General Physics I

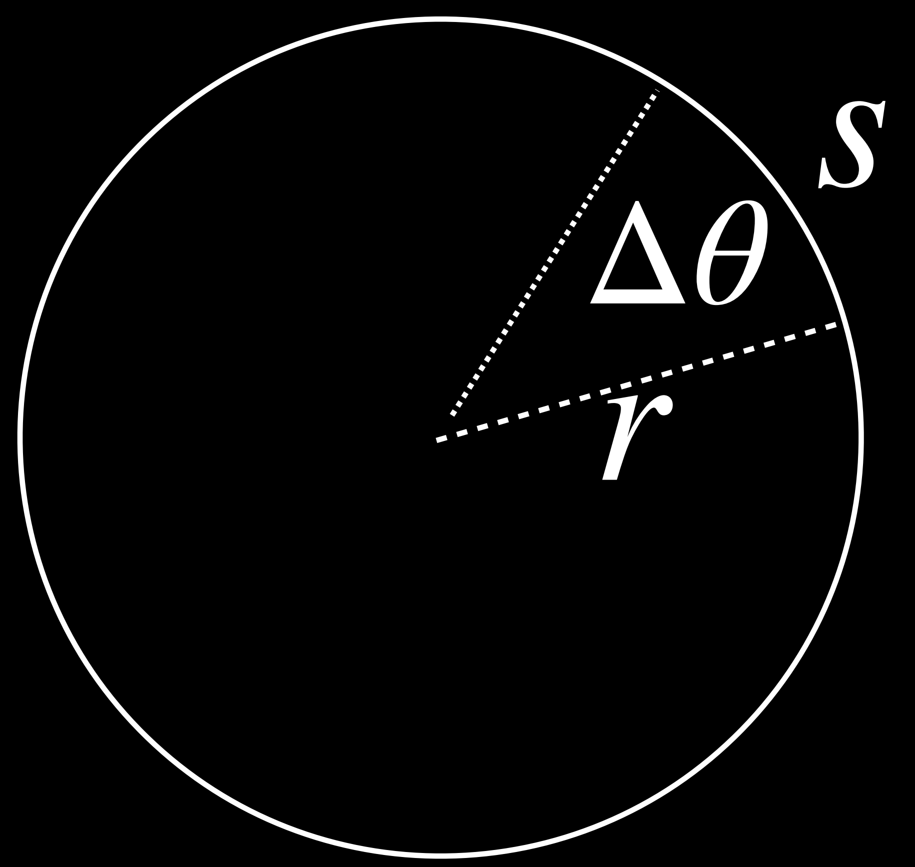
Mechanics, optics, thermal dynamics, and other basic fundamental things.

TsungChe Liu

Lecture V : Rotation

	Position	Displacement	Velocity	Acceleration
Linear	x	Δx	$\vec{v}$	$\vec{a}$
Angular	θ	$\Delta\theta$	ω	α

