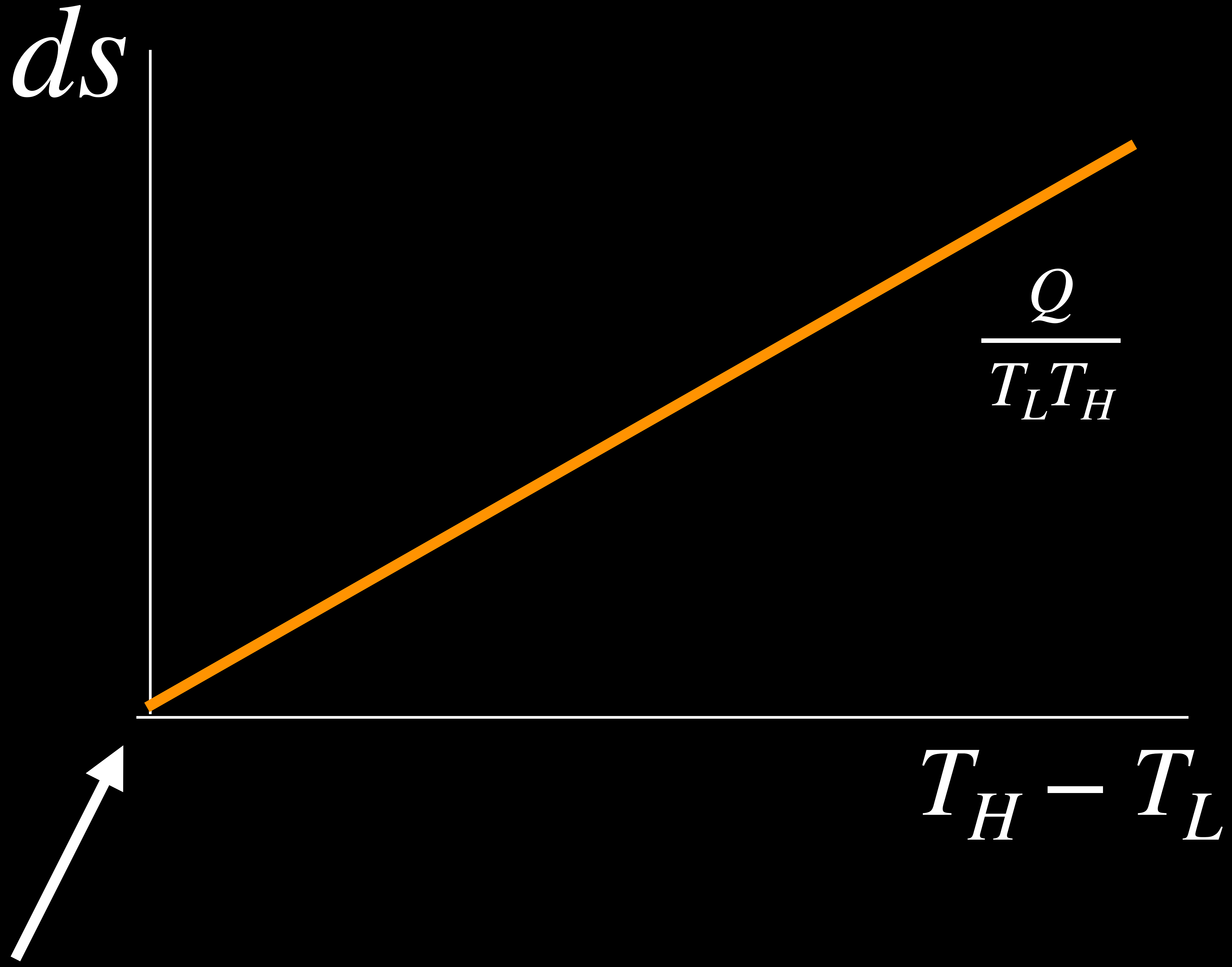
$$dS_H \equiv \frac{dQ_H}{T_H} = -\frac{Q}{T_H} \qquad \frac{dQ_L}{T_L} = \frac{Q}{T_L} \equiv dS_L$$

$$Q(\frac{1}{T_L} - \frac{1}{T_H}) \equiv dS \geq 0$$

Equalization of Temperature



$$Q\left(\frac{1}{T_L}-\frac{1}{T_H}\right) \equiv dS \geq 0$$

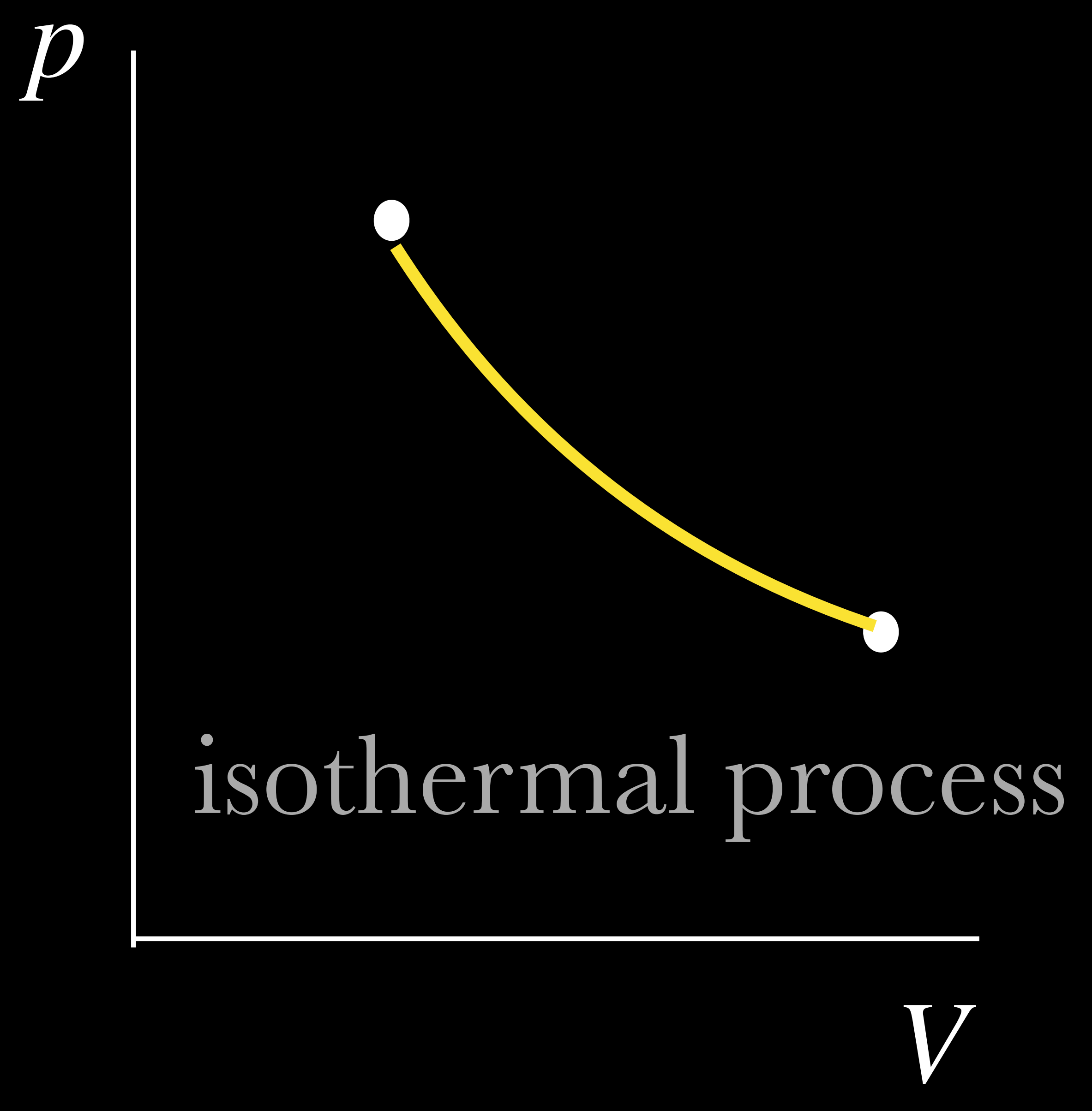


thermal equilibrium dS=0

Irreversible process

The processes do not reserve themselves.

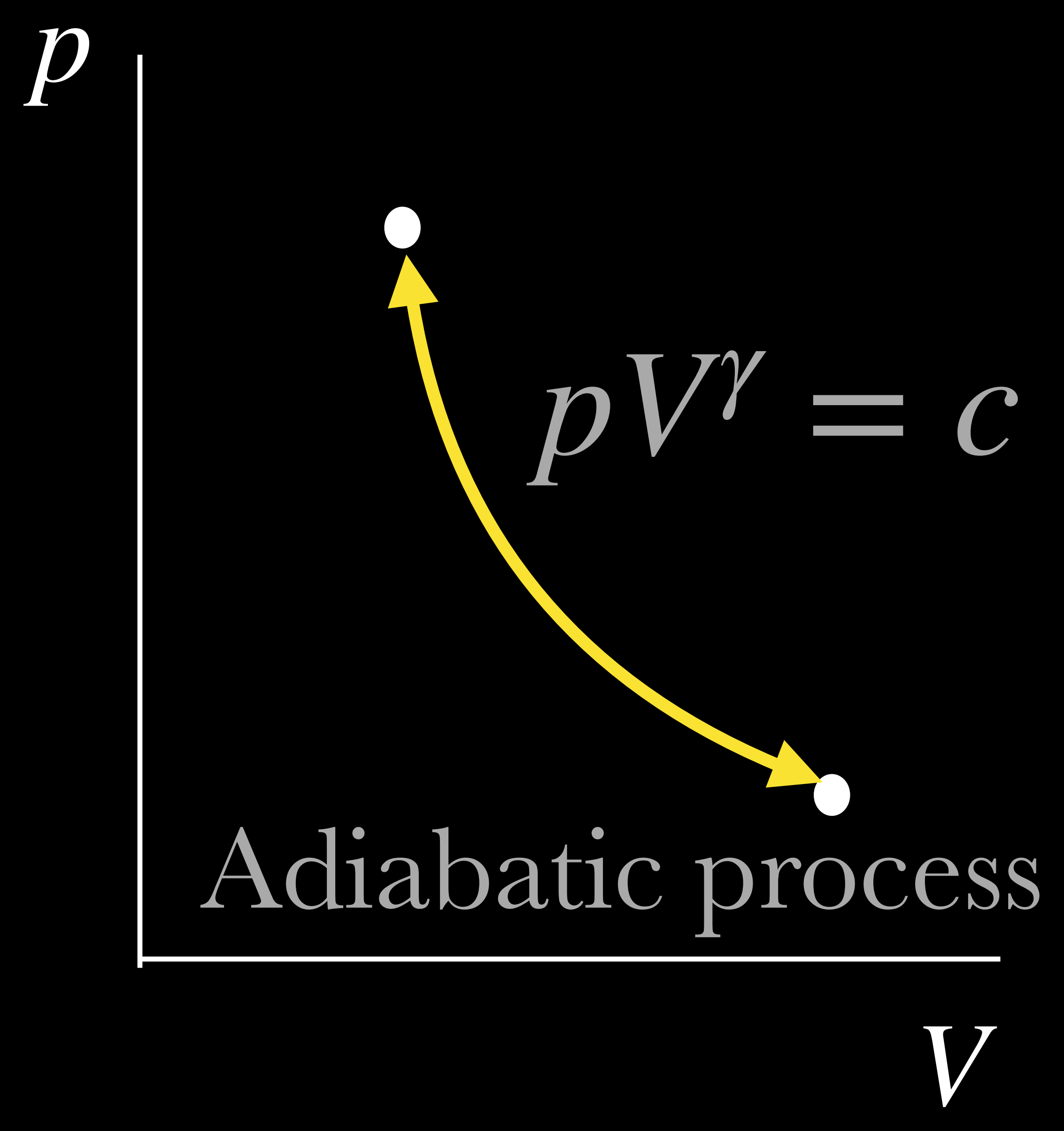
Ex: isothermal process



Reversible process

The processes only over equilibrium states.

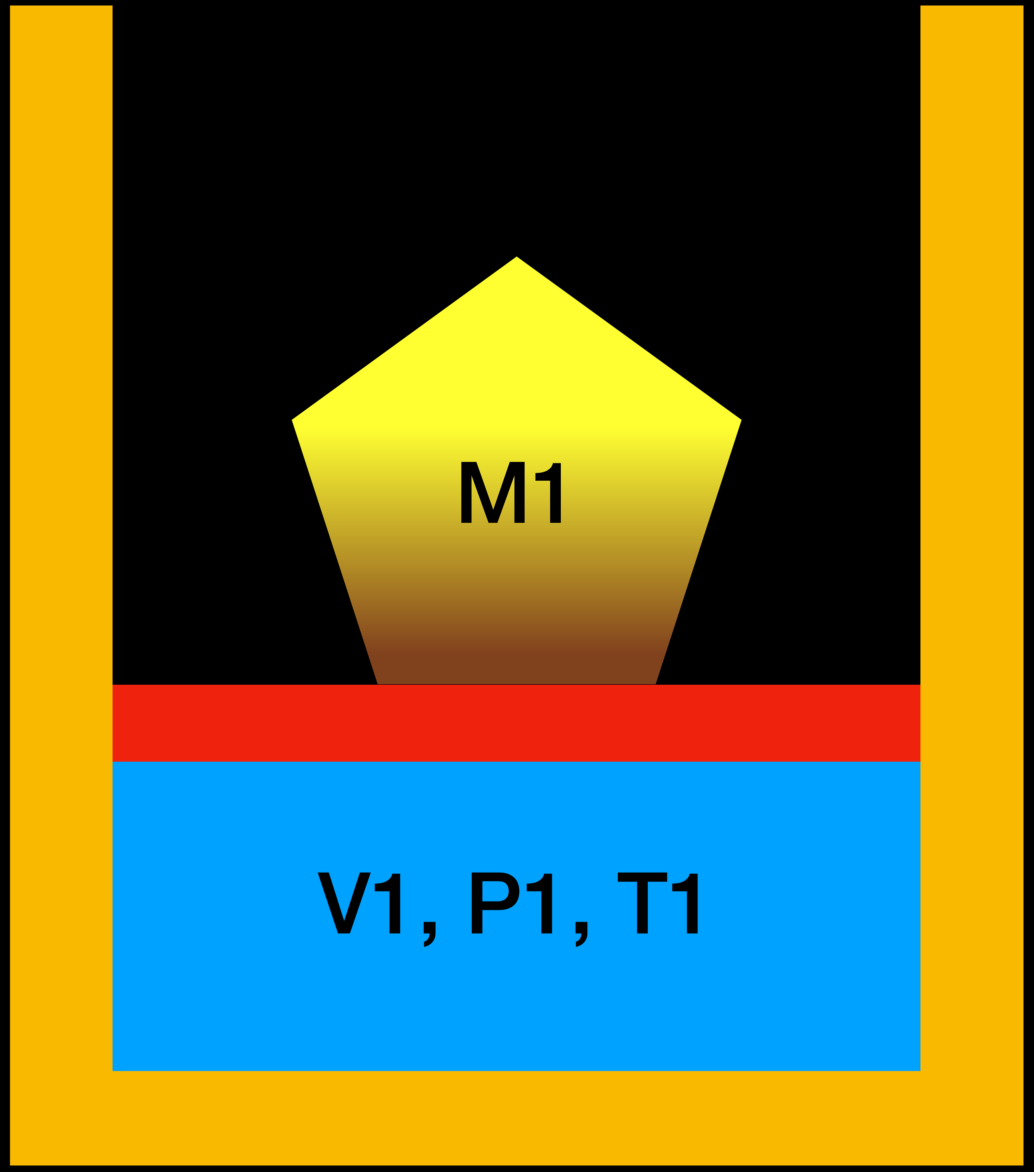
Ex: frictionless adiabatic process



Lecture VII : Reversible process

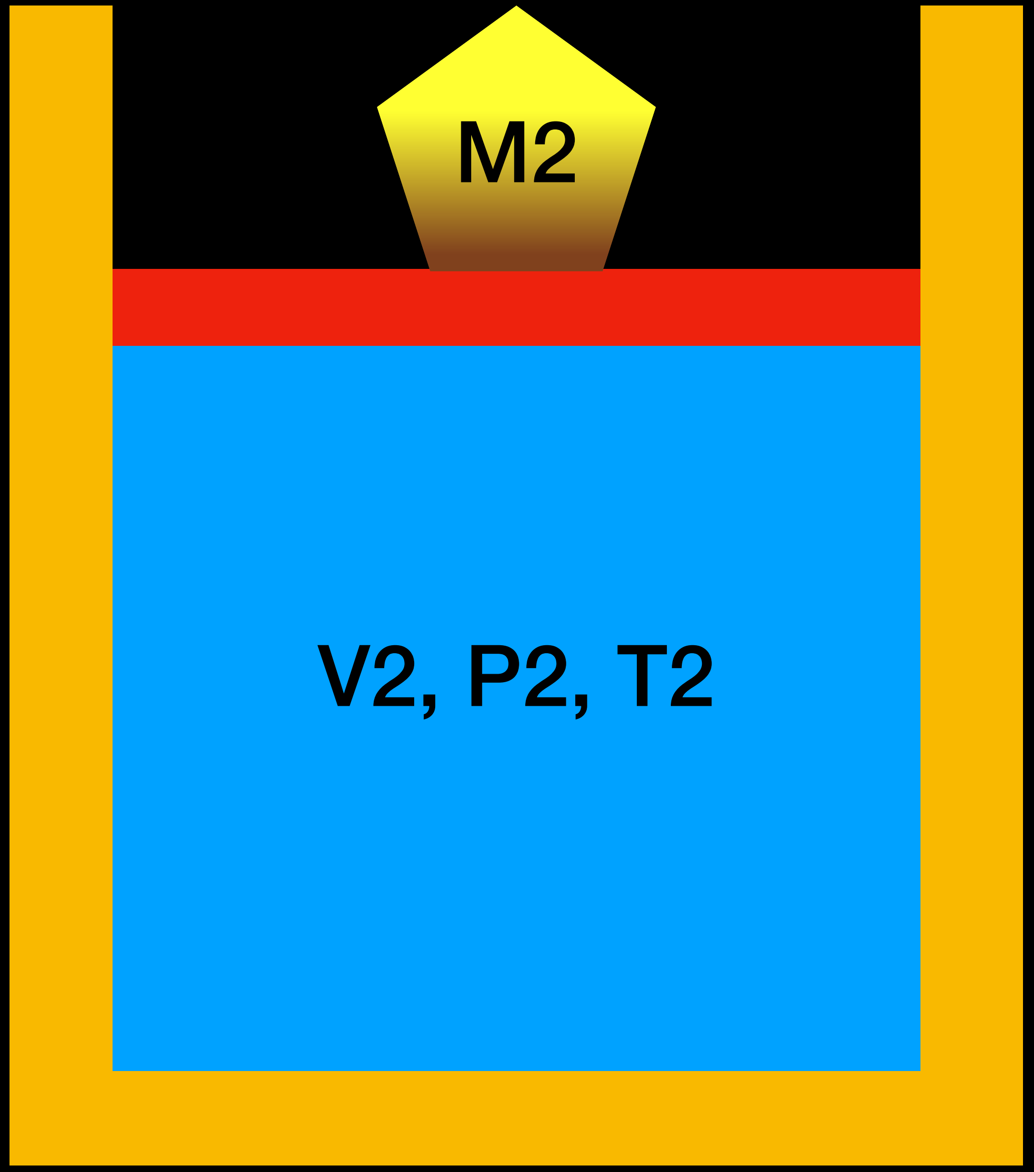
In thermodynamics, a reversible process is a process whose direction can be returned to its original position by inducing infinitesimal changes to some property of the system via its surroundings.

The processes only over equilibrium states.