Relating the linear and angular variables



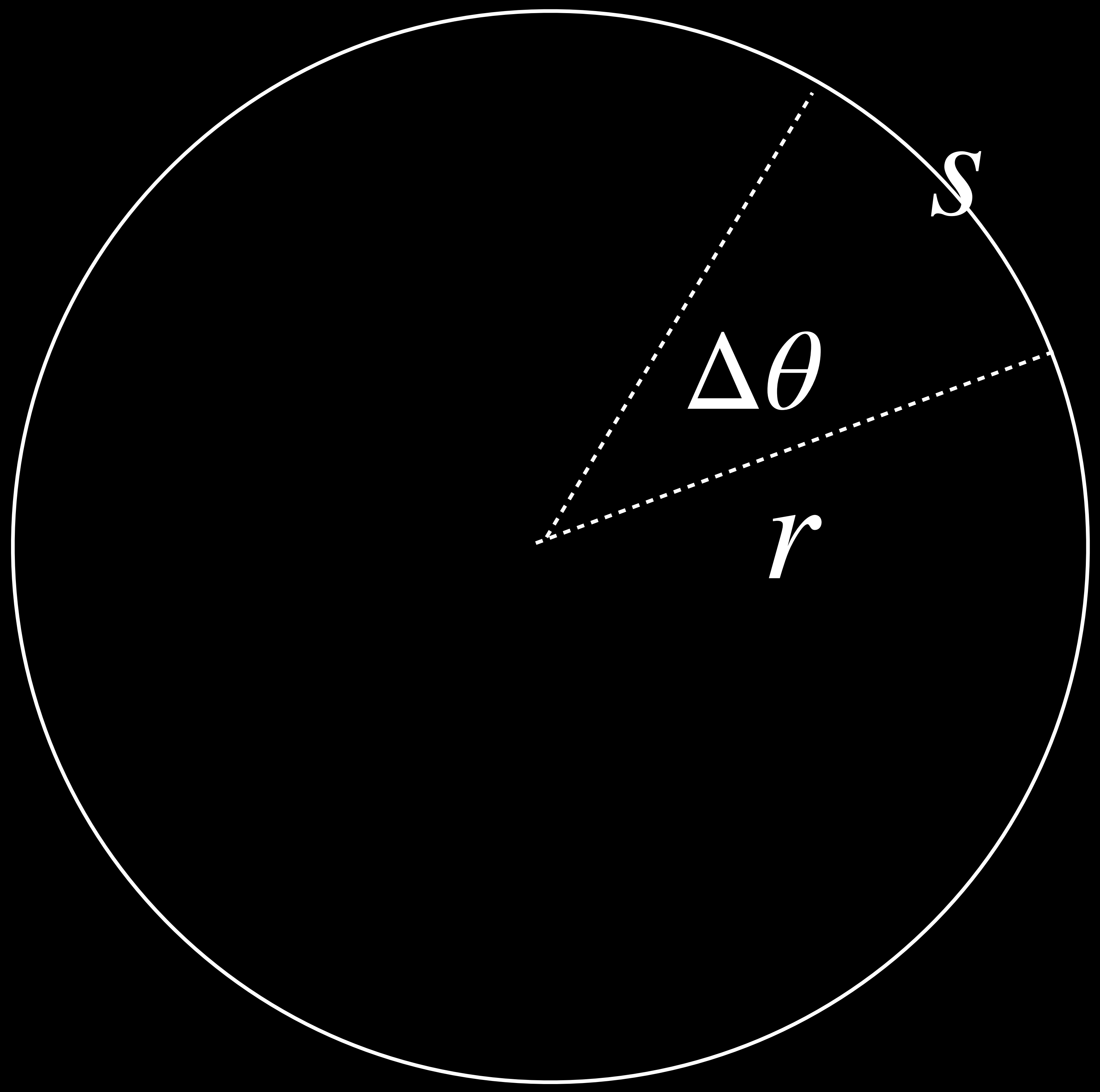
$$s = \Delta\theta r$$

$$\frac{ds}{dt} = \frac{d\theta}{dt}r \qquad \frac{dv}{dt} = \frac{d\omega}{dt}r$$

$$v = \omega r \qquad a = \alpha r$$

Lecture V : Rotation

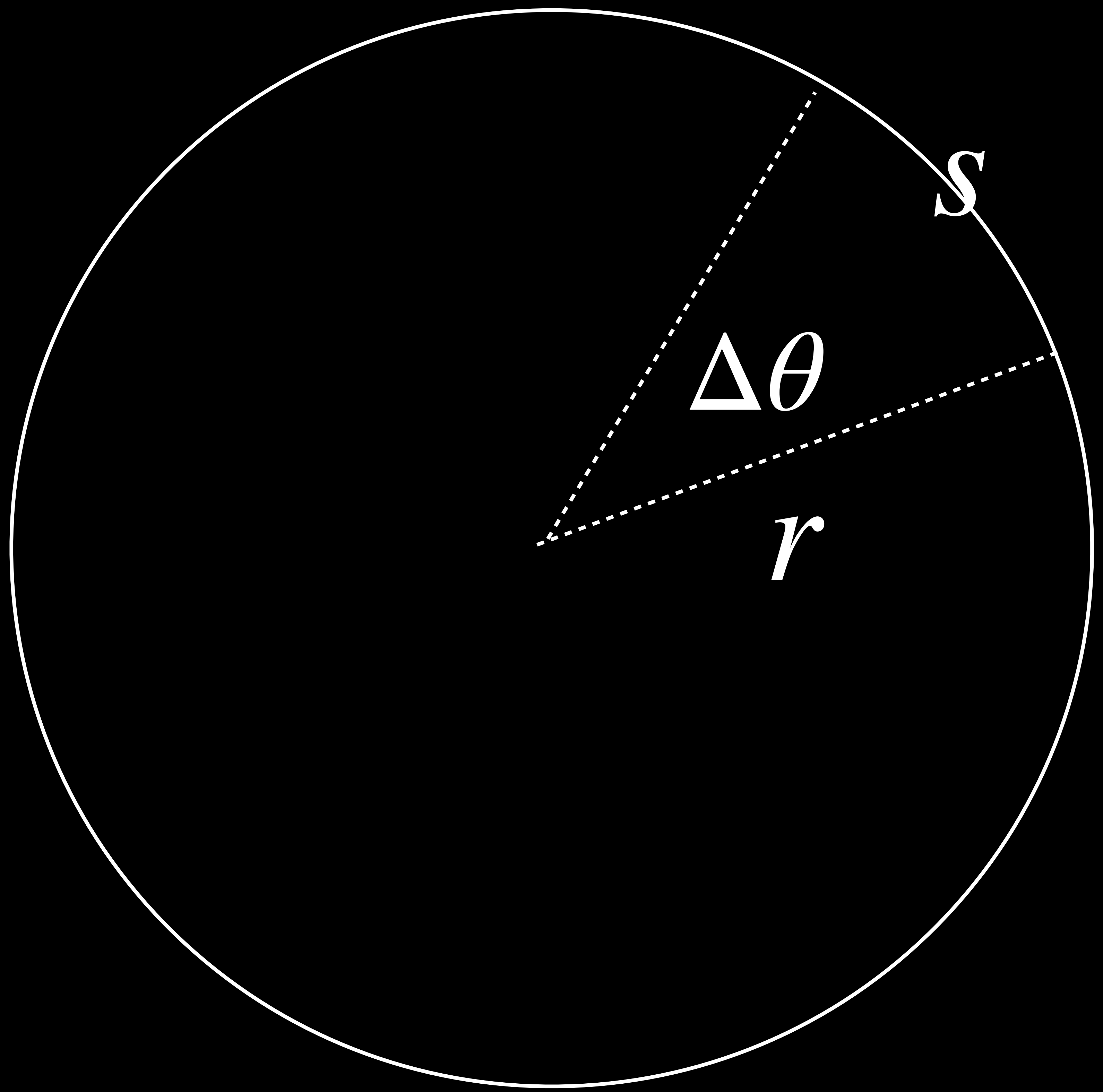
The period of revolution T : considering the $s = 2\pi r$



$$T = \frac{2\pi r}{v} = \frac{2\pi r}{\omega r}$$

$$= \frac{2\pi}{\omega}$$

Lecture V : Kinetic Energy of Rotation



$$K = \frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2} + \frac{m_3 v_3^2}{2} + \ldots$$