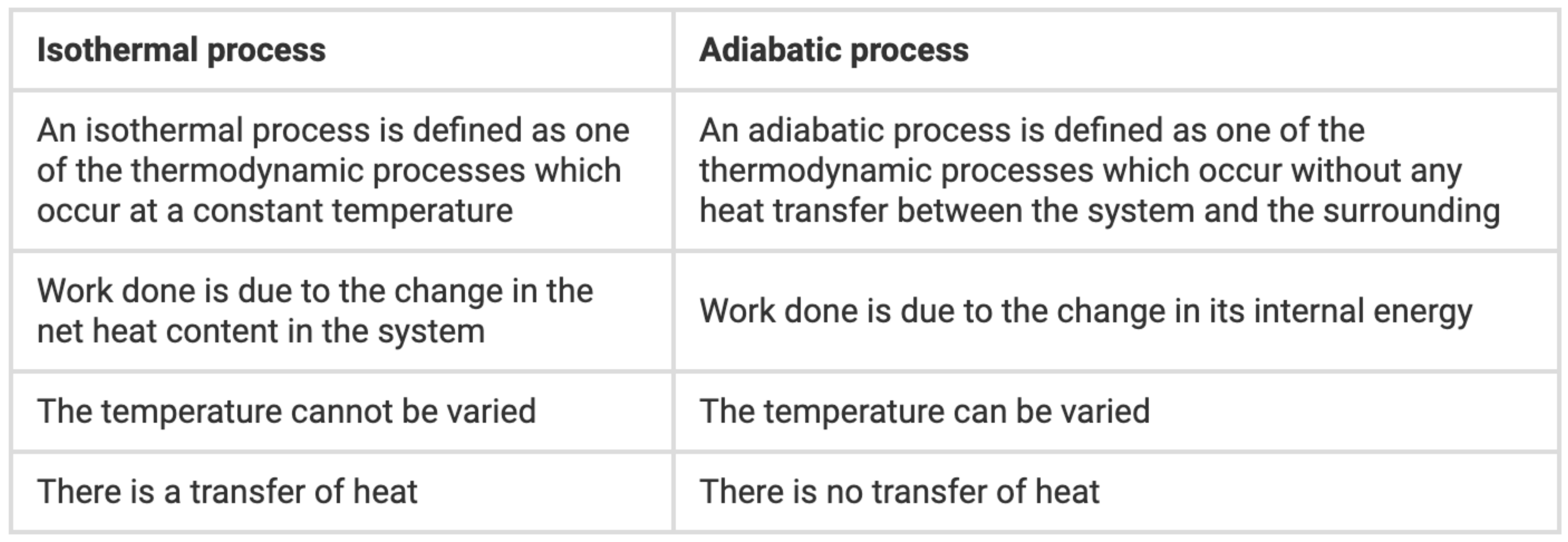


Adiabatic process

$$p_i V_i^{\gamma} = p_f V_f^{\gamma}$$



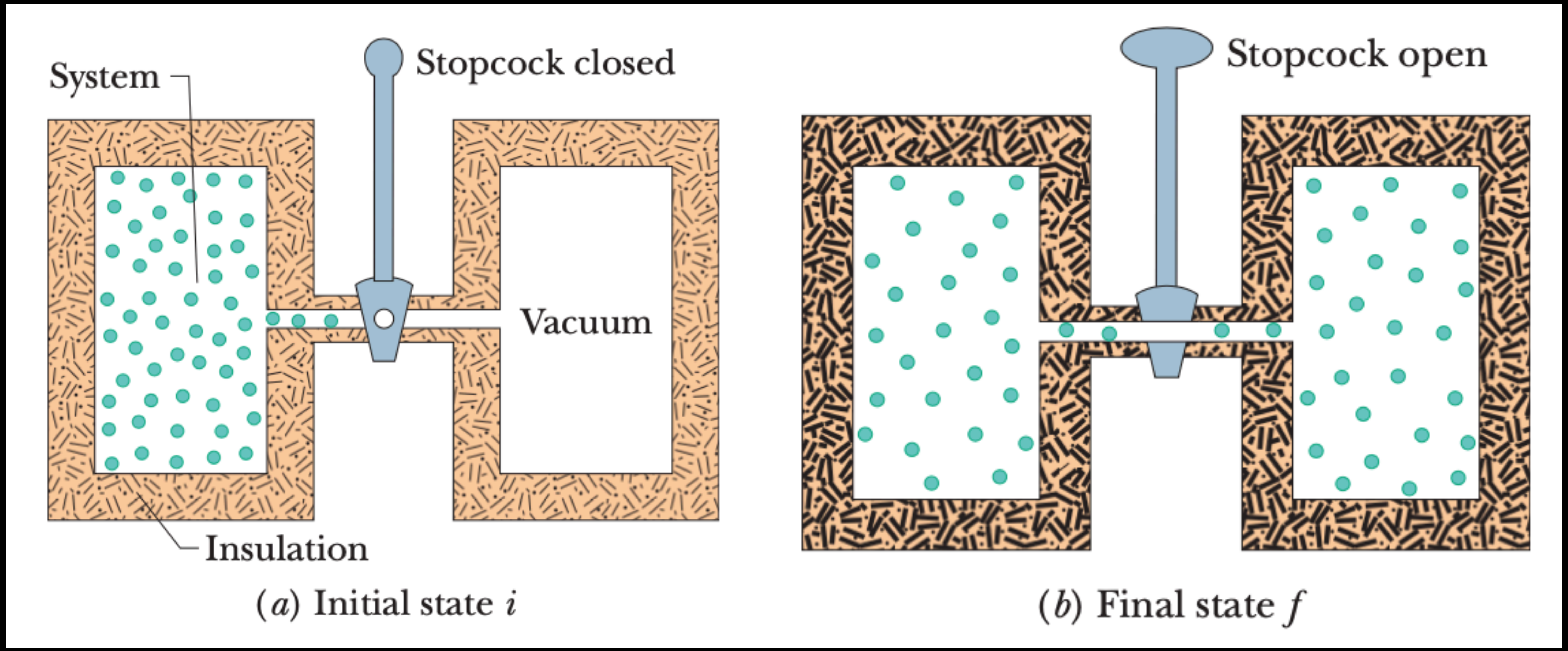
Lecture VII : Irreversible vs. Reversible



Isothermal process	**Adiabatic process**
An isothermal process is defined as one of the thermodynamic processes which occur at a constant temperature	An adiabatic process is defined as one of the thermodynamic processes which occur without any heat transfer between the system and the surrounding
Work done is due to the change in the net heat content in the system	Work done is due to the change in its internal energy
The temperature cannot be varied	The temperature can be varied
There is a transfer of heat	There is no transfer of heat

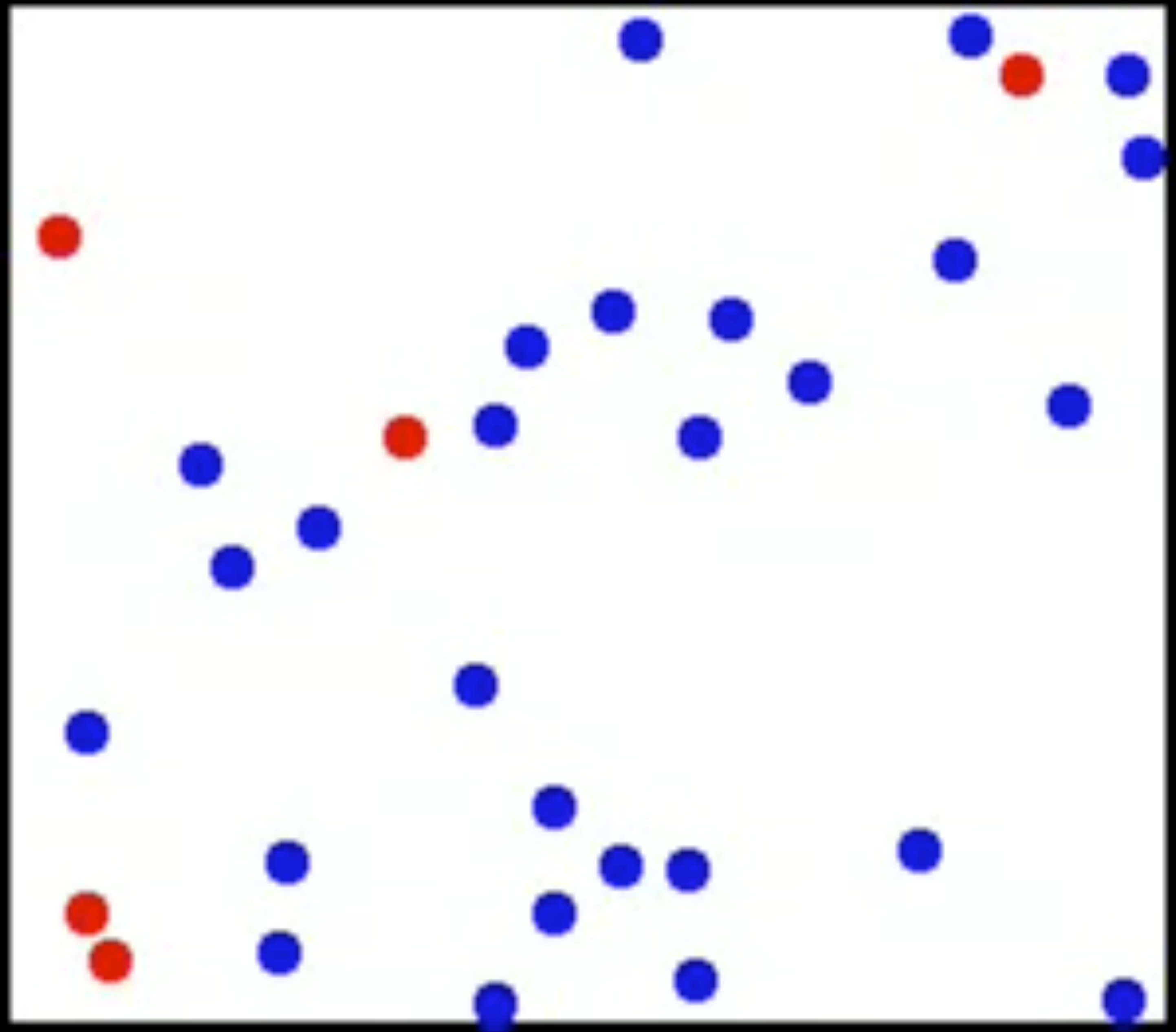
Lecture VII : Irreversible vs. Reversible

credit: wiki



(*a*) Initial state *i*

(*b*) Final state *f*



Irreversible

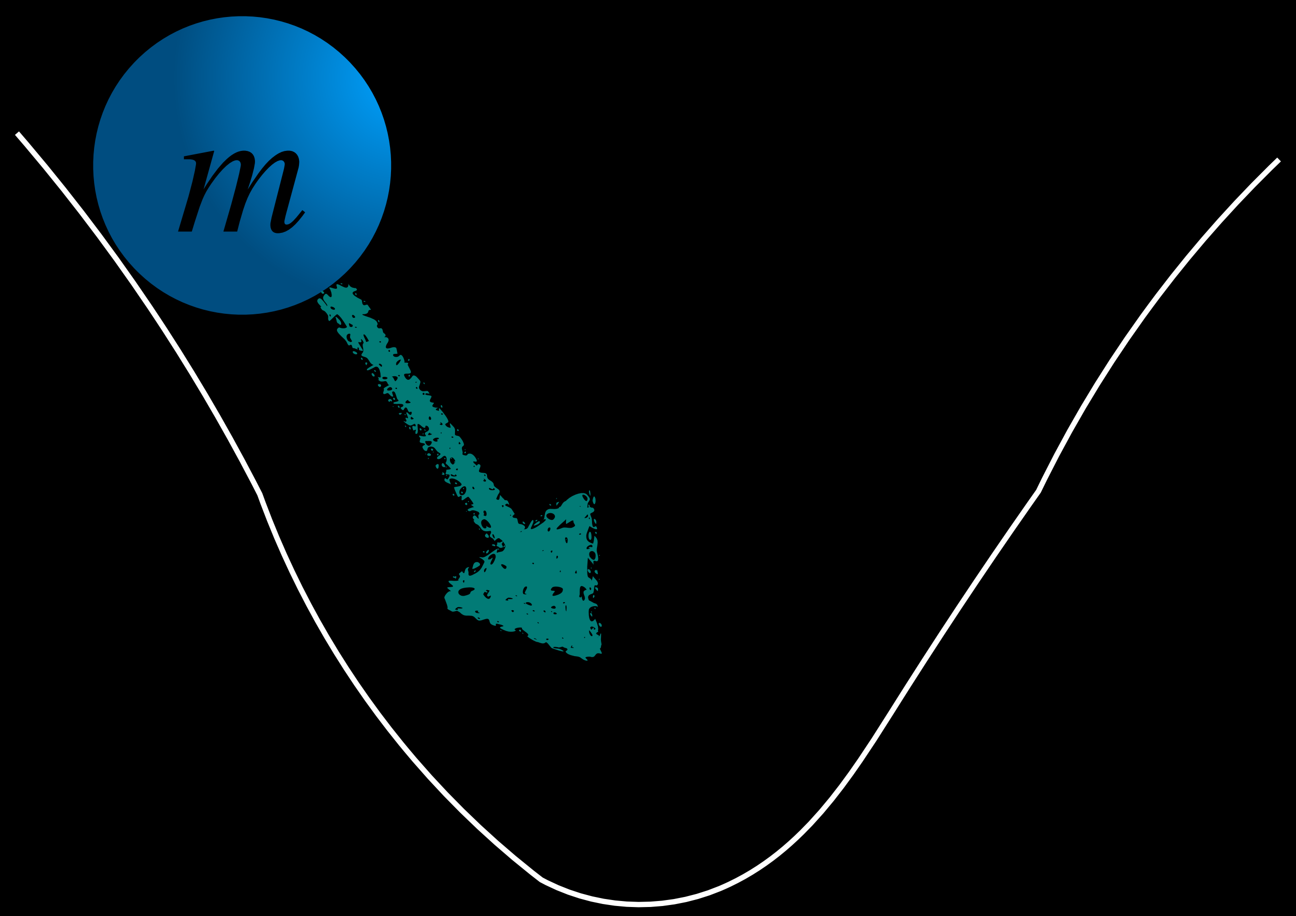
Macroscopic

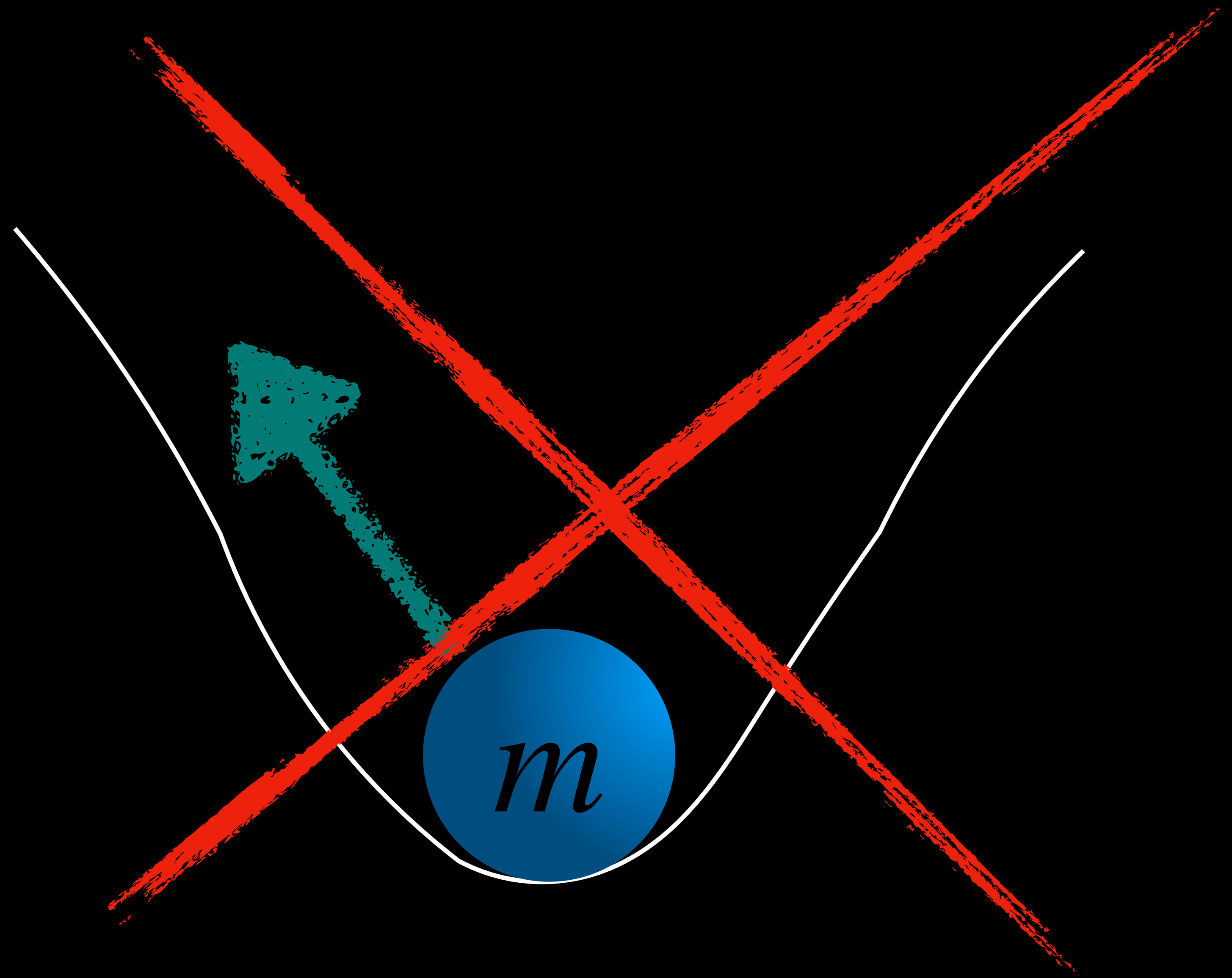
reversible

Microscopic

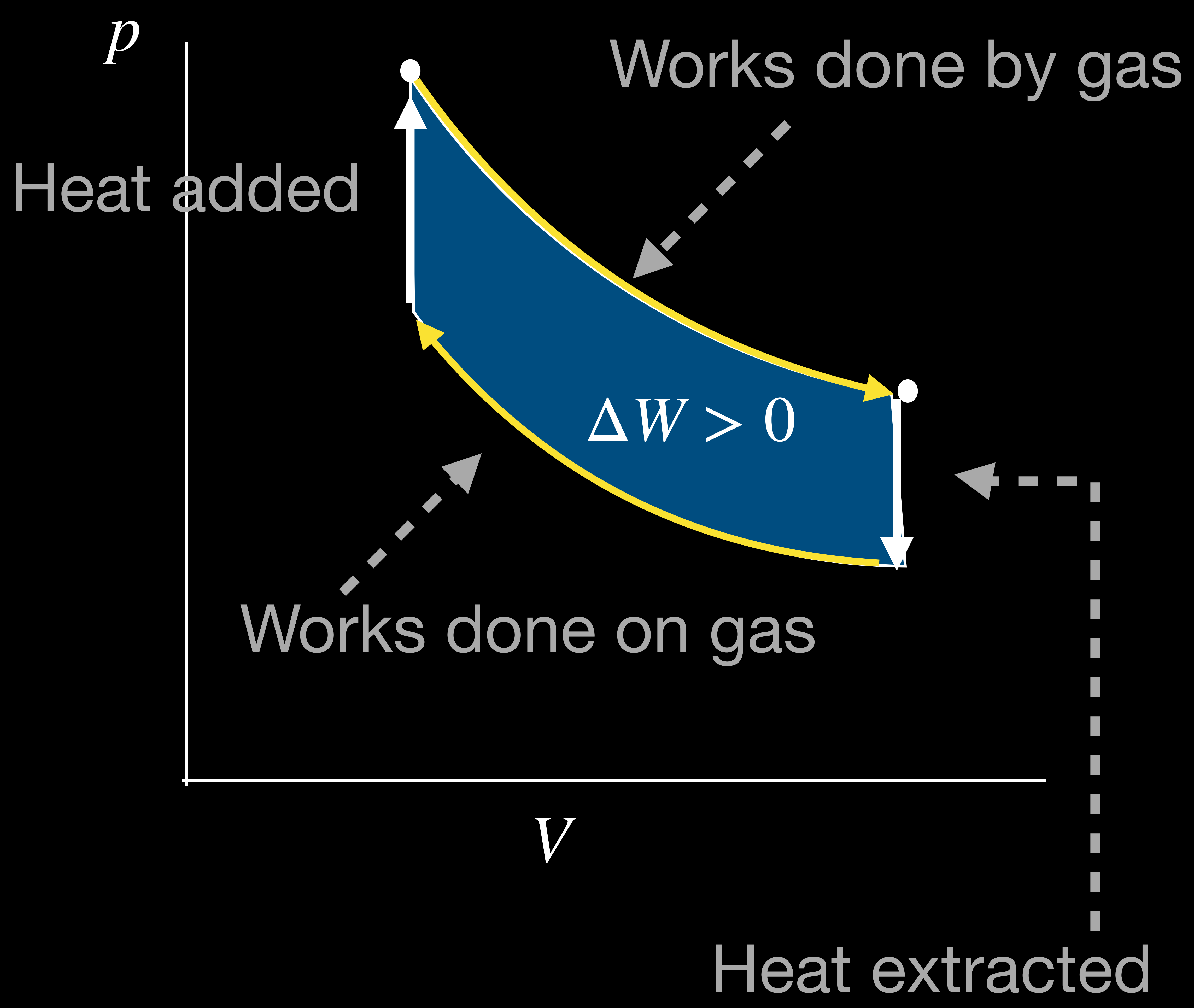
Lecture VII : Spontaneous Process

a spontaneous process is the time-evolution of a system in which it releases free energy and it moves to a lower, more thermodynamically stable energy state (equilibrium state)