$$= \sum \frac{m_i v_i^2}{2}$$

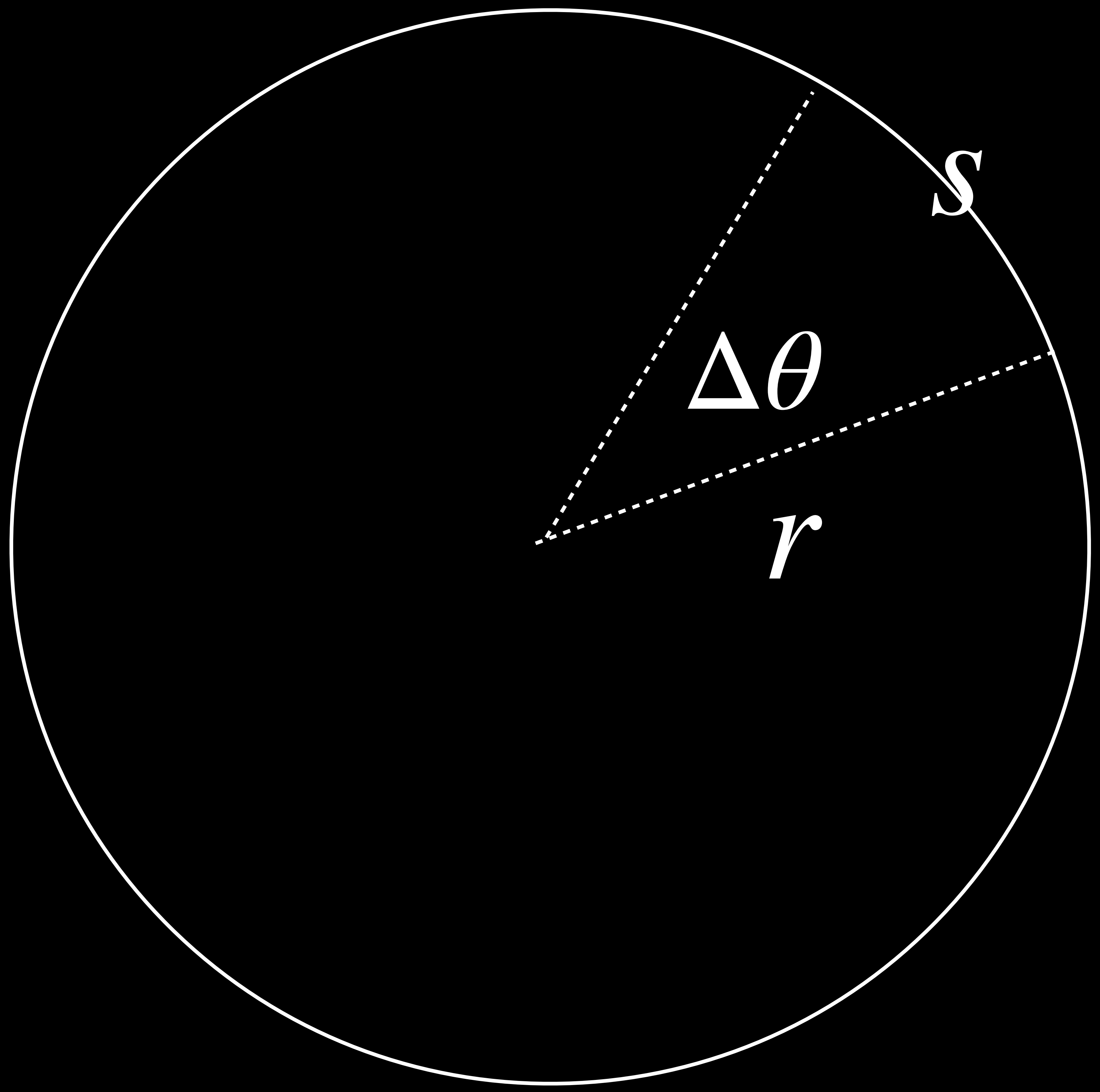
$$= \sum \frac{m_i (\omega r_i)^2}{2} = \frac{\left(\sum m_i r_i^2\right) \omega^2}{2}$$

$$= \frac{\sum m_i r_i^2 \omega^2}{2} = \frac{I\omega^2}{2}$$

Rotation inertia $I = \sum m_i r_i^2$

Lecture V : Rotation Inertia

$$I = \sum m_i r_i^2$$



$$K = \frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2} + \frac{m_3 v_3^2}{2} + \dots$$

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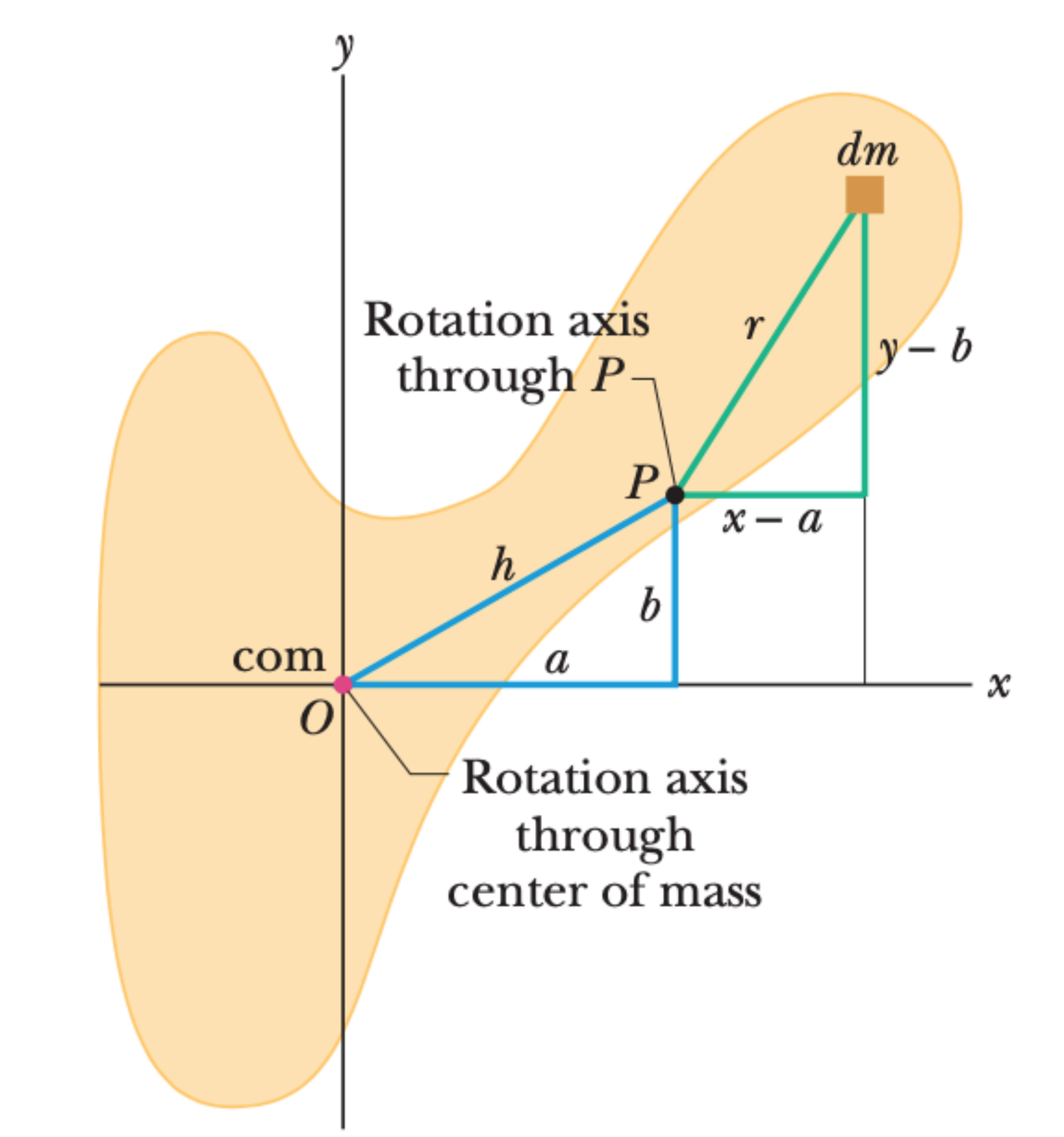
$$= \frac{\sum m_i r_i^2 \omega^2}{2} = \frac{I\omega^2}{2}$$