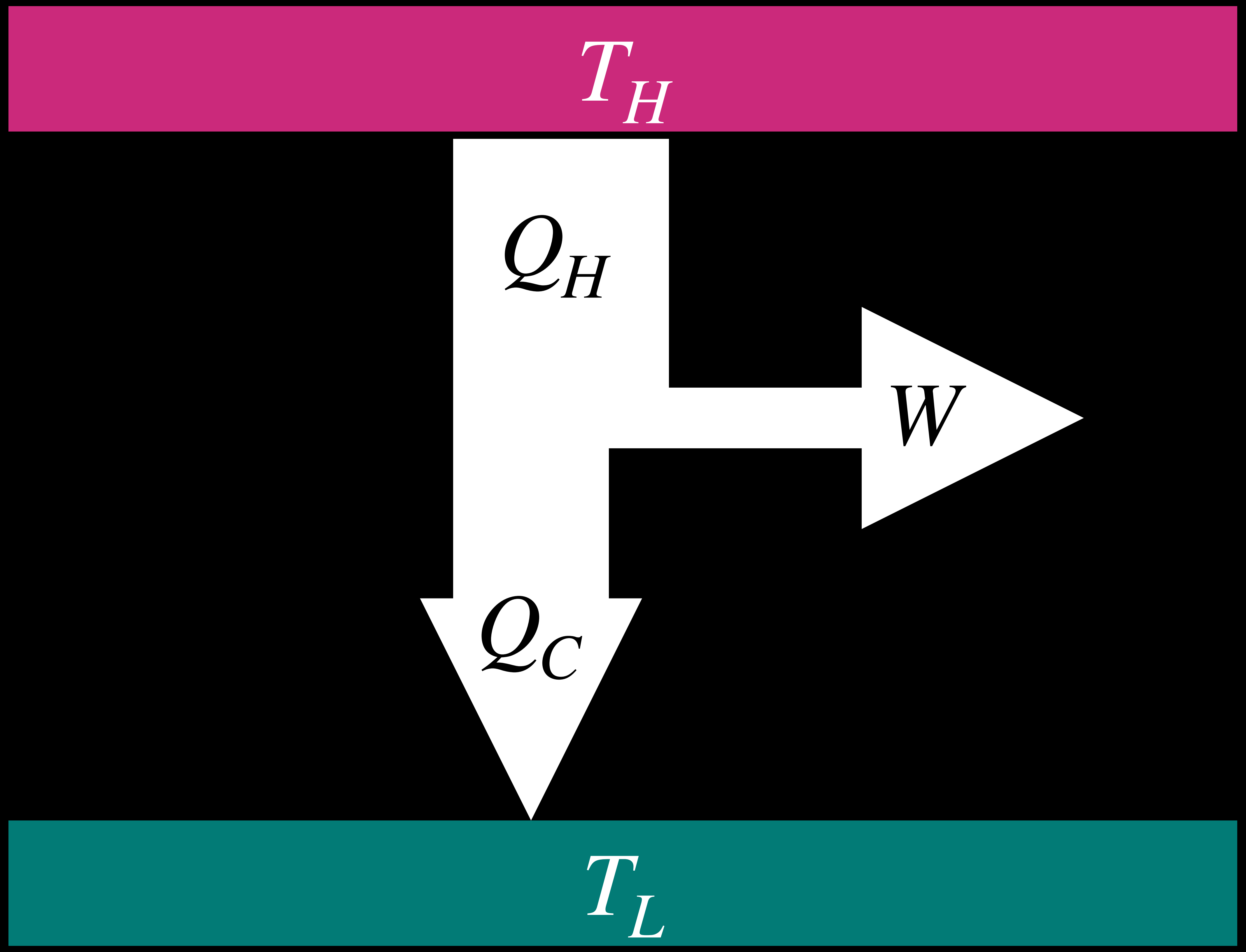




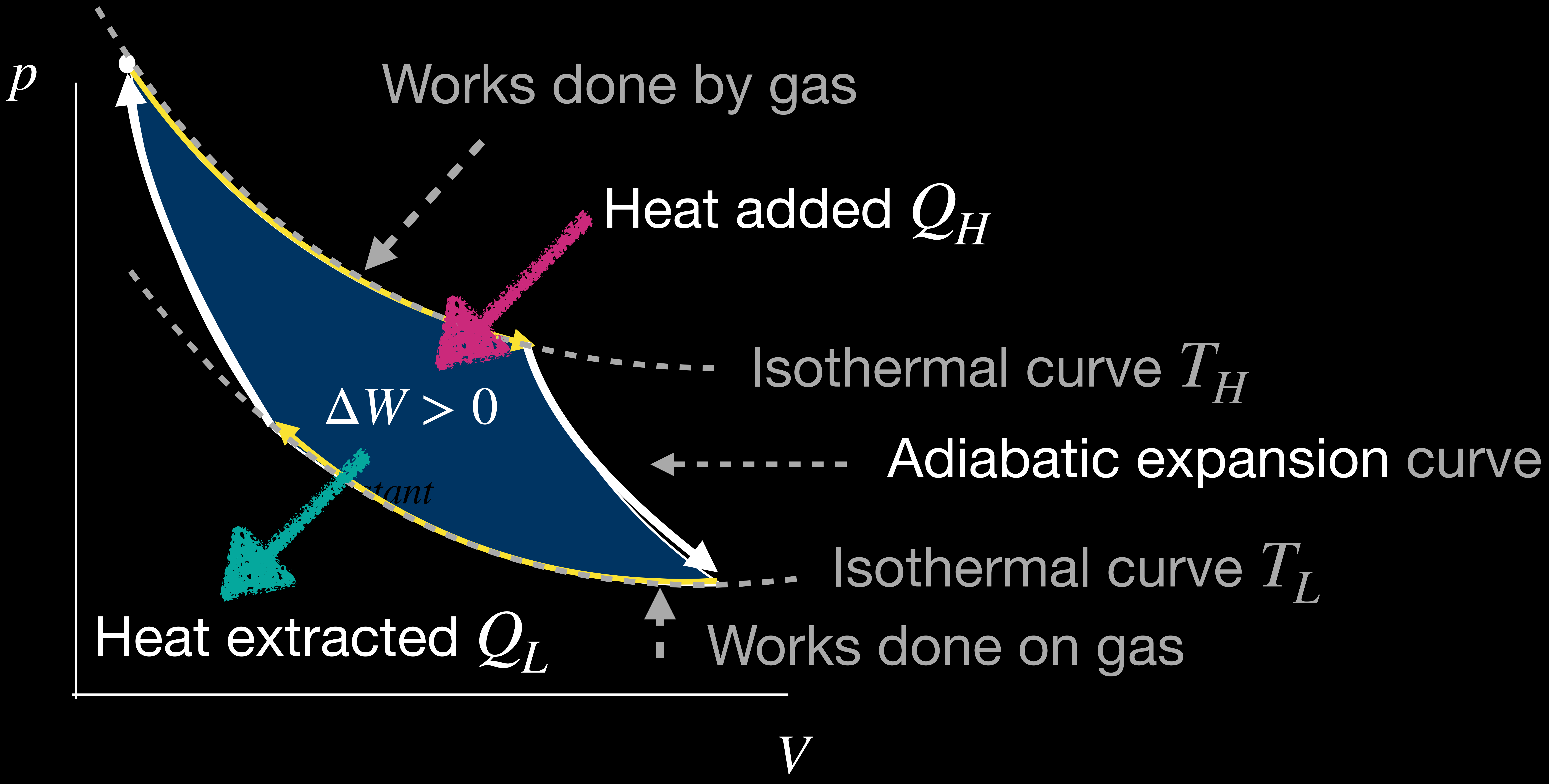
Lecture VII : Heat Engine



Energy Reservoir Model



Lecture VII : Carnot's engine



Works done by gas

Heat added Q_H

Isothermal curve T_H

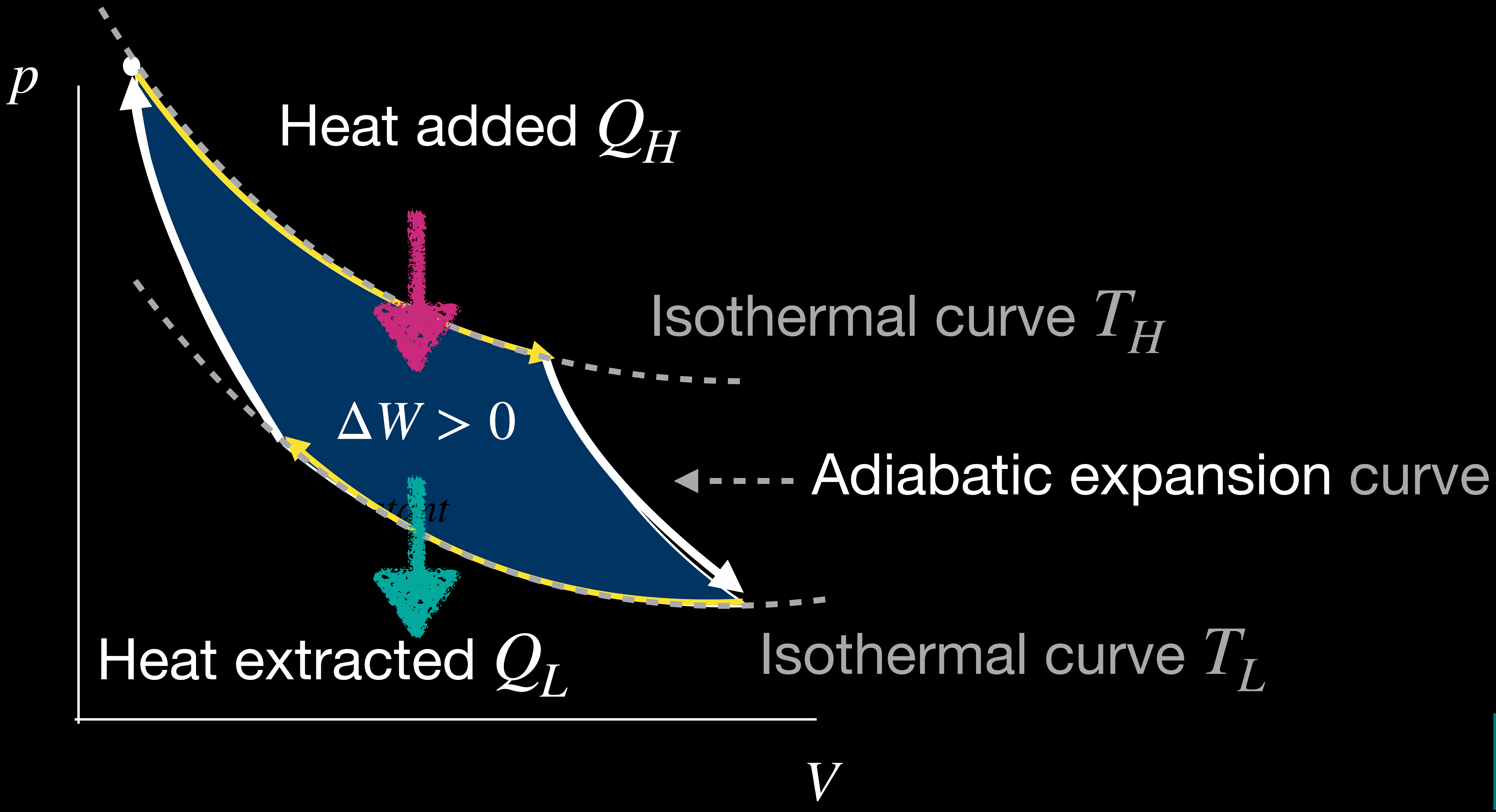
Adiabatic expansion curve

Isothermal curve T_L

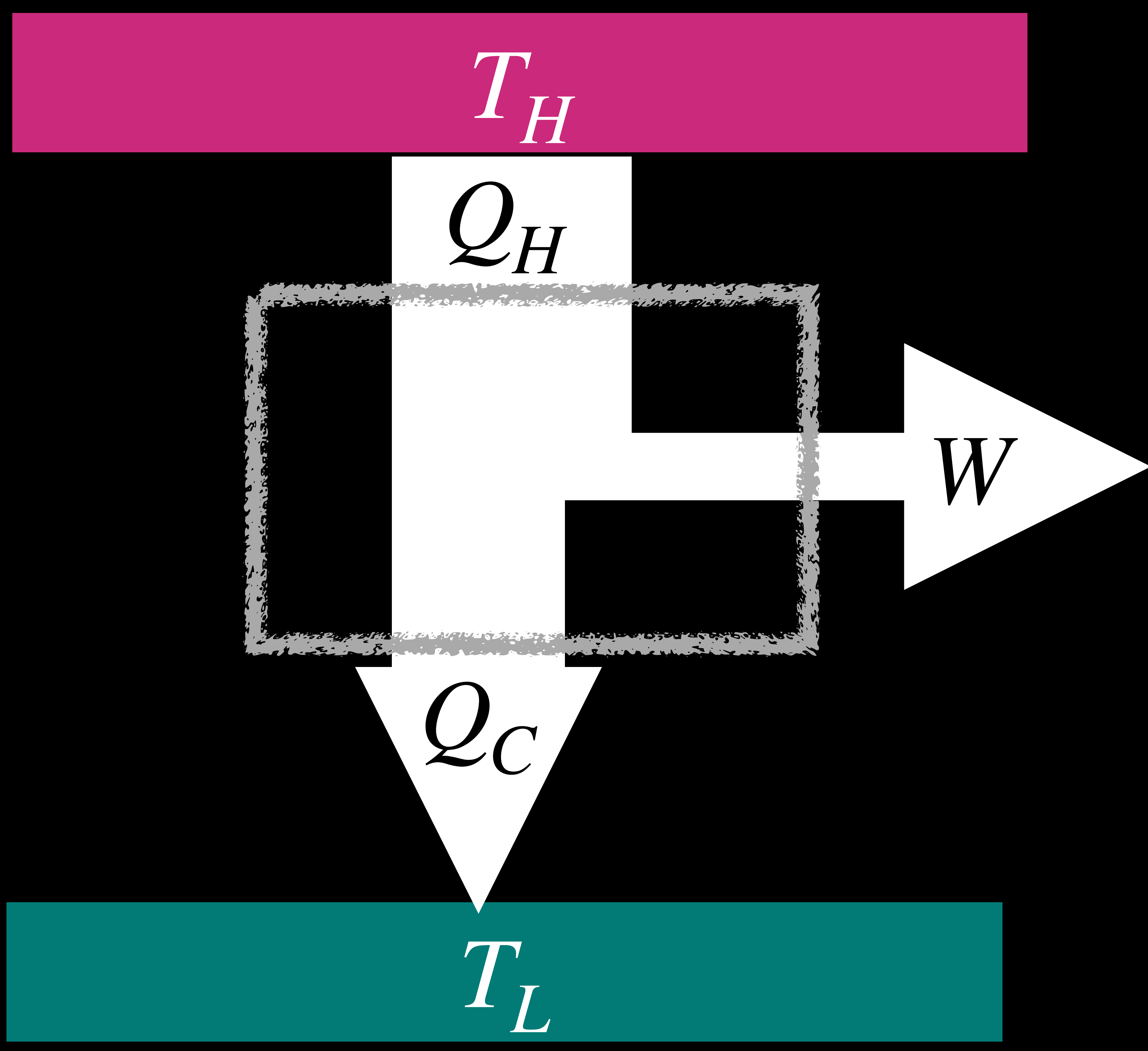
Works done on gas

Heat extracted Q_L

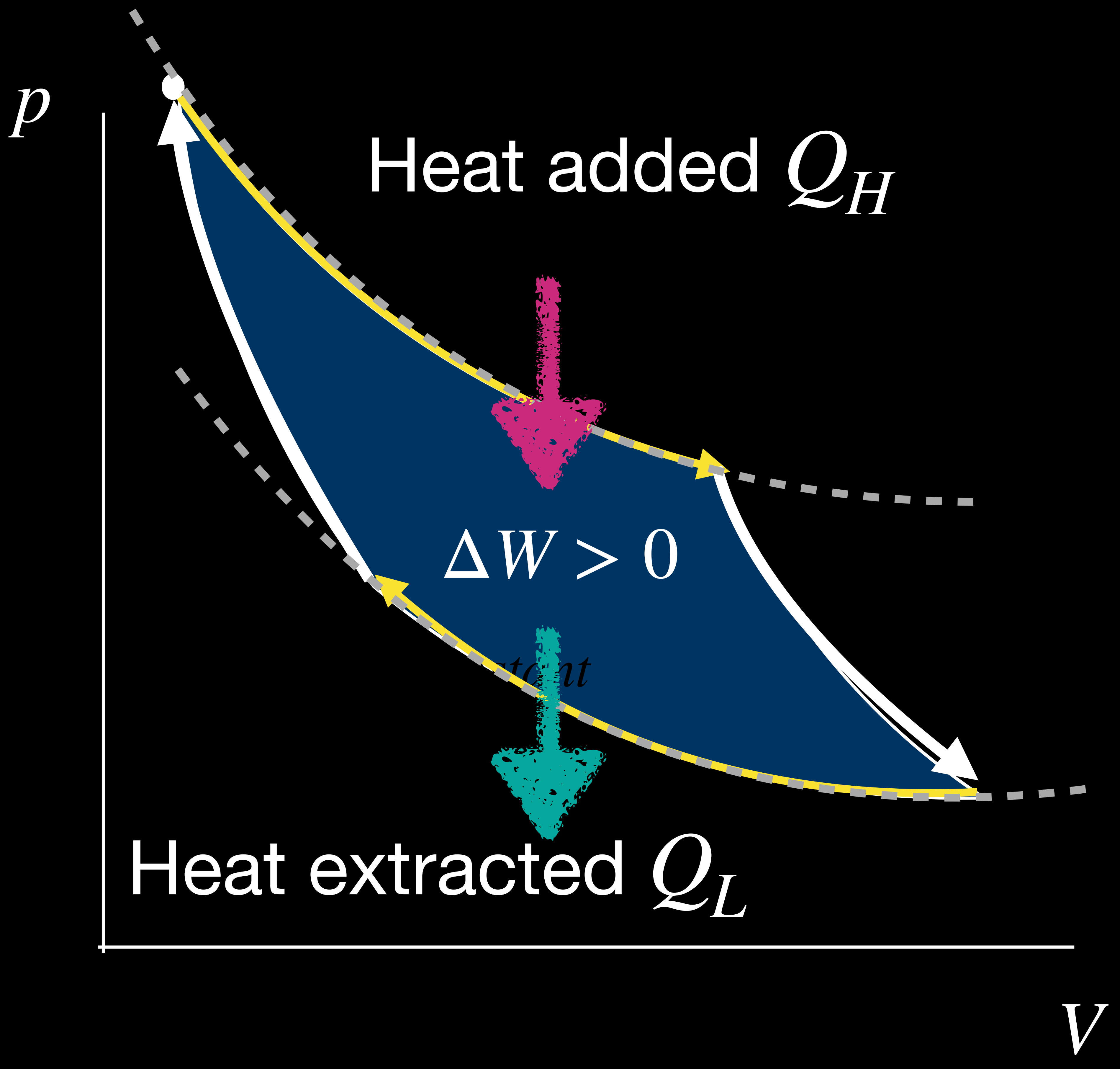
Lecture VII : Carnot's engine

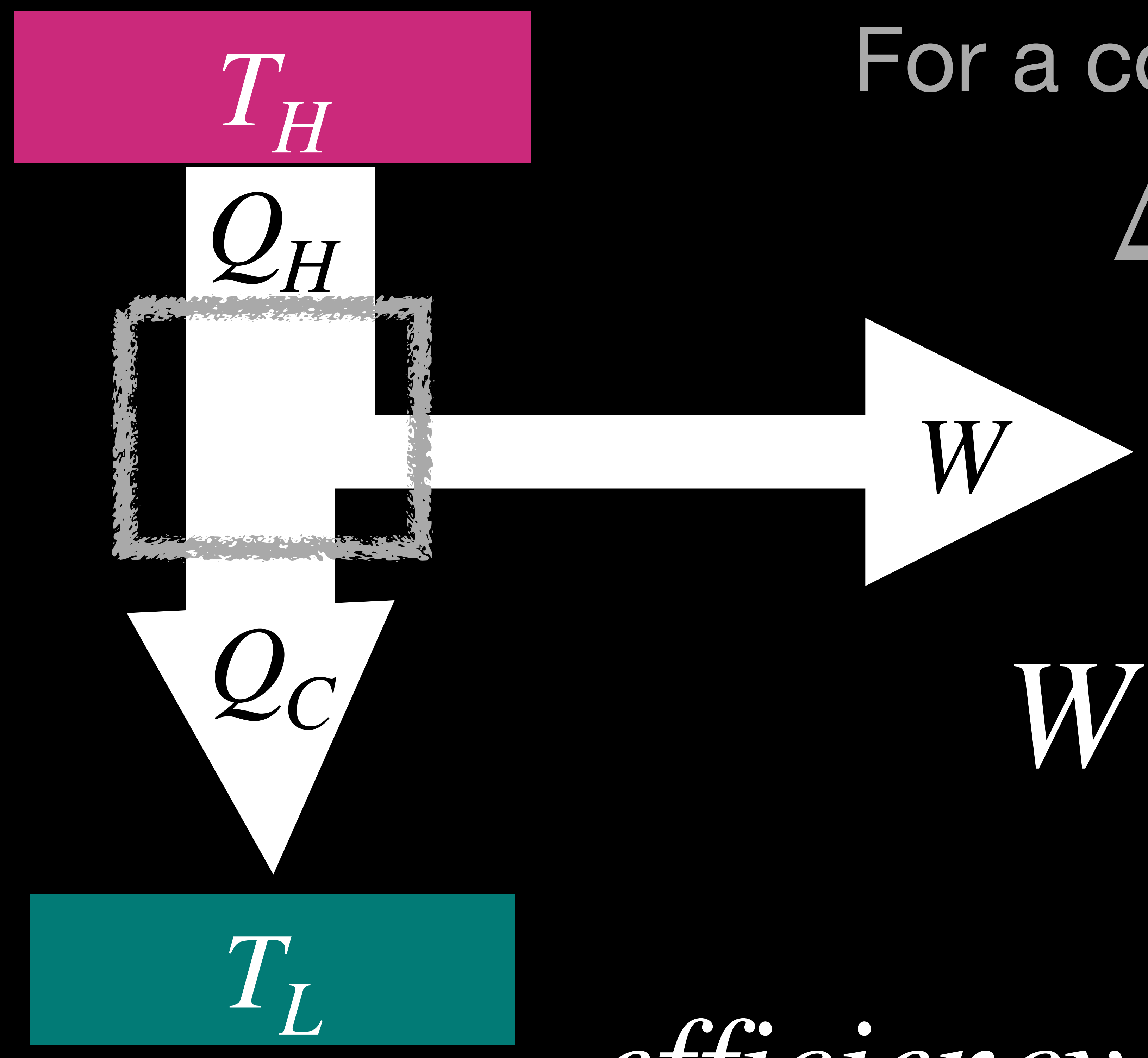


Energy Reservoir Model



Lecture VII : Carnot's engine





$$\Delta E_{int} = Q - W$$

—the first law

For a complete Carnot's cycle

$$\Delta E_{int} = 0$$

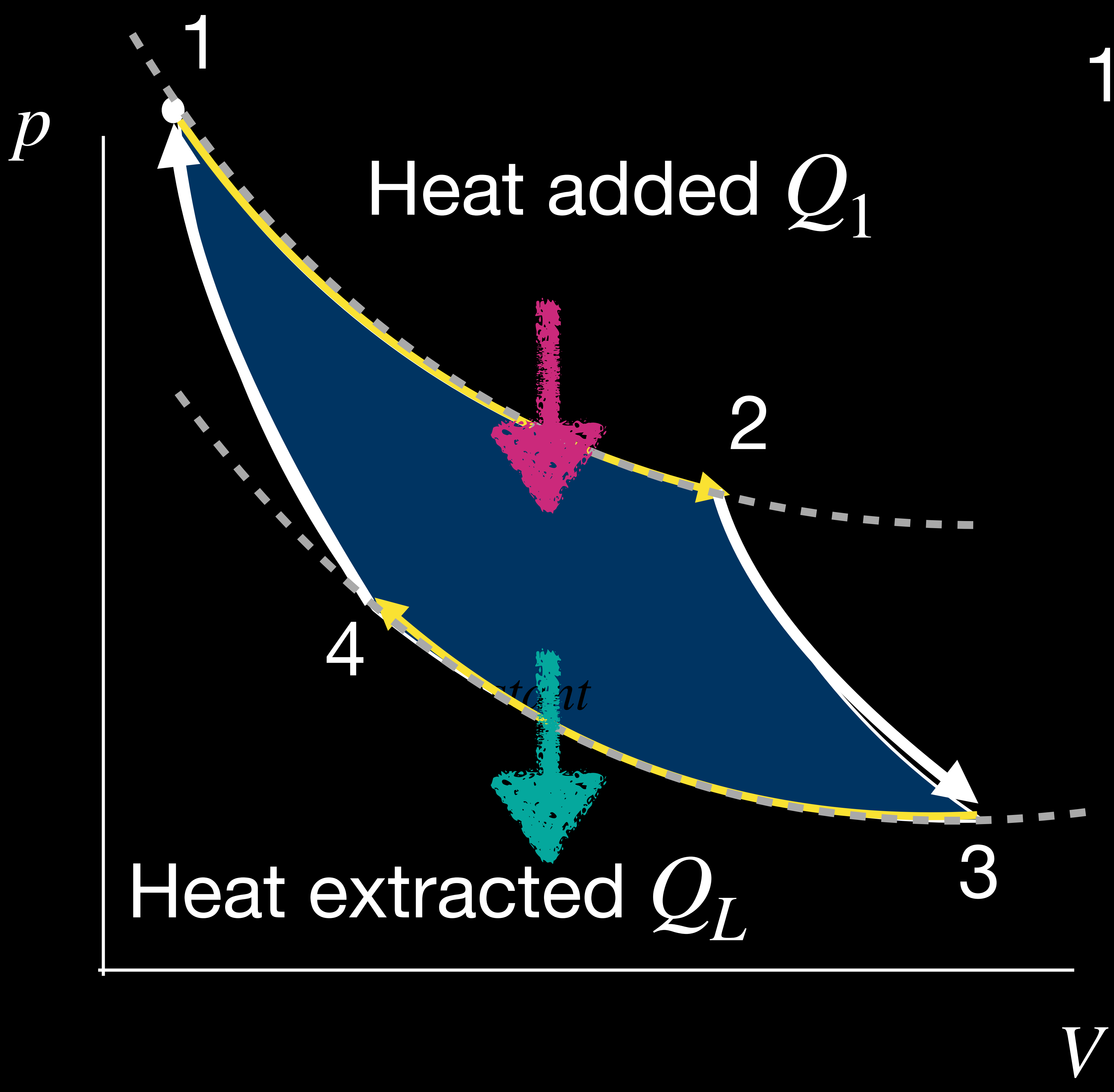
$$W = Q$$

$$W = (Q_H - Q_L)$$

$$efficiency = \frac{W}{Q_H} = 1 - \frac{Q_L}{Q_H}$$

Lecture VII : Carnot's engine (detail)

$dE_{int} = dQ - dW$



1->2: $p_1V_1 = p_2V_2 = nKT_h$ $\quad \frac{V_2}{V_1} = \frac{p1}{p2}$

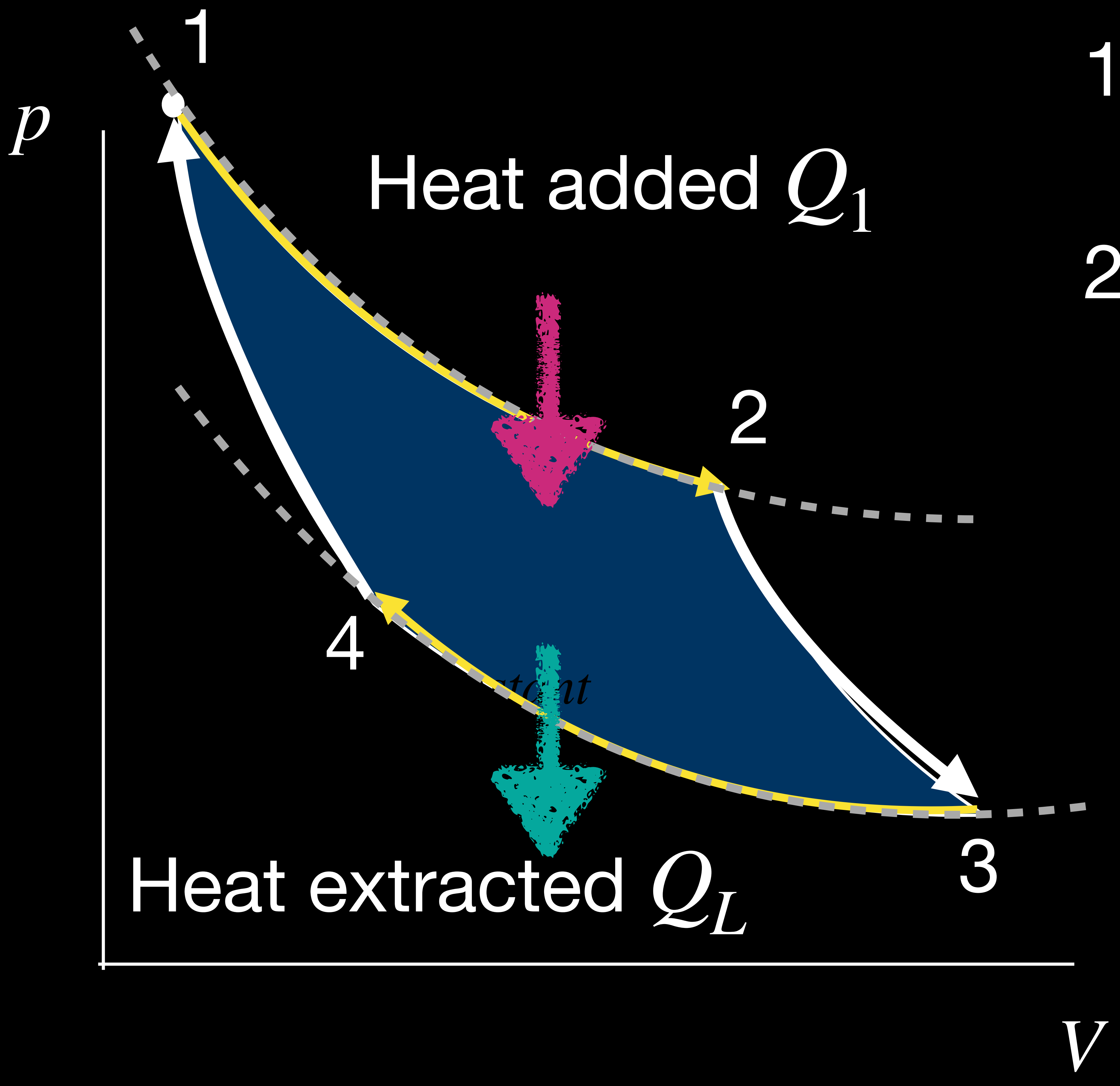
$$\Delta E_{int} = 0 = Q_1 - W_1$$

Isothermal

$$Q_1 = -W_1 = NKT_h \ln \frac{V_2}{V_1}$$

This is the amount of heat exchanged with heat bath in 1->2 step. Since $V_2 > V_1$, $dQ_1 > 0$; the amount of heat ΔQ_1 is added to the gas at the expense of the heat bath.

Lecture VII : Carnot's engine (detail) $dE_{int} = dQ - dW$



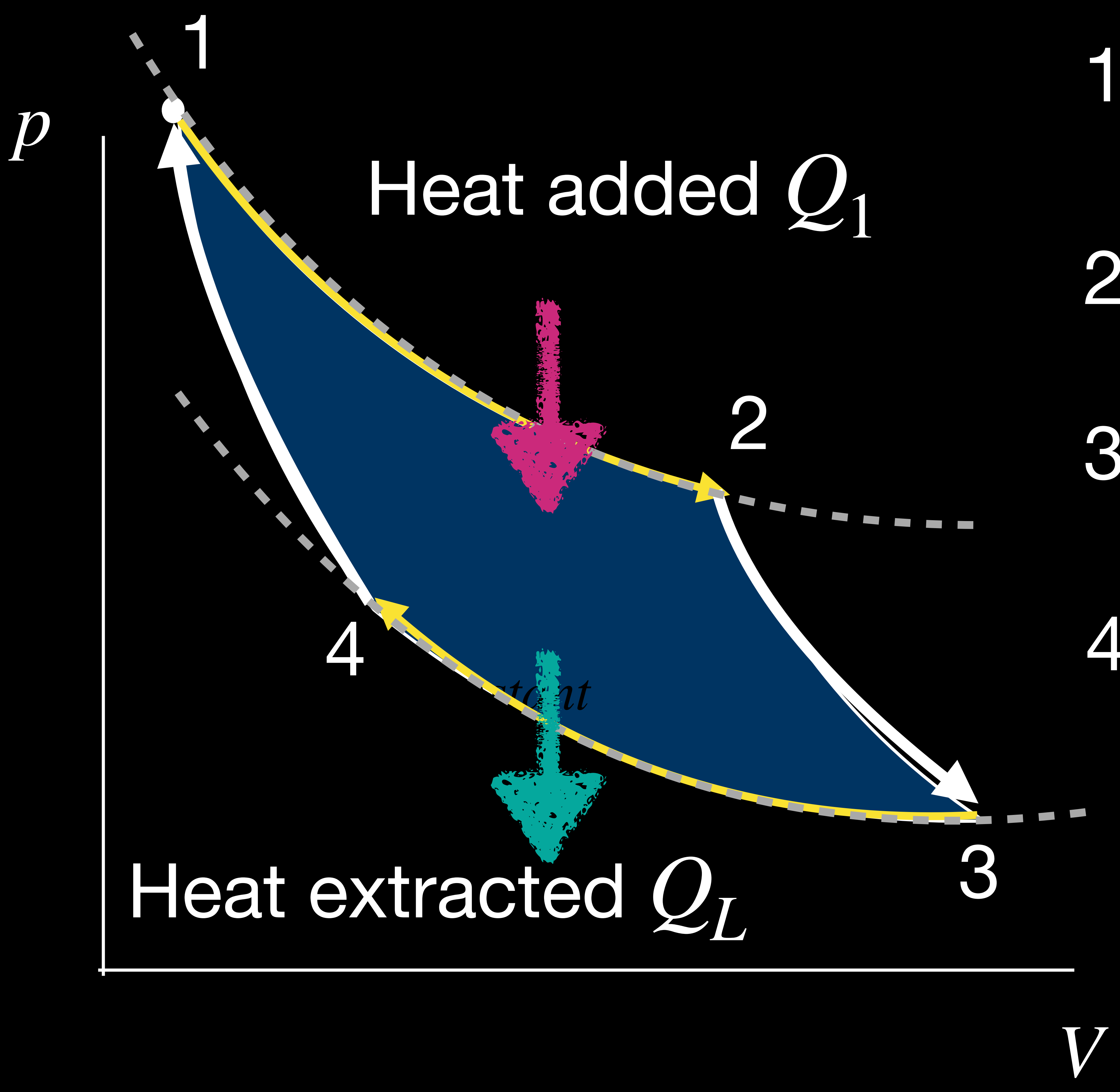
1->2: $Q_1 = -W_1 = NKT_h \ln \frac{V_2}{V_1}$ Isothermal

2->3: $\frac{V_3}{V_2} = (\frac{T_h}{T_c})^{\frac{3}{2}}$ Adiabatic

$$C_v dT = \Delta E_{int} = Q_2 - W_@ = -W_2$$

$$W_2 = C_v(T_c - T_h)$$

Lecture VII : Carnot's engine (detail) $dE_{int} = dQ - dW$



1->2: $Q_1 = -W_1 = NKT_h \ln \frac{V_2}{V_1}$ Isothermal

2->3: $W_2 = C_v(T_c - T_h)$ Adiabatic

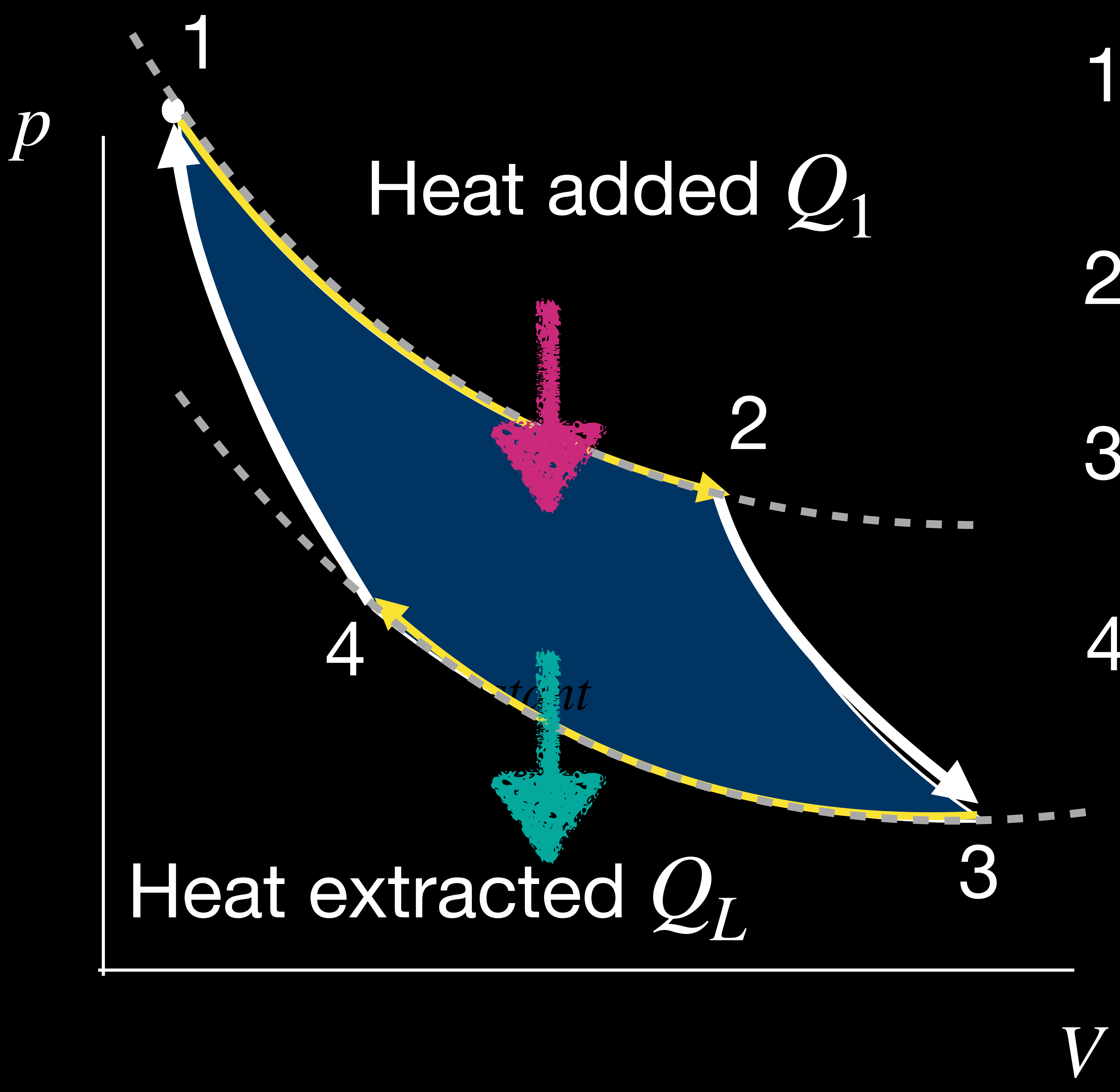
3->4: $Q_3 = -W_3 = NKT_c \ln \frac{V_4}{V_3}$ Isothermal

4->1: $W_4 = C_v(T_h - T_c)$ Adiabatic

$$\Delta E_{total} =$$

$$\underline{Q_1 - W_1} + \underline{Q_2 - W_2} + \underline{Q_3 - W_3} + \underline{Q_4 - W_4}$$

Lecture VII : Carnot's engine (detail) $dE_{int} = dQ - dW$



1->2: $Q_1 = -W_1 = NKT_h \ln \frac{V_2}{V_1}$ Isothermal

2->3: $W_2 = C_v(T_c - T_h)$ Adiabatic

3->4: $Q_3 = -W_3 = NKT_c \ln \frac{V_4}{V_3}$ Isothermal

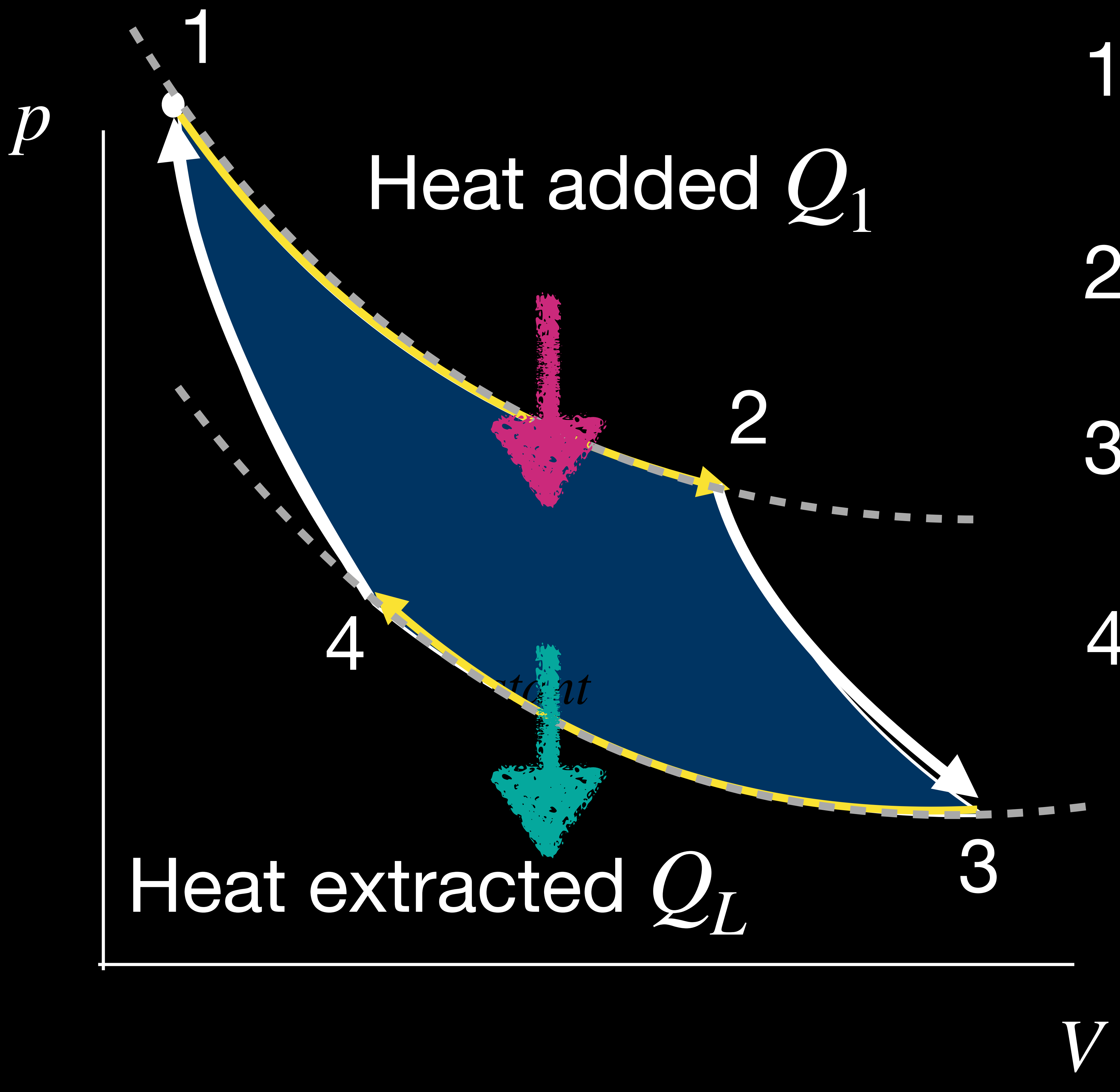
4->1: $W_4 = C_v(T_h - T_c)$ Adiabatic

$$\Delta E_{total} =$$

$$\underbrace{\cancel{Q_1 - W_1}} + \underbrace{\cancel{Q_2} - W_2}_{= dW_4} + \underbrace{\cancel{Q_3 - W_3}} + \underbrace{\cancel{Q_4} - W_4}_{= dW_2}$$

$$= 0$$

Lecture VII : Carnot's engine (detail) $dE_{int} = dQ - dW$



1->2: $Q_1 = -W_1 = NKT_h \ln \frac{V_2}{V_1}$ Isothermal

2->3: $W_2 = C_v(T_c - T_h)$ Adiabatic

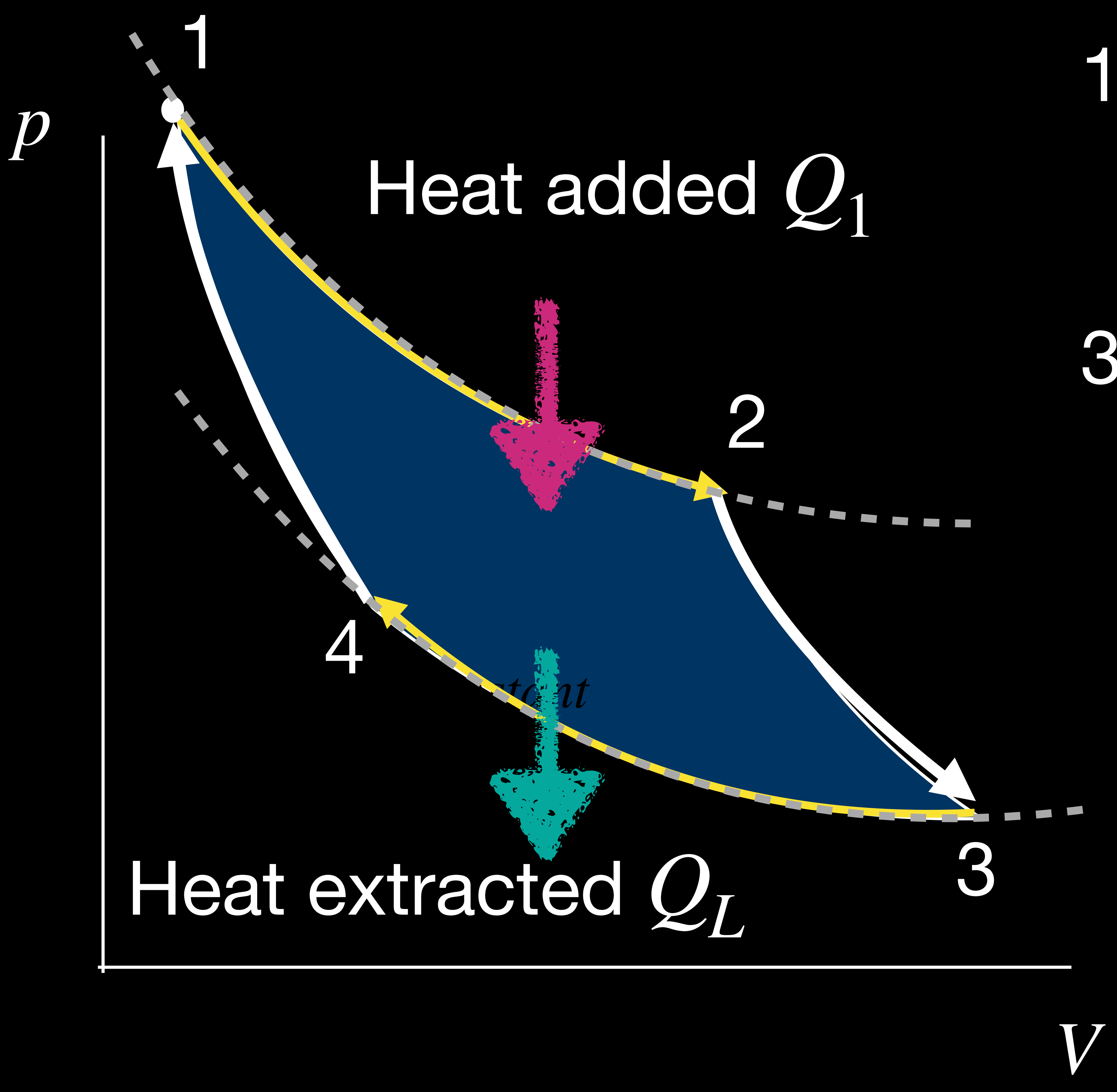
3->4: $Q_3 = -W_3 = NKT_c \ln \frac{V_4}{V_3}$ Isothermal

4->1: $W_4 = C_v(T_h - T_c)$ Adiabatic

$$\frac{V_3}{V_2} = (\frac{T_h}{T_c})^{\frac{3}{2}} \qquad \frac{V_1}{V_4} = (\frac{T_c}{T_h})^{\frac{3}{2}}$$

$$\frac{V_2}{V_1} = \frac{V_3}{V_4}$$

Lecture VII : Carnot's engine (detail) $dE_{int} = dQ - dW$



1->2: $Q_1 = -W_1 = NKT_h \ln \frac{V_2}{V_1}$ Isothermal

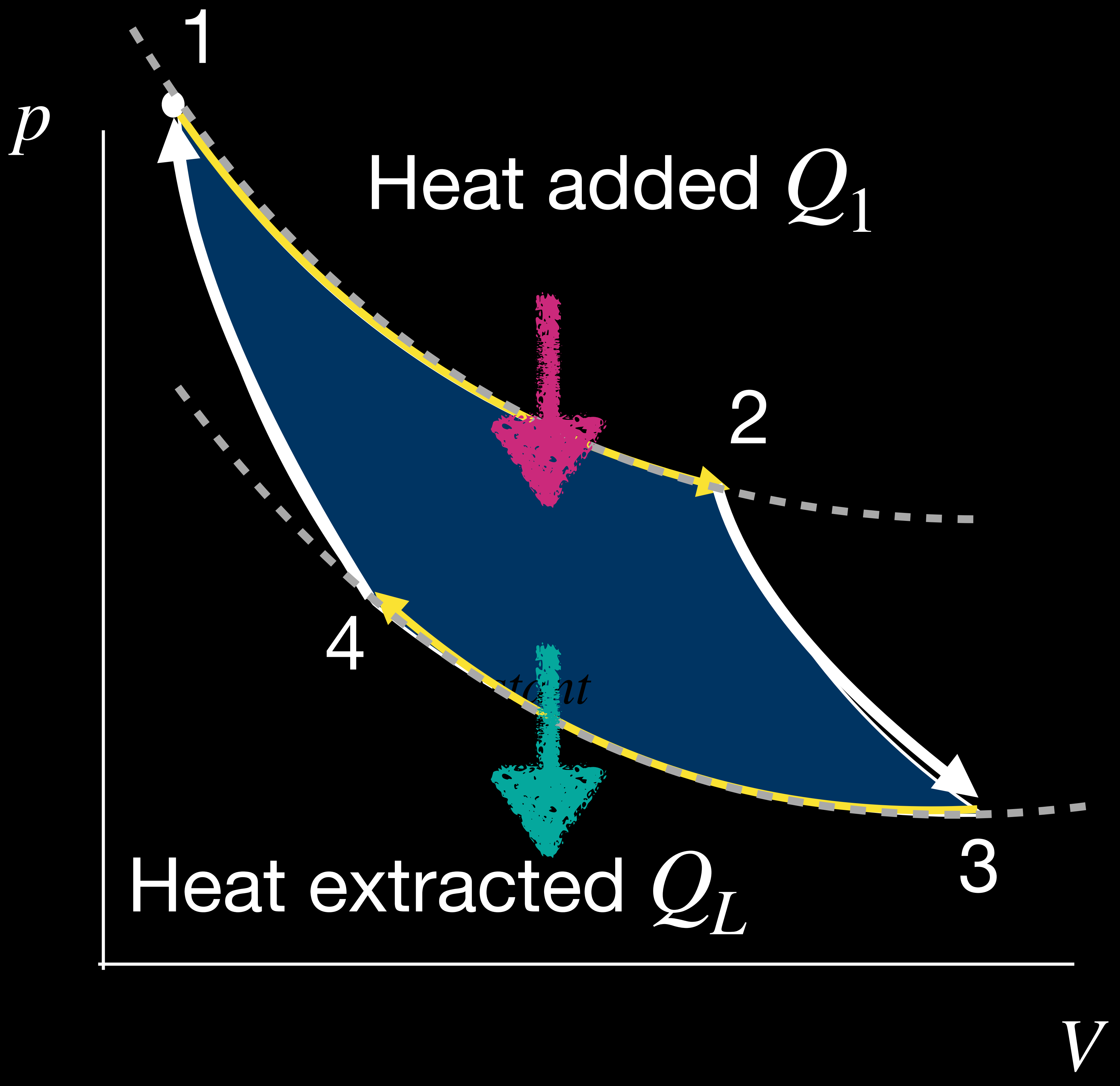
3->4: $Q_3 = -W_3 = NKT_c \ln \frac{V_4}{V_3}$ Isothermal

$$\frac{V_2}{V_1} = \frac{V_3}{V_4}$$

$$\frac{Q_1}{T_h} + \frac{Q_3}{T_c} = 0$$

It is valid for any reversible cyclic process

Lecture VII : Carnot's engine (detail) $dE_{int} = dQ - dW$



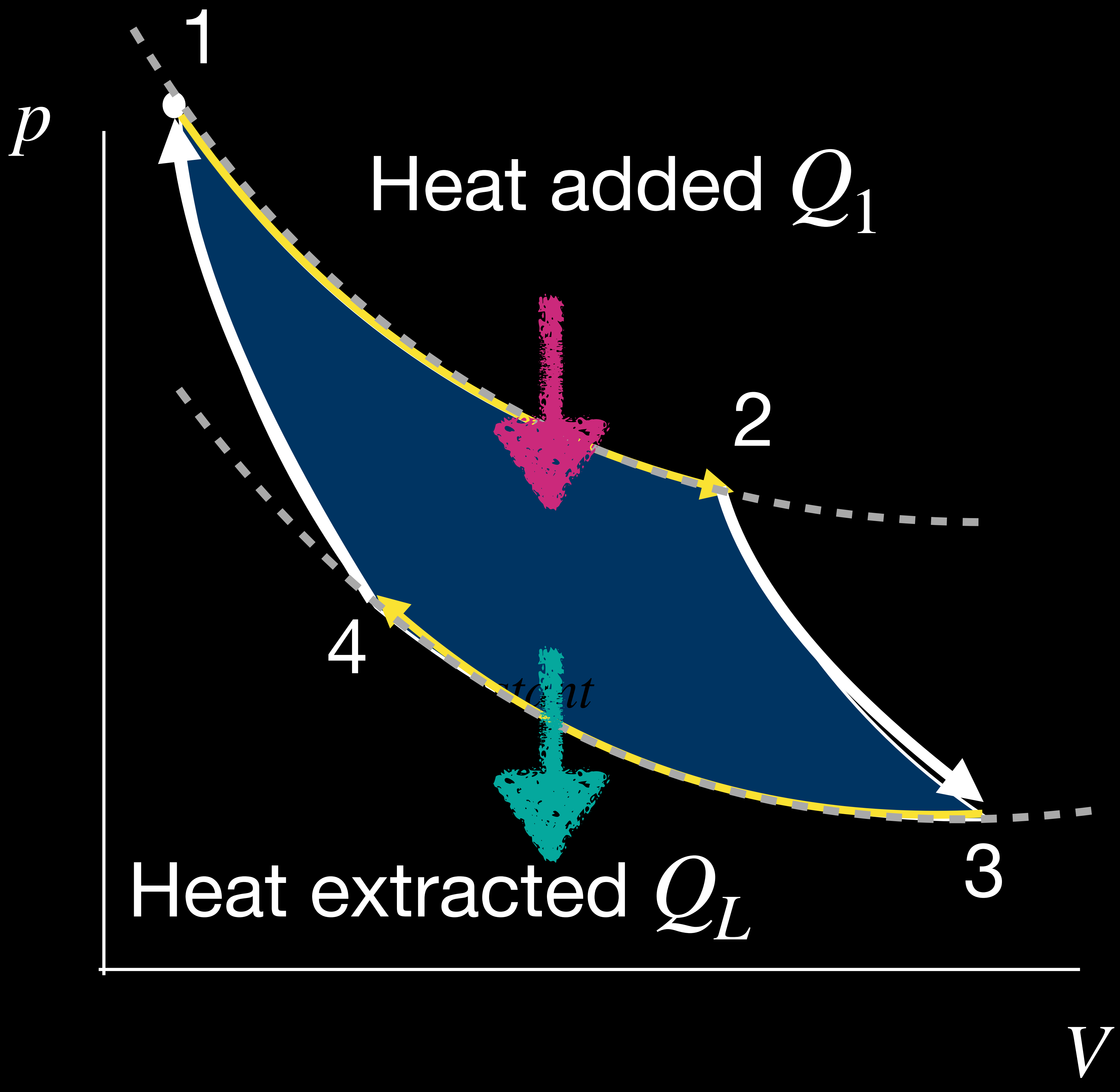
$$\frac{Q_1}{T_h} + \frac{Q_2}{T_c} = 0 \longrightarrow \oint \frac{dQ_{rev}}{T} = 0$$

$\frac{dQ_{rev}}{T}$ is for any reversible thermodynamic process

The extensive state function is the **entropy**

$$dS \equiv \frac{dQ_{rev}}{T}$$

Lecture VII : efficiency of Carnot's engine



$$W_{total} = W_1 + W_2 + W_3 + W_4$$

$$W_1 = NKT_h \ln \frac{V_2}{V_1} \quad W_3 = NKT_c \ln \frac{V_4}{V_3}$$

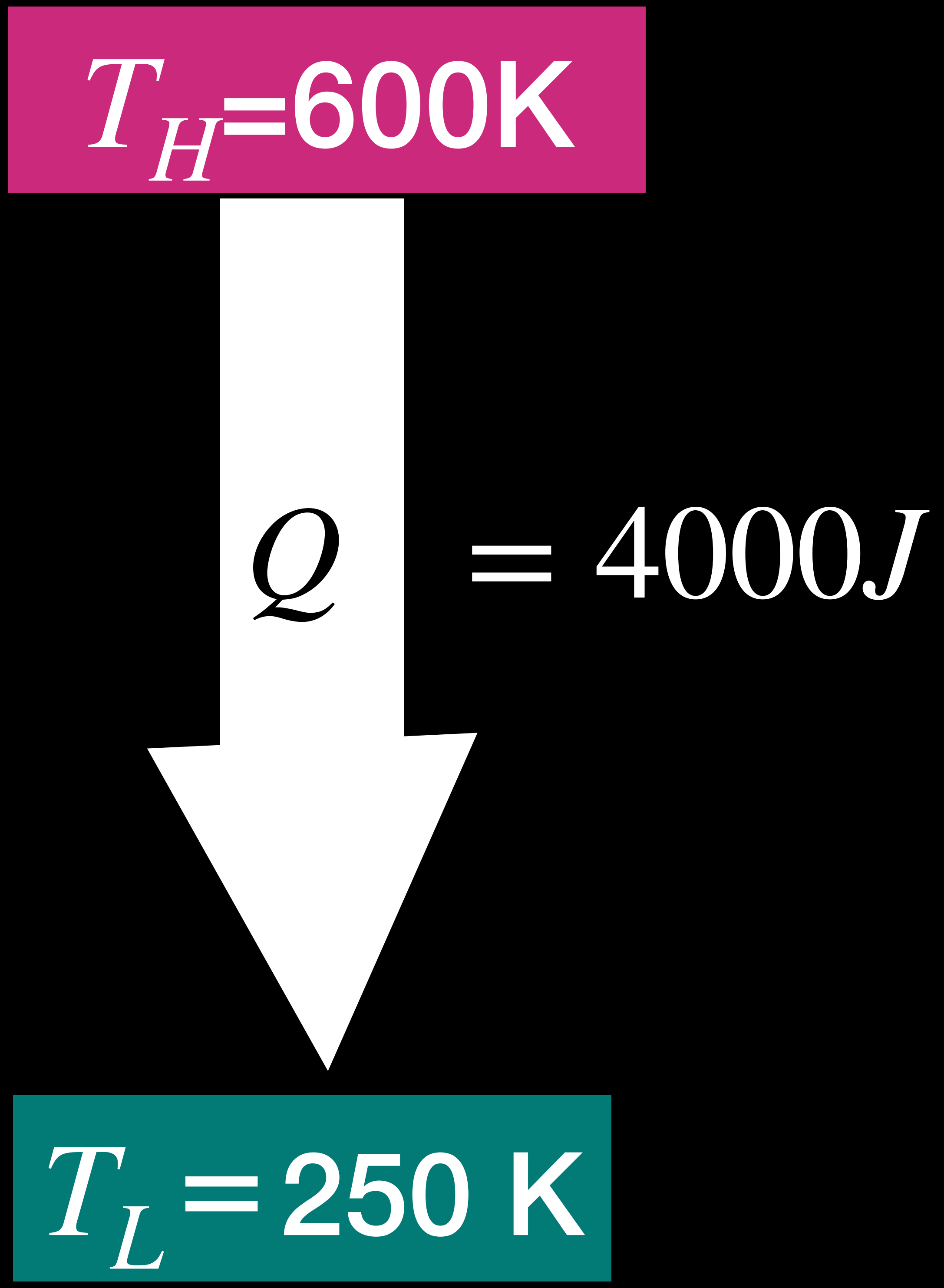
$$\frac{V_2}{V_1} = \frac{V_3}{V_4}$$

$$W_{total} = NK(T_h - T_c) \ln \frac{V_2}{V_1} = Q_1 + Q_3$$

$$\eta = \frac{W_{total}}{Q_1} = \frac{Q_1 + Q_3}{Q_1} = 1 - \frac{T_c}{T_h}$$

Lecture VII : example:

Calculate the total change in entropy if 4000 J of heat transfer occurs from a hot reservoir at Th = 600 K to a cold reservoir at Tc = 250 K , assuming there is no temperature change in either reservoir.



the heat transfer from the hot reservoir,

$$\Delta S_H = \frac{4000J}{600K} = -6.67 \ J/K$$

the heat transfer from the hot reservoir,

$$\Delta S_C = \frac{4000J}{250K} = 16 \ J/K$$

$$\Delta S_{total} = \Delta S_H + \Delta S_C = -6.67 \ J/K + 16 \ J/K = 9.33 J/K$$

example: $\eta = \dfrac{W_{total}}{Q_1} = 1 - \dfrac{T_c}{T_h}$

(a) Calculate the work output of a Carnot engine operating between temperatures of 600 K and 100 K for 4000 J of heat transfer to the engine. (b) Now suppose that the 4000 J of heat transfer occurs first from the 600 K reservoir to a 250 K reservoir (without doing any work, and this produces the increase in entropy calculated above) before transferring into a Carnot engine operating between 250 K and 100 K. What work output is produced?

(a) $$\eta = 1 - \frac{100\ K}{600\ K} = 0.833$$

$$W_{total} = \eta Q_H = 0.833 \times 4000\ J = 3333J$$

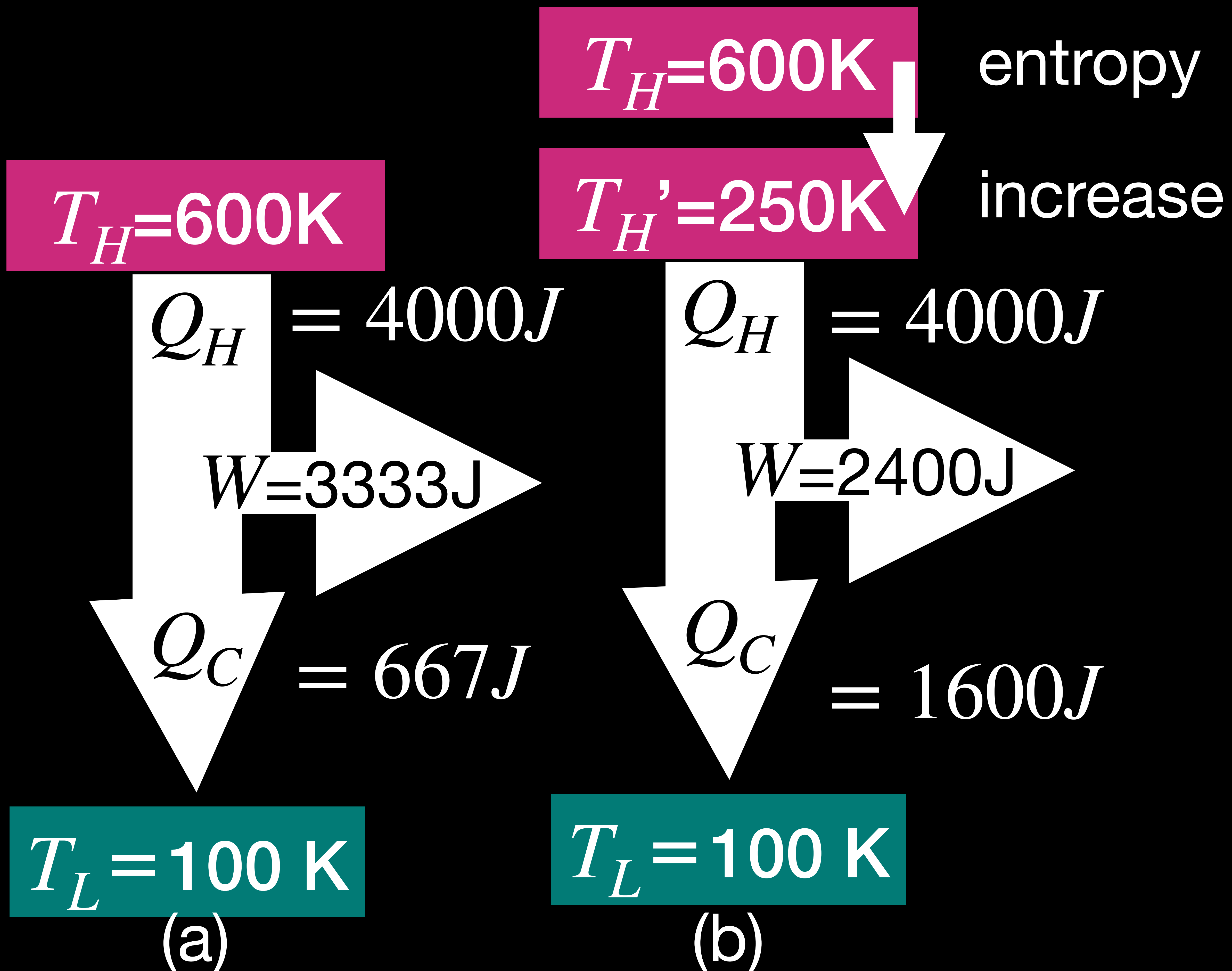
(b) $$\eta' = 1 - \frac{100\ K}{250\ K} = 0.6$$

$$W'_{total} = \eta' Q_H = 0.6 \times 4000\ J = 2400J$$

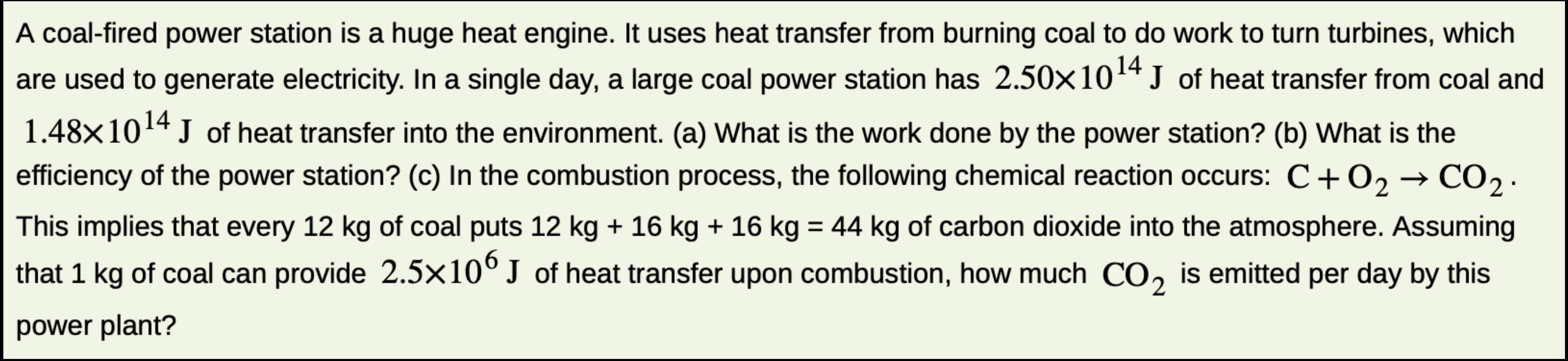
$$W_{unavail} = W_{total} - W'_{total} = 933\ J$$

$$W_{unavail} = \Delta S / T_C = \Delta S / 100\ K = 933\ J$$

$$\Delta S = 9.33\ J$$



Lecture VII : example: coal-fired power station



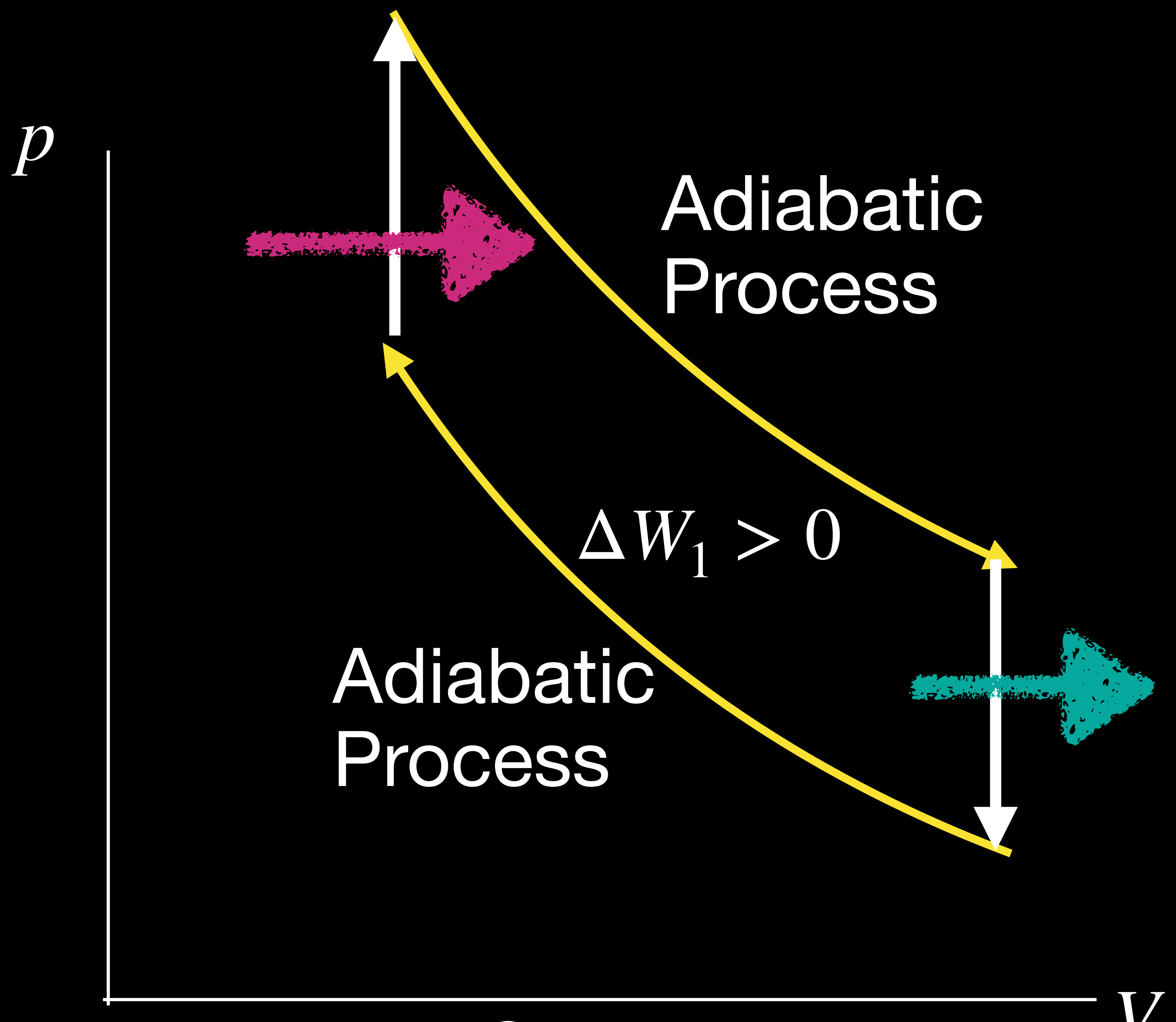
A coal-fired power station is a huge heat engine. It uses heat transfer from burning coal to do work to turn turbines, which are used to generate electricity. In a single day, a large coal power station has $2.50\times10^{14}\ \mathrm{J}$ of heat transfer from coal and $1.48\times10^{14}\ \mathrm{J}$ of heat transfer into the environment. (a) What is the work done by the power station? (b) What is the efficiency of the power station? (c) In the combustion process, the following chemical reaction occurs: $\mathrm{C}+\mathrm{O}_2\rightarrow\mathrm{CO}_2$. This implies that every 12 kg of coal puts 12 kg + 16 kg + 16 kg = 44 kg of carbon dioxide into the atmosphere. Assuming that 1 kg of coal can provide $2.5\times10^{6}\ \mathrm{J}$ of heat transfer upon combustion, how much CO_2 is emitted per day by this power plant?

$$W = Q_H - Q_L = 2.50\times10^{14}\ J - 1.48\times10^{14}\ J = 1.02\times10^{14}\ J$$

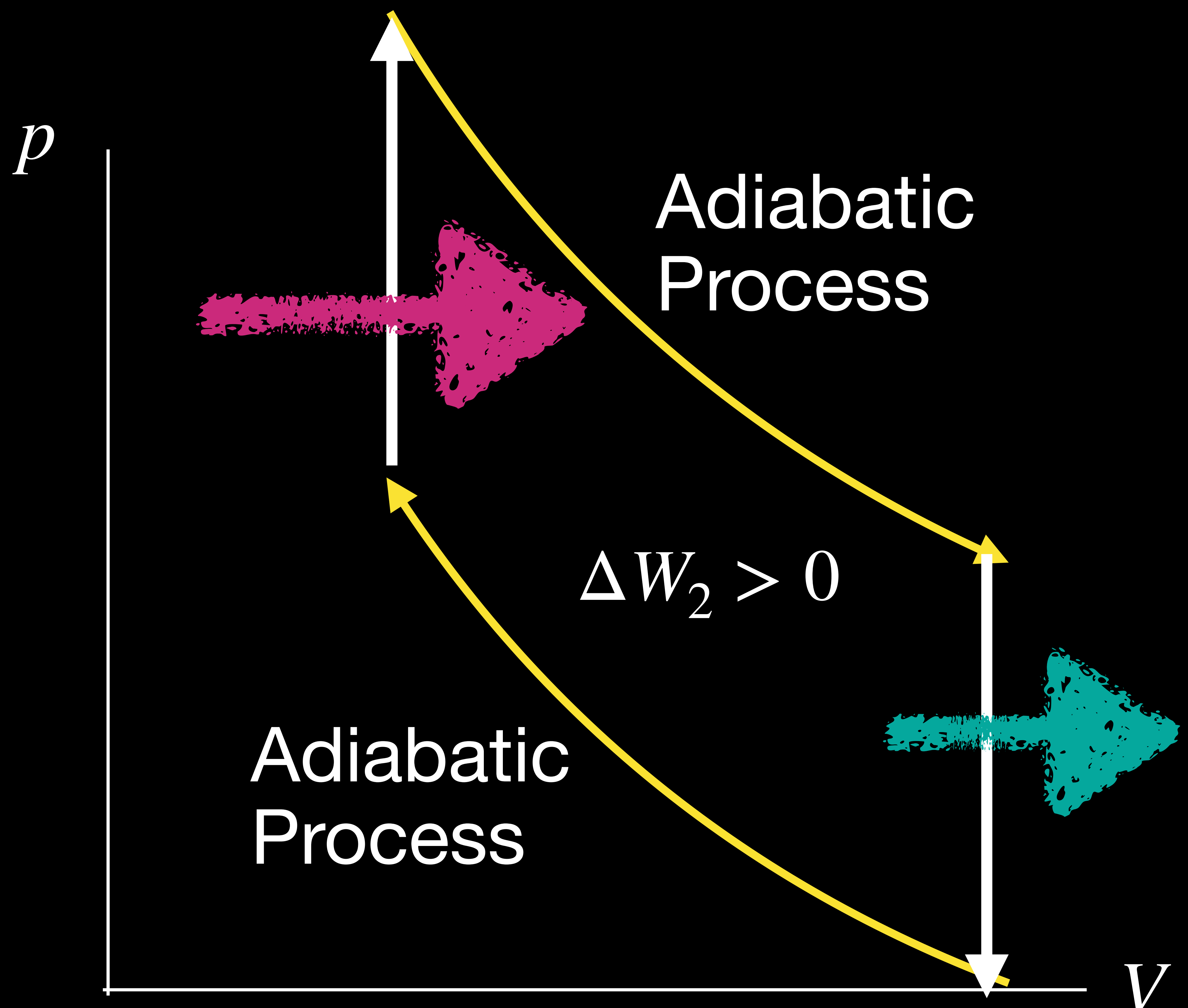
$$eff = \frac{W}{Q_H} = (1.02\times10^{14}\ J)/2.50\times10^{14}\ J \sim 0.408$$

$$\frac{2.50\times10^{14}\ J}{2.5\times10^{6}\ J}\ coal\frac{44g\ CO_2}{12g\ Coal} = \ 3.7\times10^{8}\ kg\ coal$$

Lecture VII : Otto's cycle



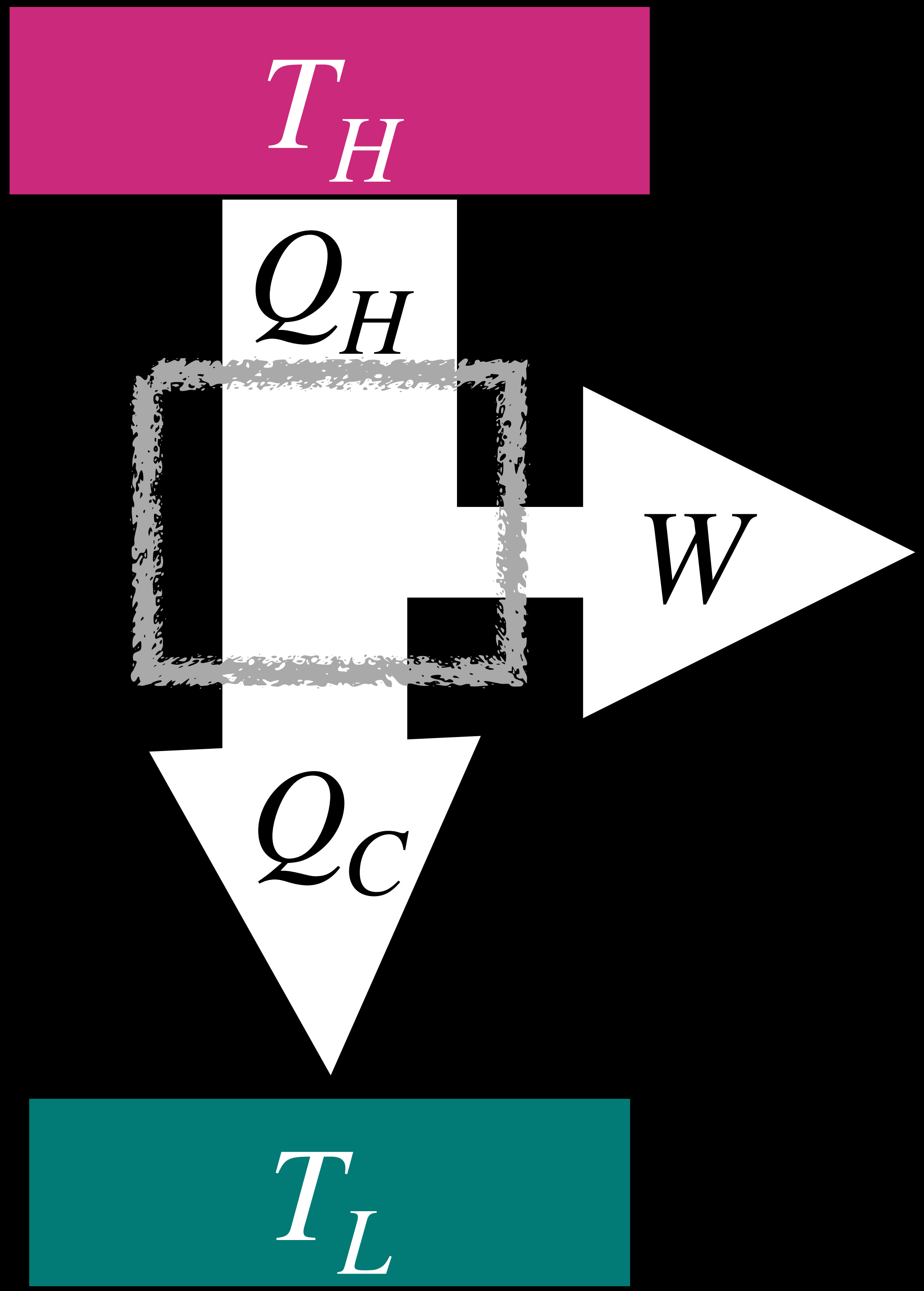
Case 1



Case 2

$$\Delta W_2 > \Delta W_1$$

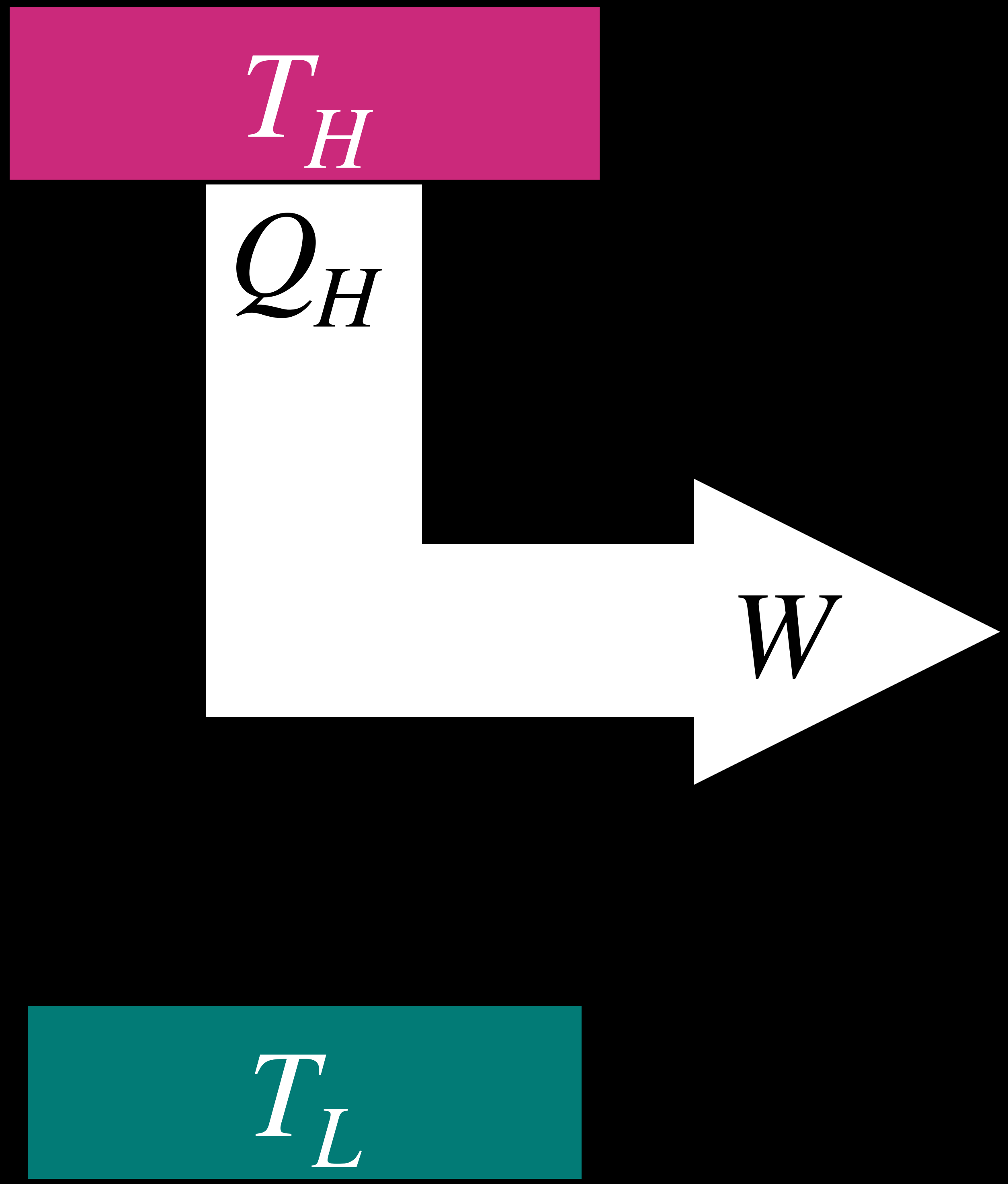
Lecture VII : heat engine



All real heat engines lose some heat to the environment

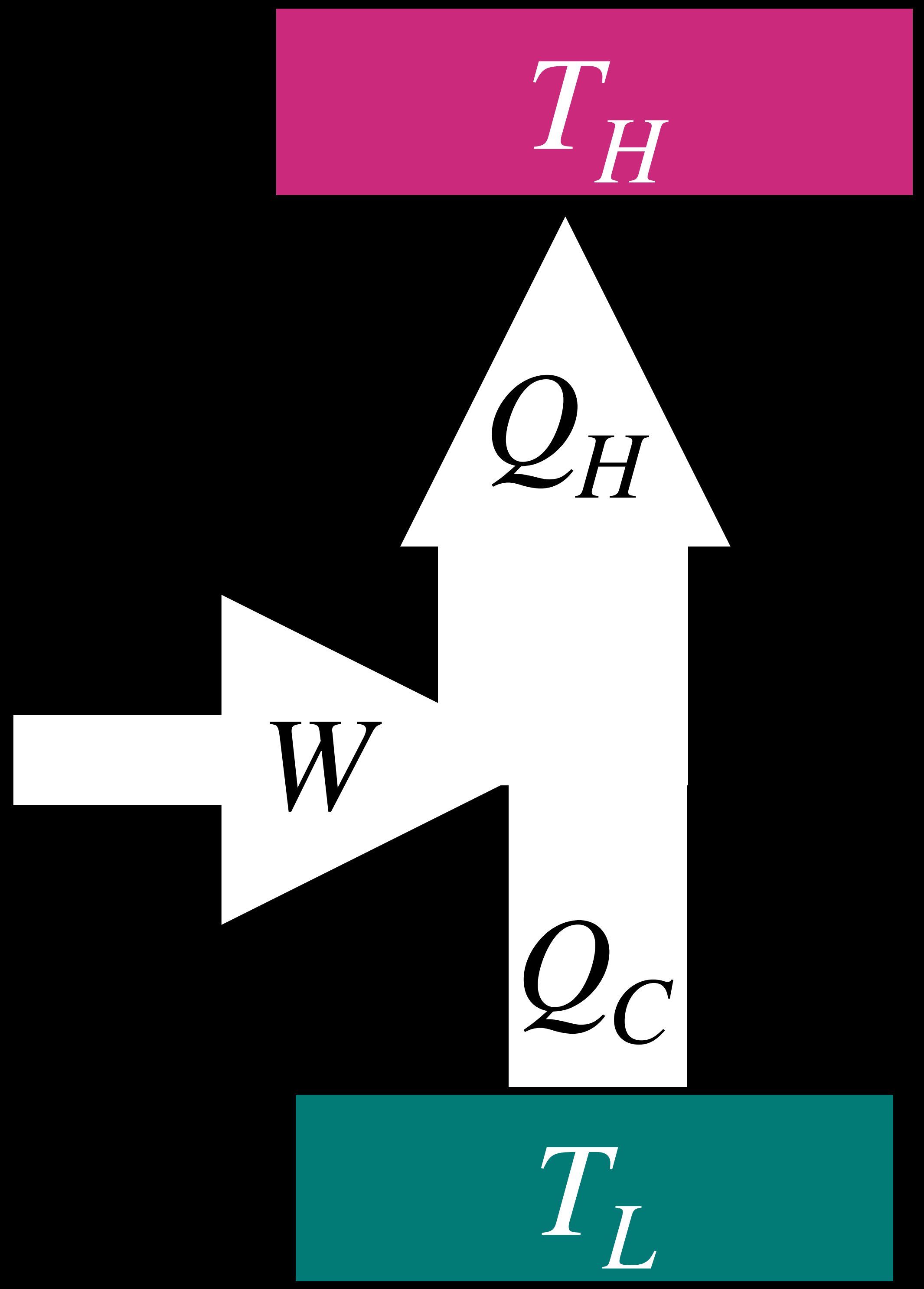
$$\text{eff} = \frac{W}{Q_H}$$

maximum for the Carnot's cycle

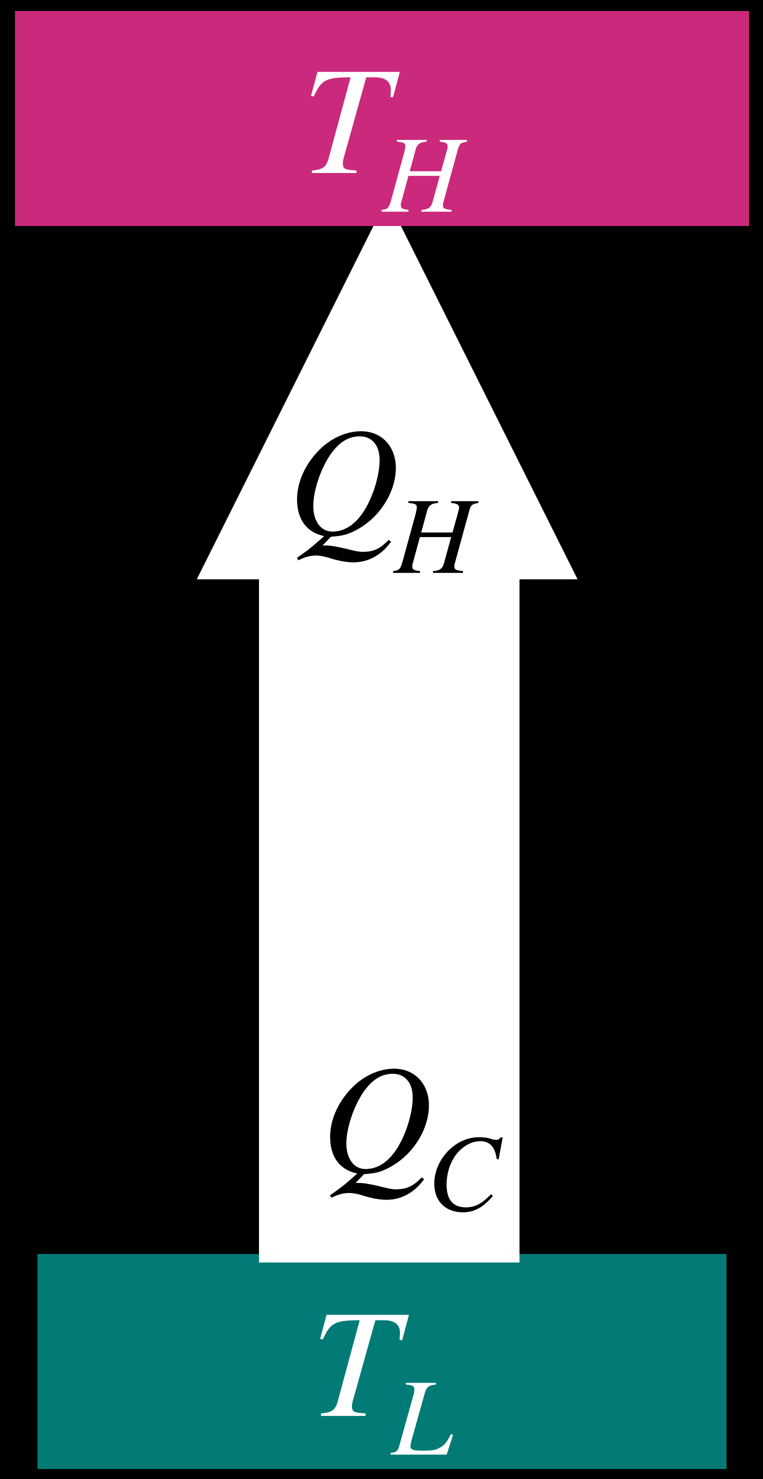


Extracting heat Q_H and using it all to do work W would constitute a perfect heat engine. Forbidden by the second law

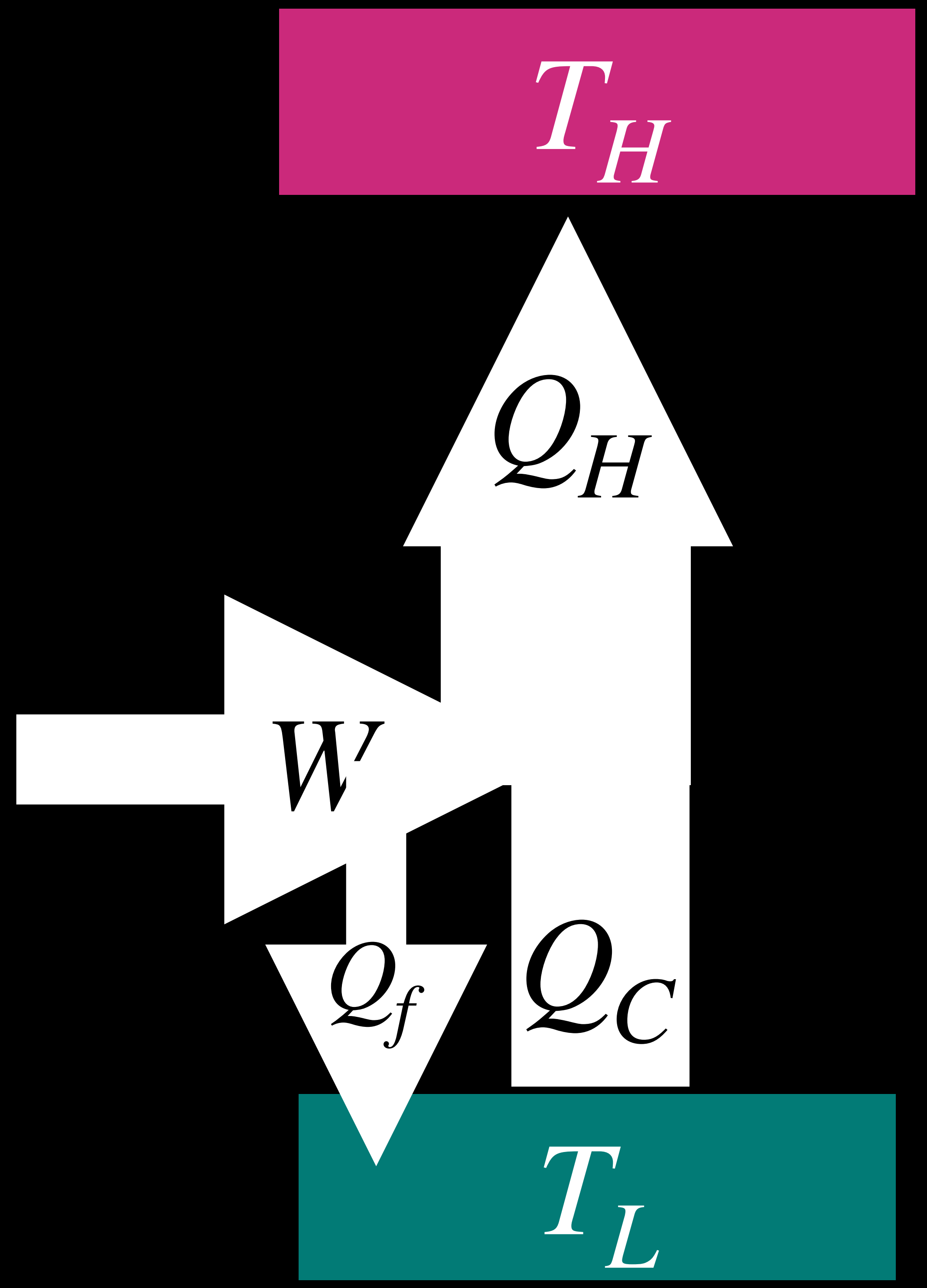
Lecture VII : Refrigerator



All real refrigerators require work to get heat to flow from a cold area to a warmer area.

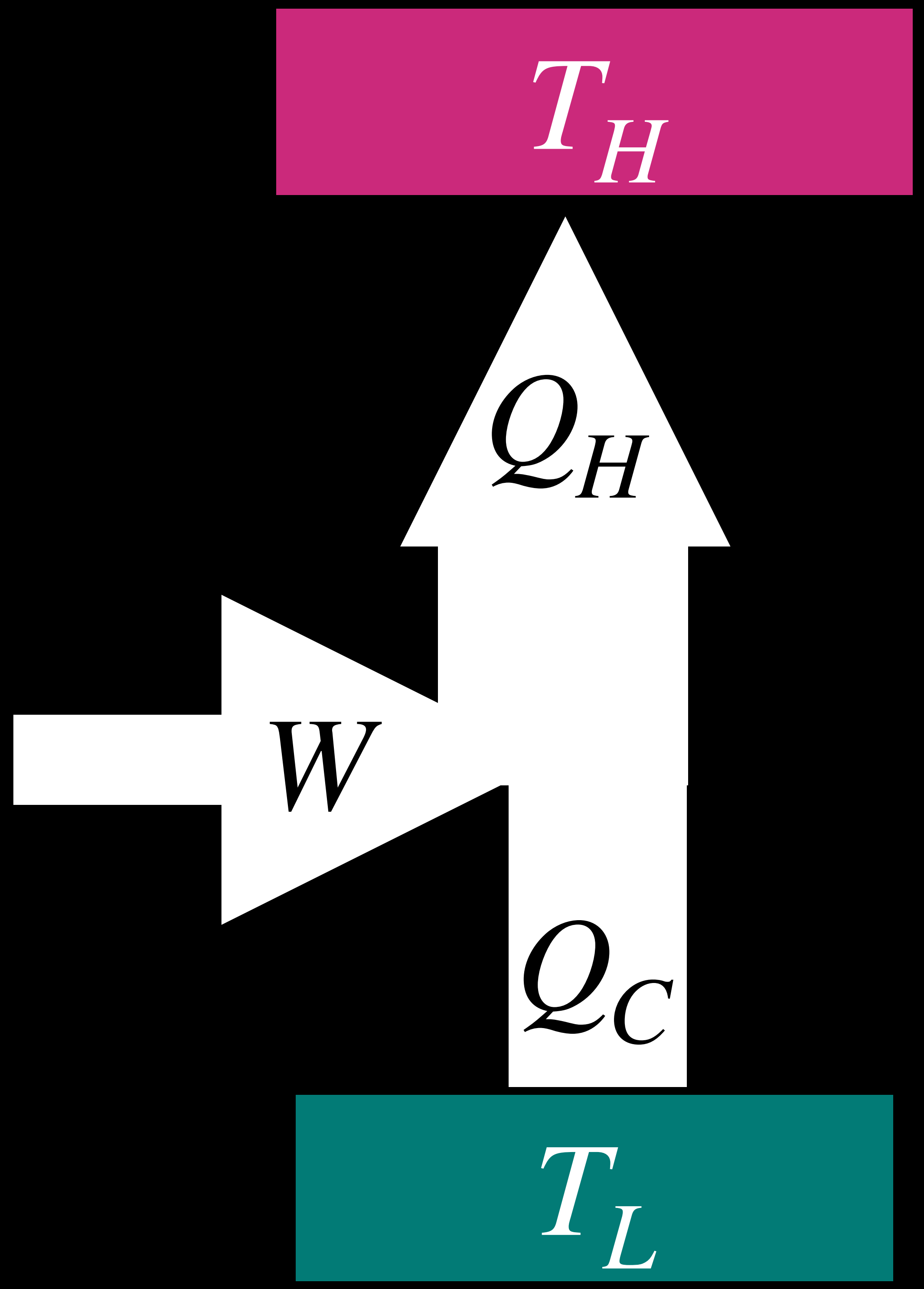


Spontaneous flow of heat from a cold area to a hot area would constitute a perfect refrigerator forbidden by second law



Real case: work done against friction is lost to cold reservoir

Lecture VII : Refrigerator



$$K = \frac{|Q_L|}{|W|}$$, Coefficient of refrigerator performance

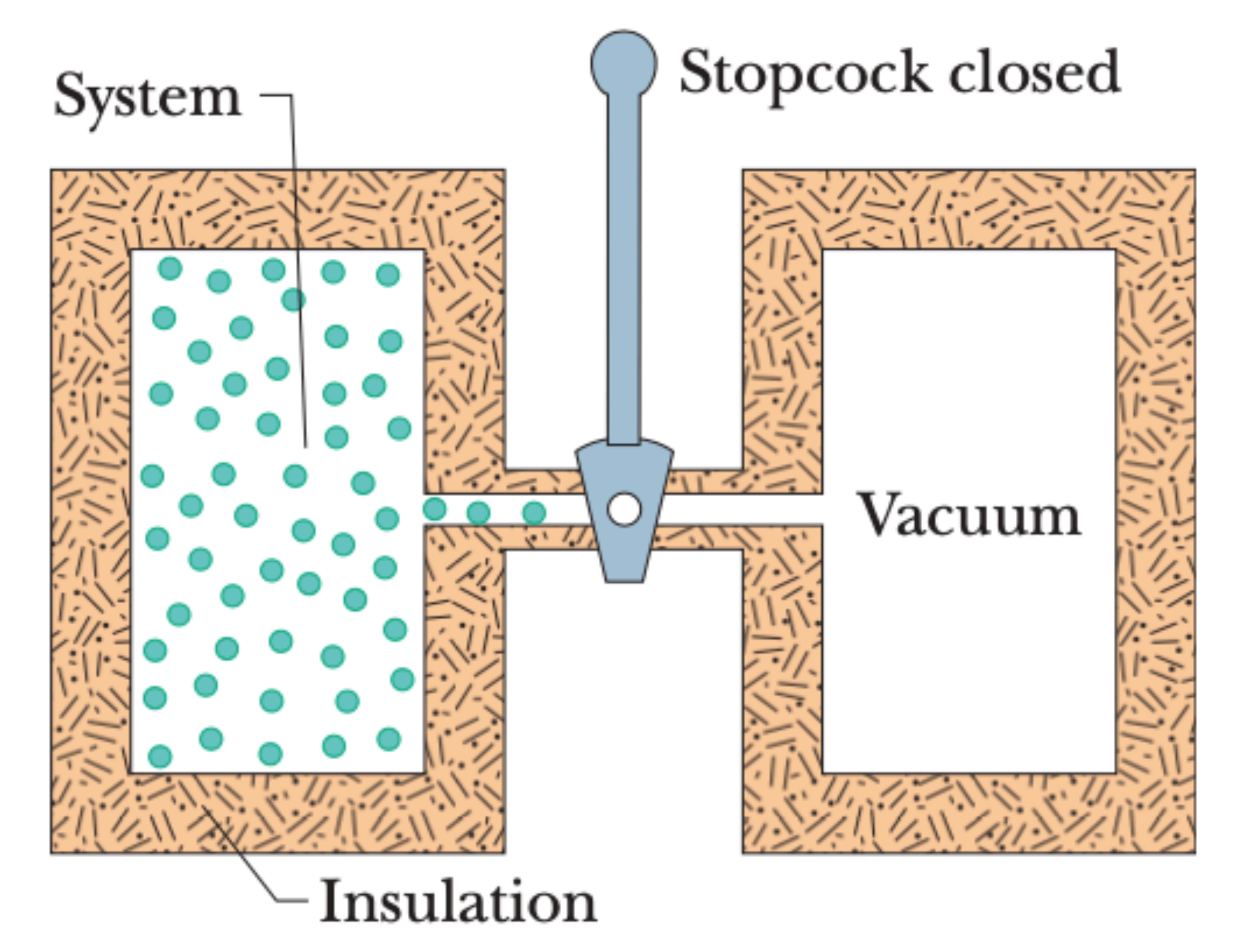
Carnot refrigerator is a Carnot engine operating in reverse.

$$K_C = \frac{|Q_L|}{|Q_H| - |Q_L|}$$

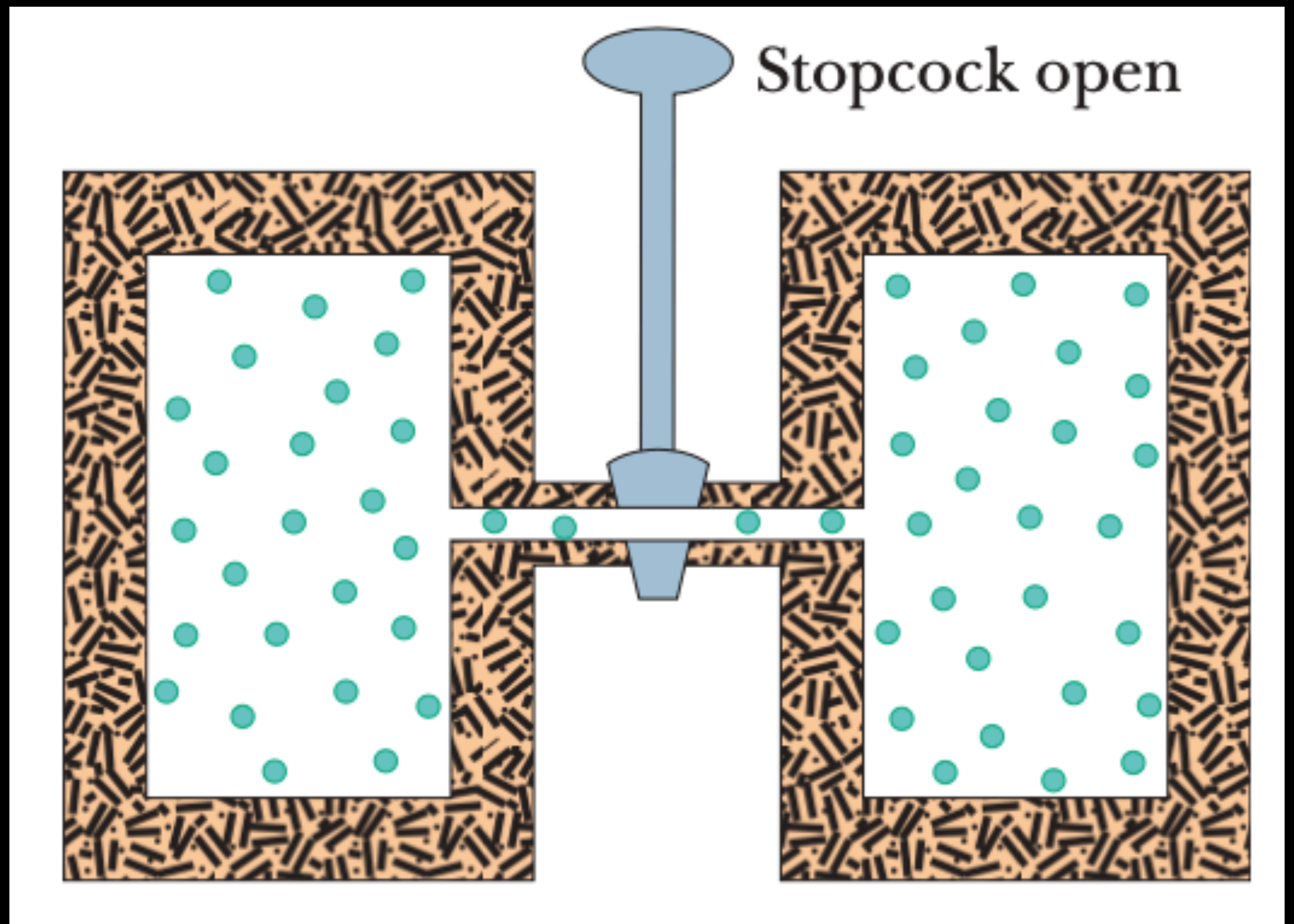
All real refrigerators require work to get heat to flow from a cold area to a warmer area.

Lecture VII : Entropy(熵ㄕㄤ)

If an irreversible process occurs in a closed system, the entropy **S** of the system always increases; it never decreases.



(*a*) Initial state *i*

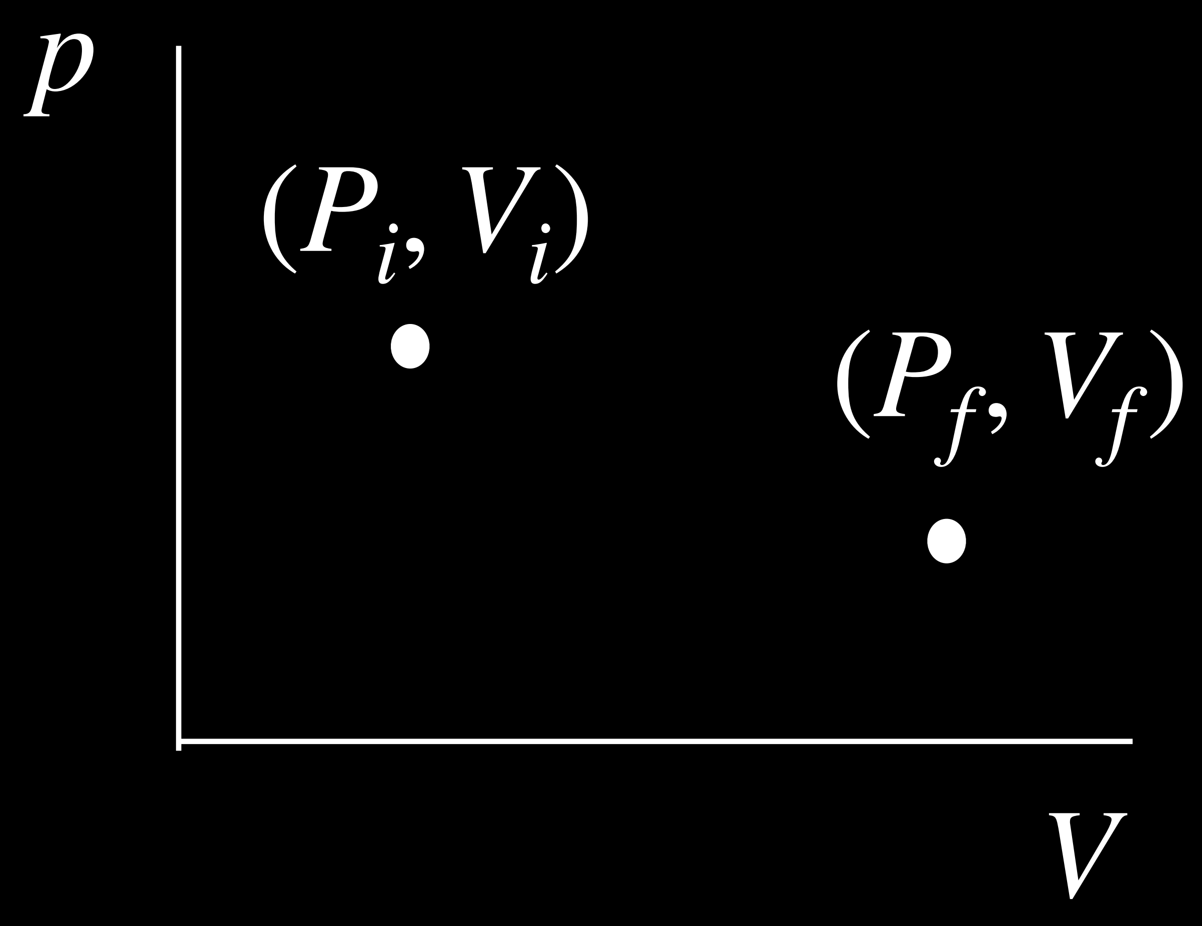


(*b*) Final state *f*

Lecture VII : Entropy(熵 ㄕㄤ)

If an irreversible process occurs in a closed system, the entropy **S** of the system always increases; it never decreases.

Pressure and volume are state properties, properties that depend only on the state of the gas and not on how it reached that state. Other state properties are temperature and energy. We now assume that the gas has still another state property—its entropy.

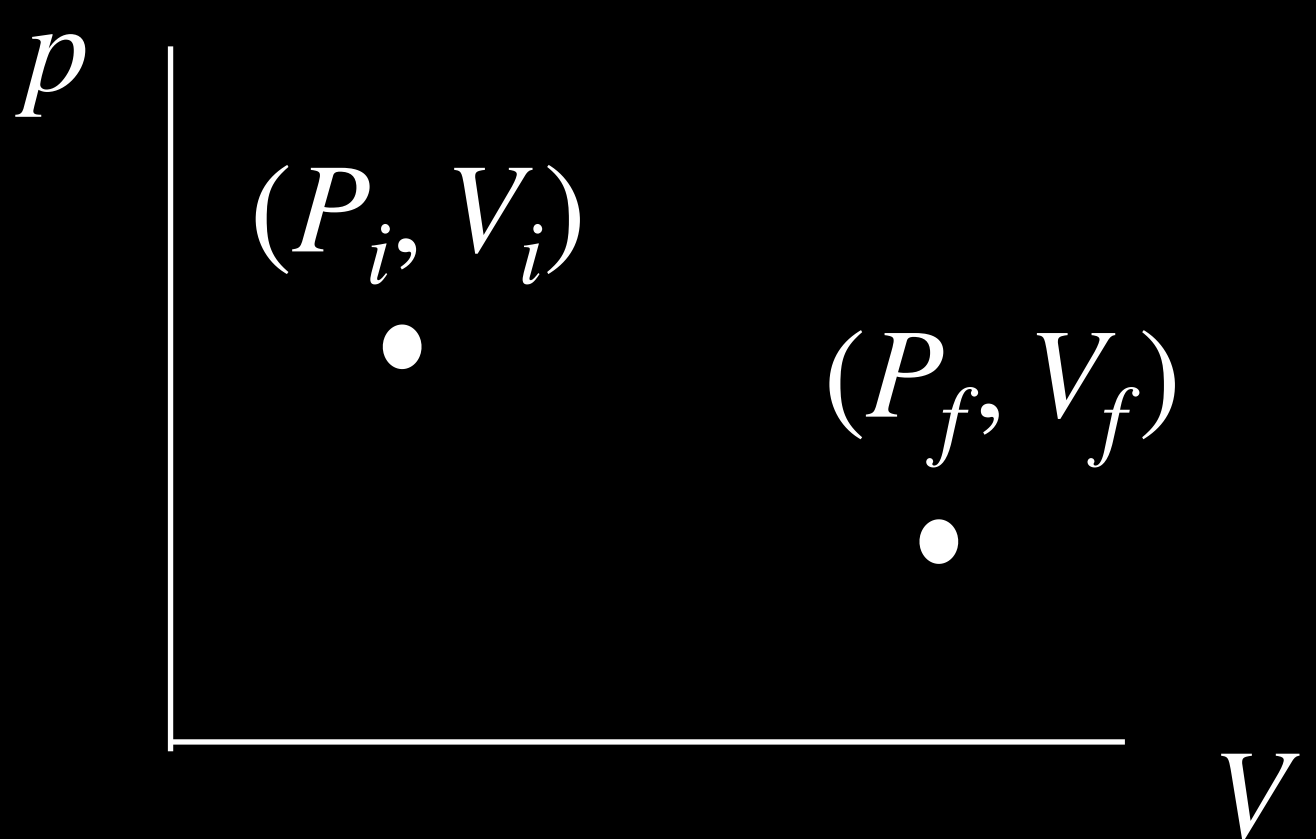


Lecture VII : Entropy(熵 ㄕ ㄤ)

The change of the entropy dS: entry of a system during a process that takes the system from a initial state I to finial state f

As the gas rushes to fill the entire container, the pressure, temperature, and volume of the gas fluctuate unpredictably. In other words, they do not have a sequence of well-defined equilibrium values during the intermediate stages of the change from initial state i to final state f. Thus, we cannot trace a pV path for the free expansion on the p-V plot(right plot), and we cannot find a relation between Q and T that allows us to integrate as Eq. (1) requires.

$$\Delta S = S_f - S_i = \int_i^f \frac{dQ}{T} \quad (1)$$



Lecture VII : Entropy

$$\Delta S = S_f - S_i = \int_i^f \frac{dQ}{T}$$

$$\int_i^f dQ = Q$$

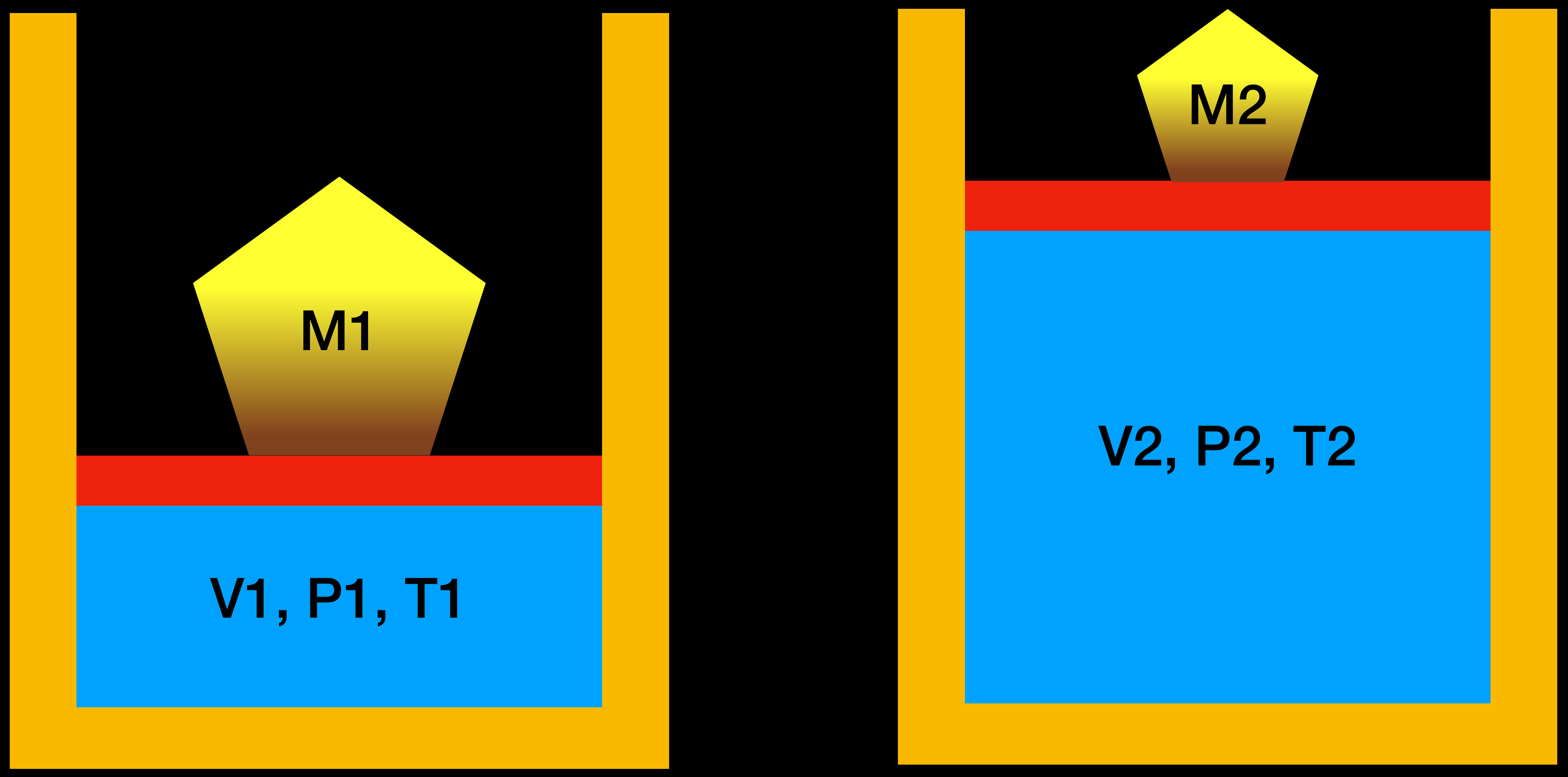
where Q is the total energy transferred as heat during the process

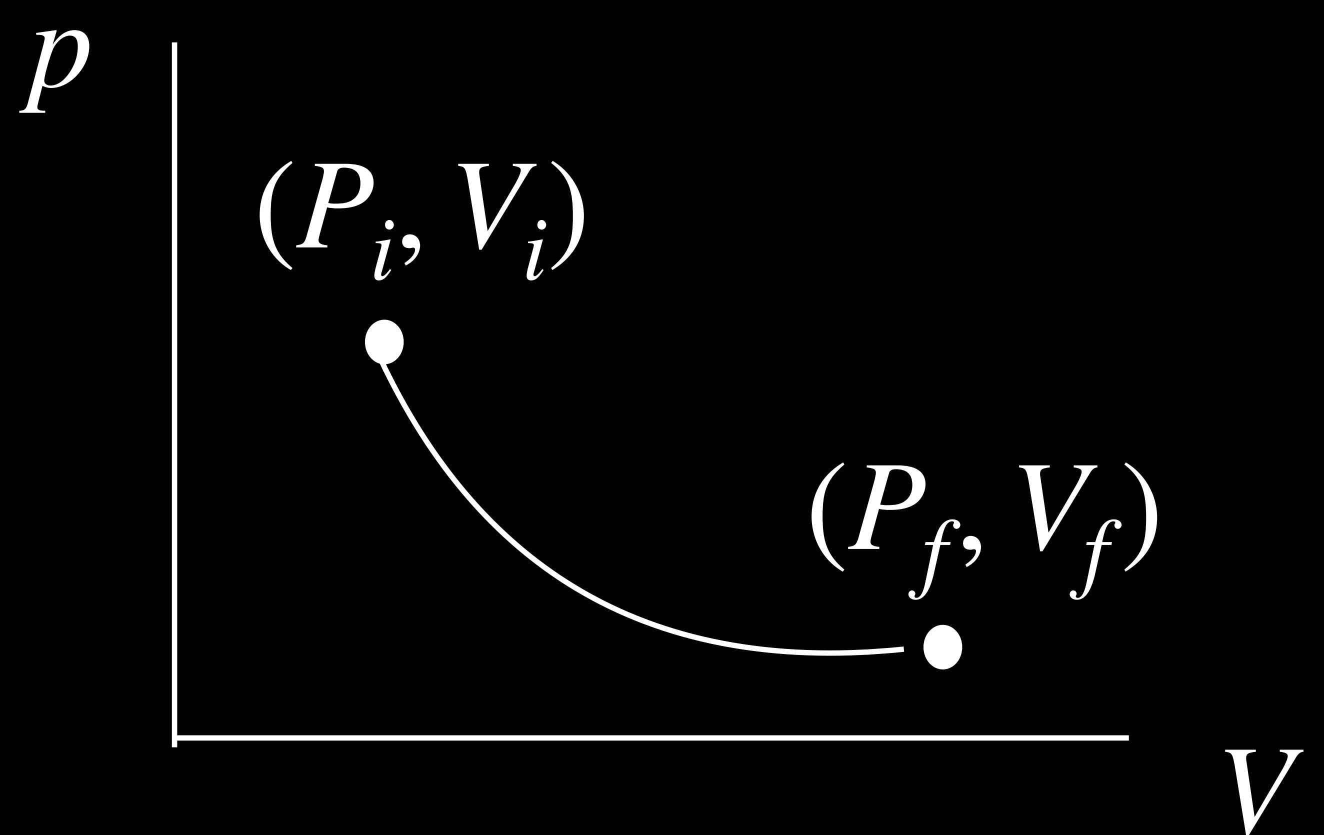
$$\Delta S = S_f - S_i = \frac{Q}{T}$$

To find the entropy change for an irreversible process, replace that process with any reversible process that connects the same initial and final states. Calculate the entropy change for this reversible process

Adiabatic process

$$p_i V_i^{\gamma} = p_f V_f^{\gamma}$$





Lecture VII : The second law of thermodynamics

Clausius statement

Heat can never pass from a colder to a warmer body without some other change, connected therewith, occurring at the same time.

不可能把熱量從低溫物體傳遞到高溫物體而不產生其他影響

Kelvin statement

It is impossible for a self-acting machine, unaided by any external agency, to convey heat from one body to another at a higher temperature. It is impossible, by means of inanimate material agency, to derive mechanical effect from any portion of matter by cooling it below the temperature of the coldest of the surrounding objects.

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The entropy of any isolated system increases in any spontaneous process and remains unchanged in an equilibrium (reversible) process.

孤立熱力學系統從一個平衡態到另一平衡態的過程中，其熵永不減少：若過程可逆，則熵不變；若不可逆，則熵增加。

Lecture VII : The second law of thermodynamics

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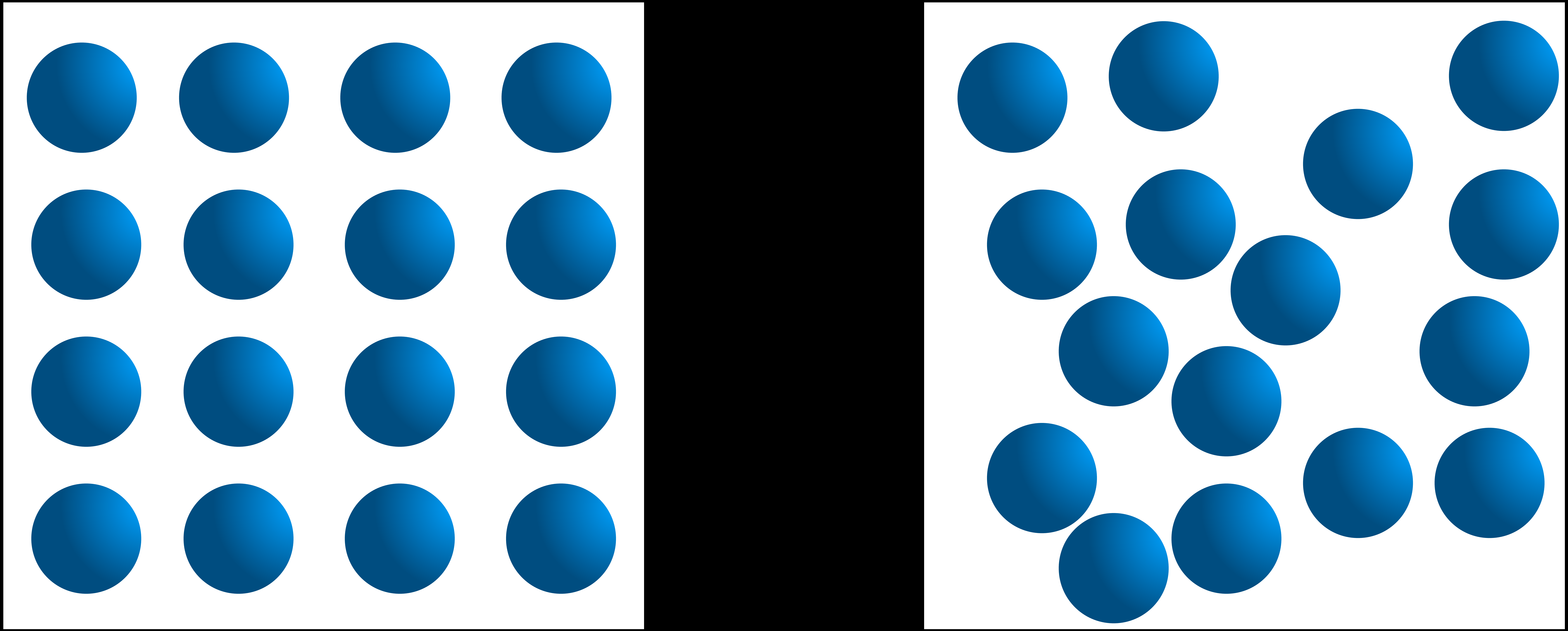
For isolated system in equilibrium it holds that

$$dS = 0, \quad S = S_{max}$$

For irreversible process it holds that

$$dS > 0$$

Lecture VII : Order and Disorder

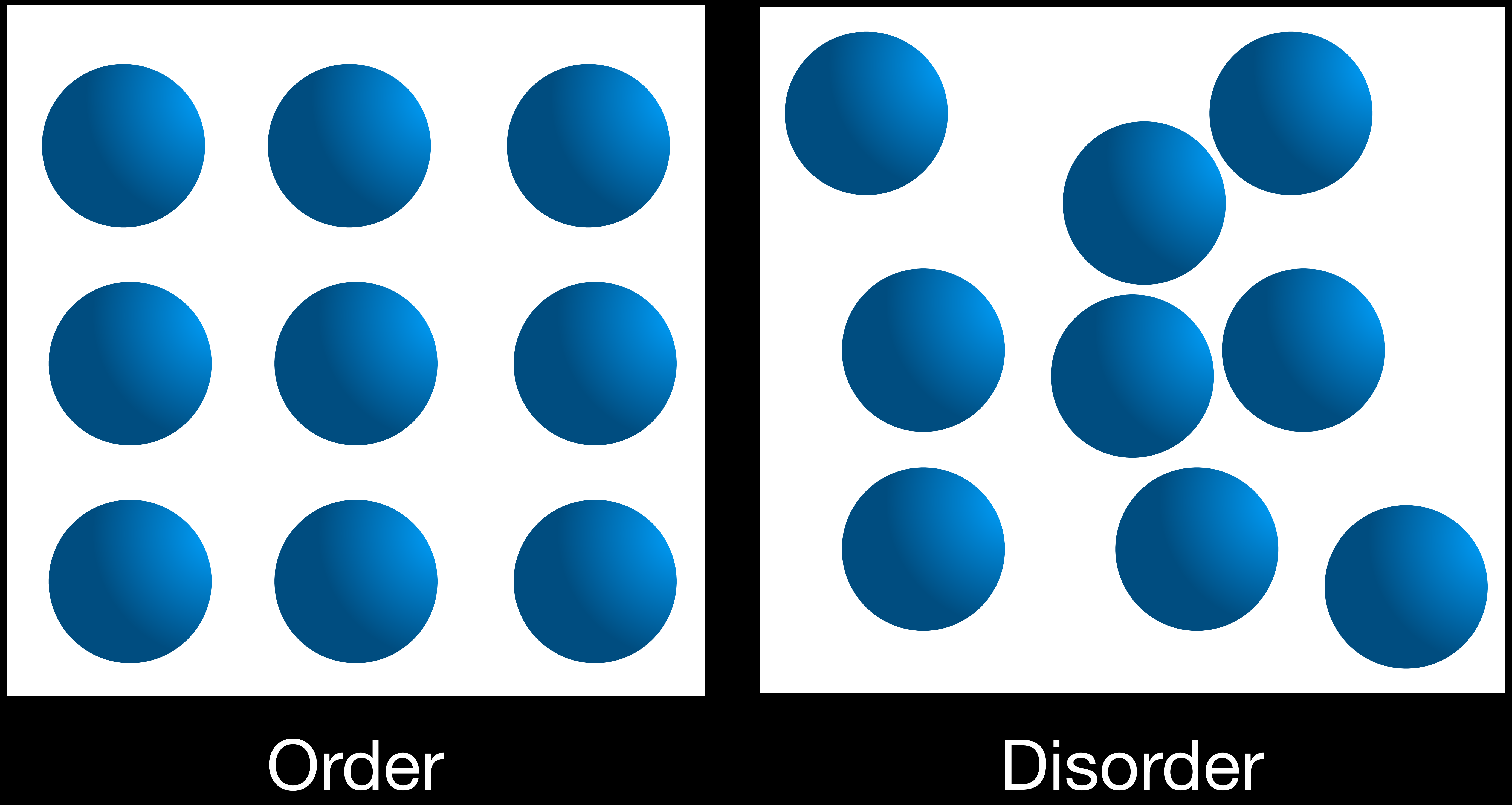


Which one has bigger entropy?

Lecture VII : example $\Delta S \equiv \frac{Q}{T}$

Find the increase in entropy of 1.00 kg of ice originally at 273.15 K that is melted to form water at 273.15 K .

latent heat of fusion= 334 kJ/kg



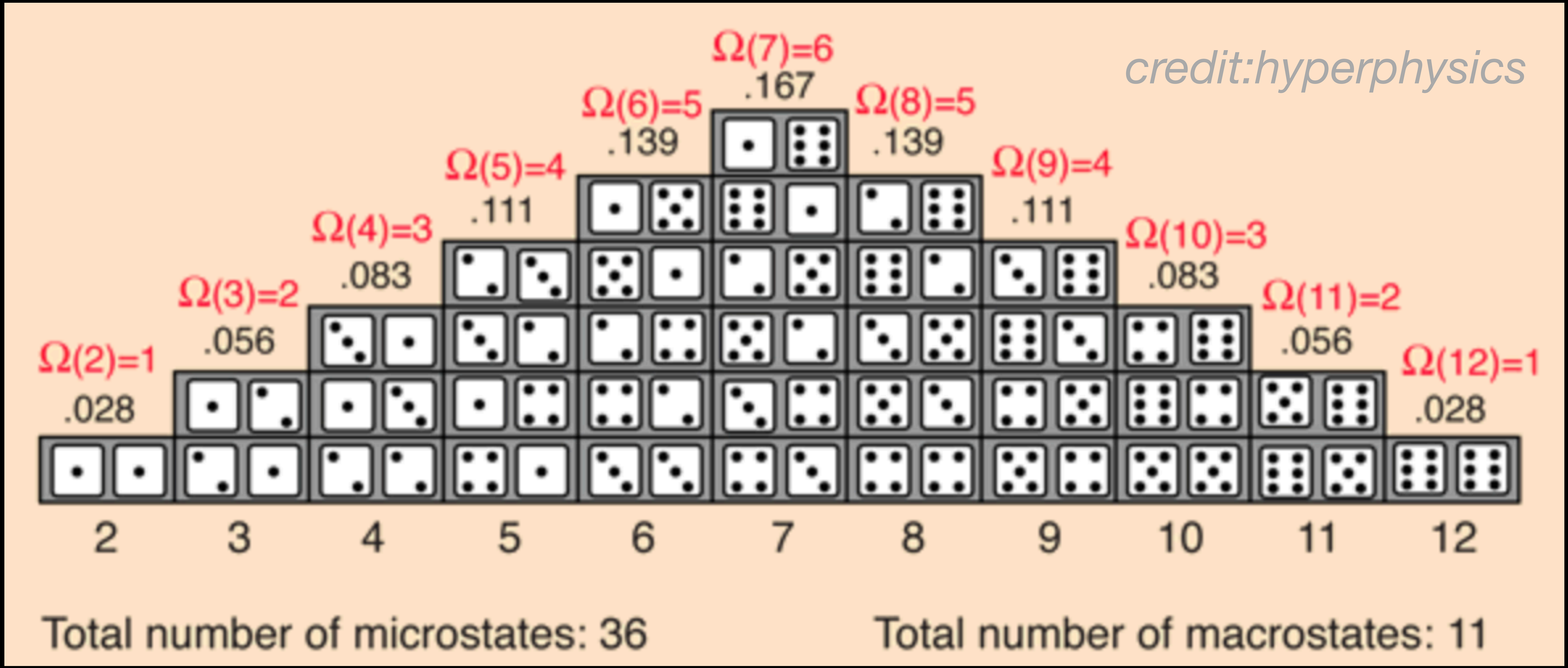
$$\Delta S \equiv \frac{Q}{T}$$

$$= \frac{1\ kg\ 334\ kJ/kg}{T}$$

$$= \frac{334\ kJ}{273.16K}$$

$$= 1.22 \times 10^3 \text{ J/K}$$

Lecture VII : Entropy Summary



credit:hyperphysics

Total number of microstates: 36

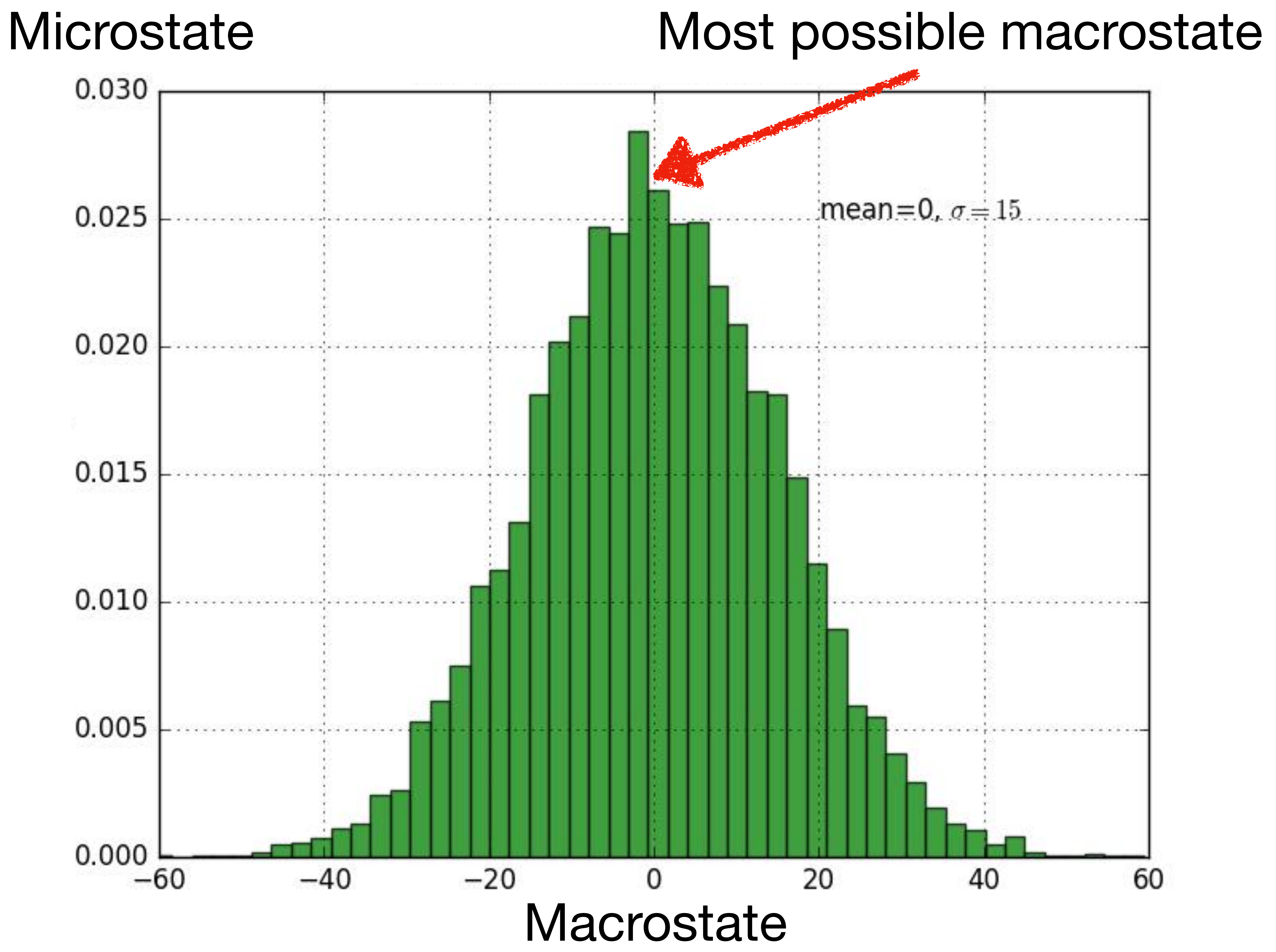
Total number of macrostates: 11

MultiPlicity = Ω

Entropy $= S = k \ln \Omega$

K: Boltzmann’s constant

Lecture VII : Normal distribution (bell curve)



MultiPlicity = Ω

Ω:

the number of possible microstates available to a system in a given macrostate

$$S = k \ln \Omega$$

系統微觀粒子的無序程度的度量

example: toss coins

$$S = k \ln \Omega$$

Suppose you toss 100 coins starting with 60 heads and 40 tails, and you get the most likely result, 50 heads and 50 tails. What is the change in entropy?

$$\Delta S = S\mathrm{f} - S\mathrm{i} = k\ln\Omega_f - k\ln\Omega_i$$

$$= 1.38 \times 10^{-23}\ J/K\ (\ln 10^{29} - 1.4 \times \ln 10^{28})$$

$$= 2.7 \times 10^{-23}\ J/K$$

Heads	Tails	N of microstates
100	0	1
99	1	100
95	5	7.5*10^7
90	10	1.7*10^13
75	25	2.4*10^23
60	40	1.4*10^28
55	45	6.1*10^28
51	49	9.9*10^28
50	50	10^29

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孤立系統的自發過程總是從熱力學機率小的宏觀狀態向熱力學機率大的宏觀狀態轉變

Lecture VII : Entropy Summary

- a state variable whose change is defined for a reversible process at T where Q is the heat absorbed.
- a measure of the amount of energy which is unavailable to do work.
- a measure of the disorder of a system.
- a measure of the multiplicity of a system.