Lecture V : parallel-Axis Theorem

$$I = \int r^2 dm = \int [(x-a)^2 + (y-b)^2] dm$$

$$= \int [(x^2+y^2) - \underbrace{2ax}_{=0} - \underbrace{2by}_{=0} + (a^2+b^2)] dm$$

$$= \int [r^2 + h^2] dm \quad = I_{com} + Mh^2$$

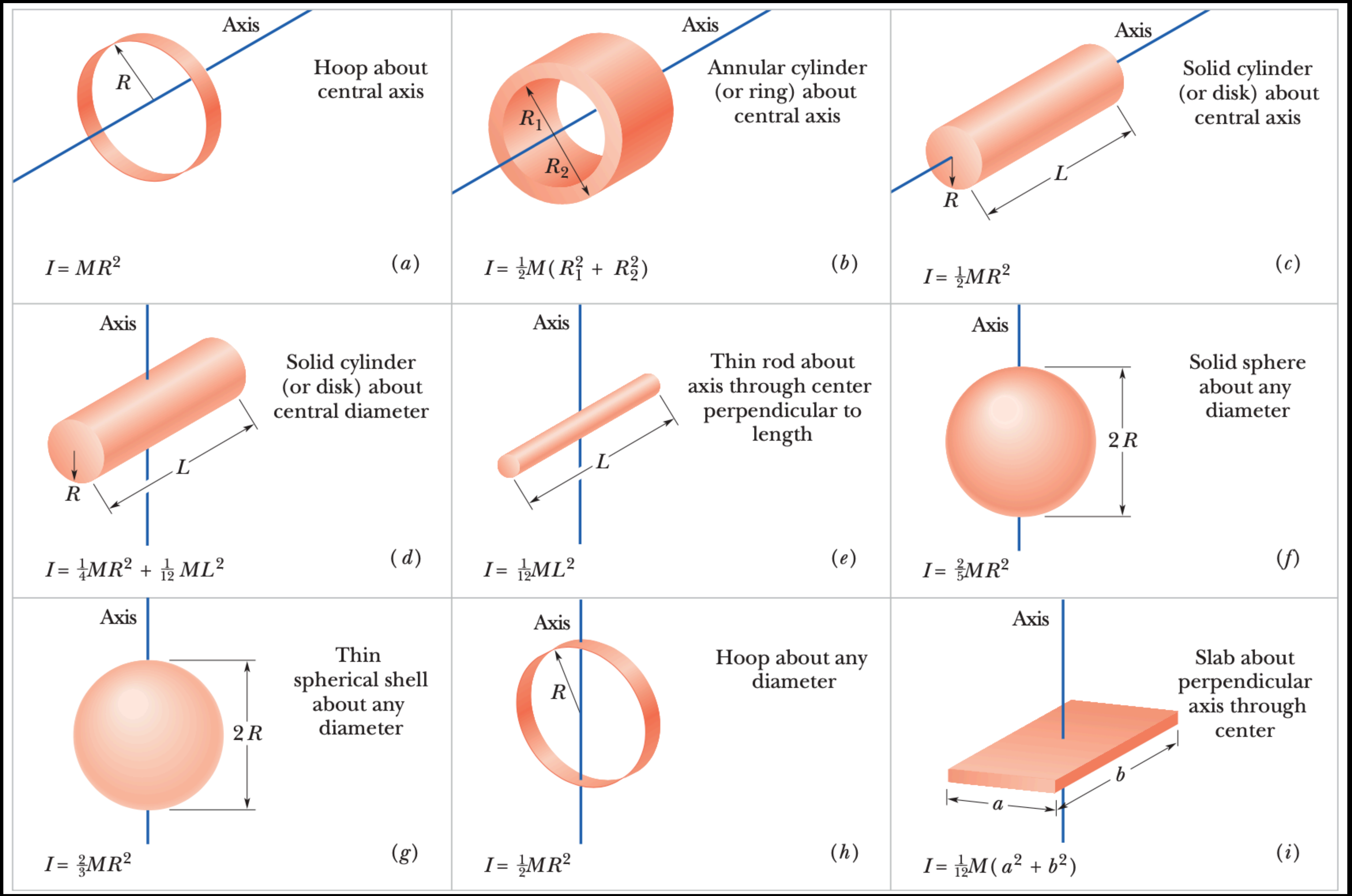


Lecture V : Rotation Inertia

$$I = \sum m_i r_i^2$$

$$= \int r^2 dm$$

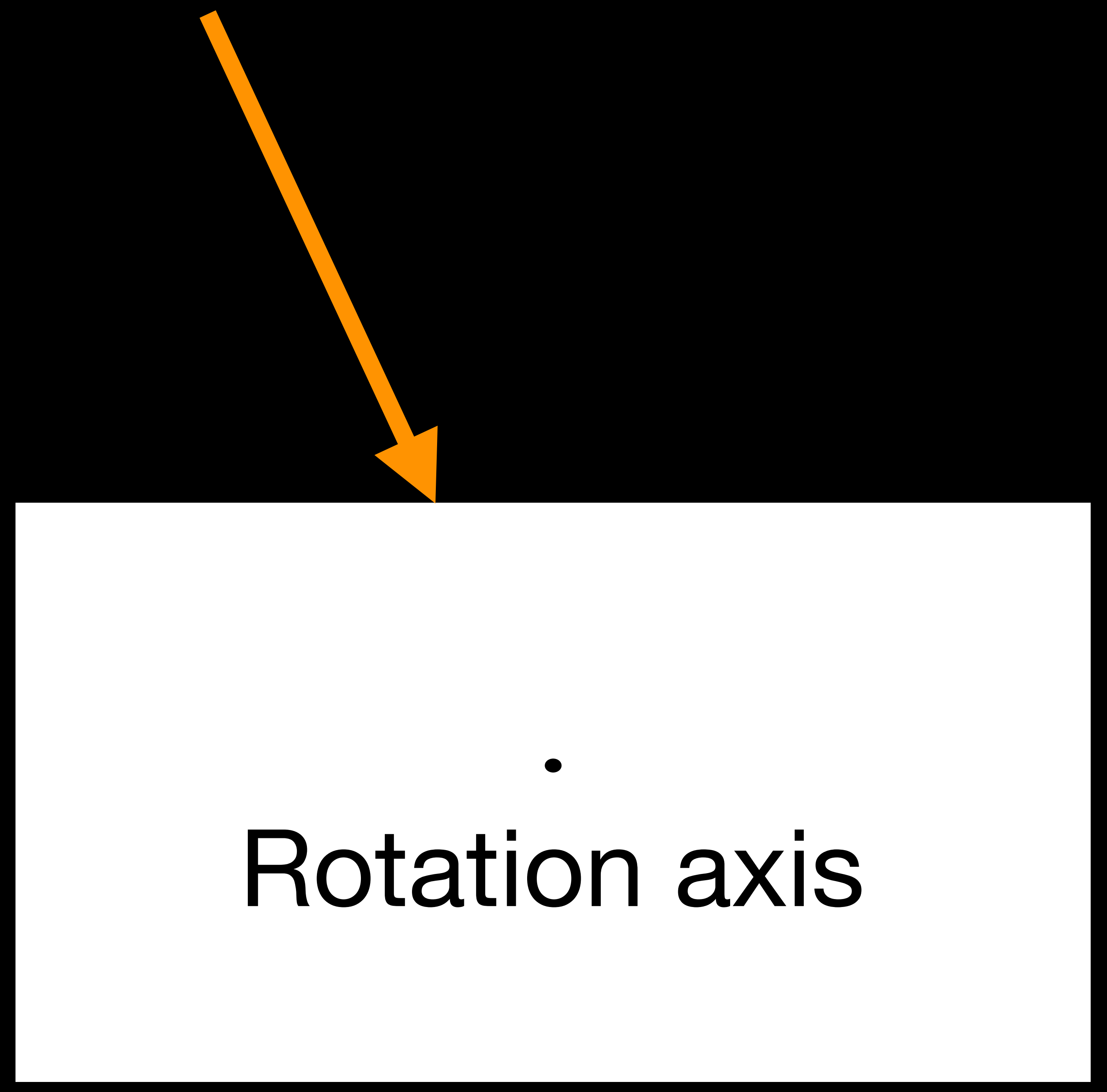
$$\int_i^f r^2 dm$$

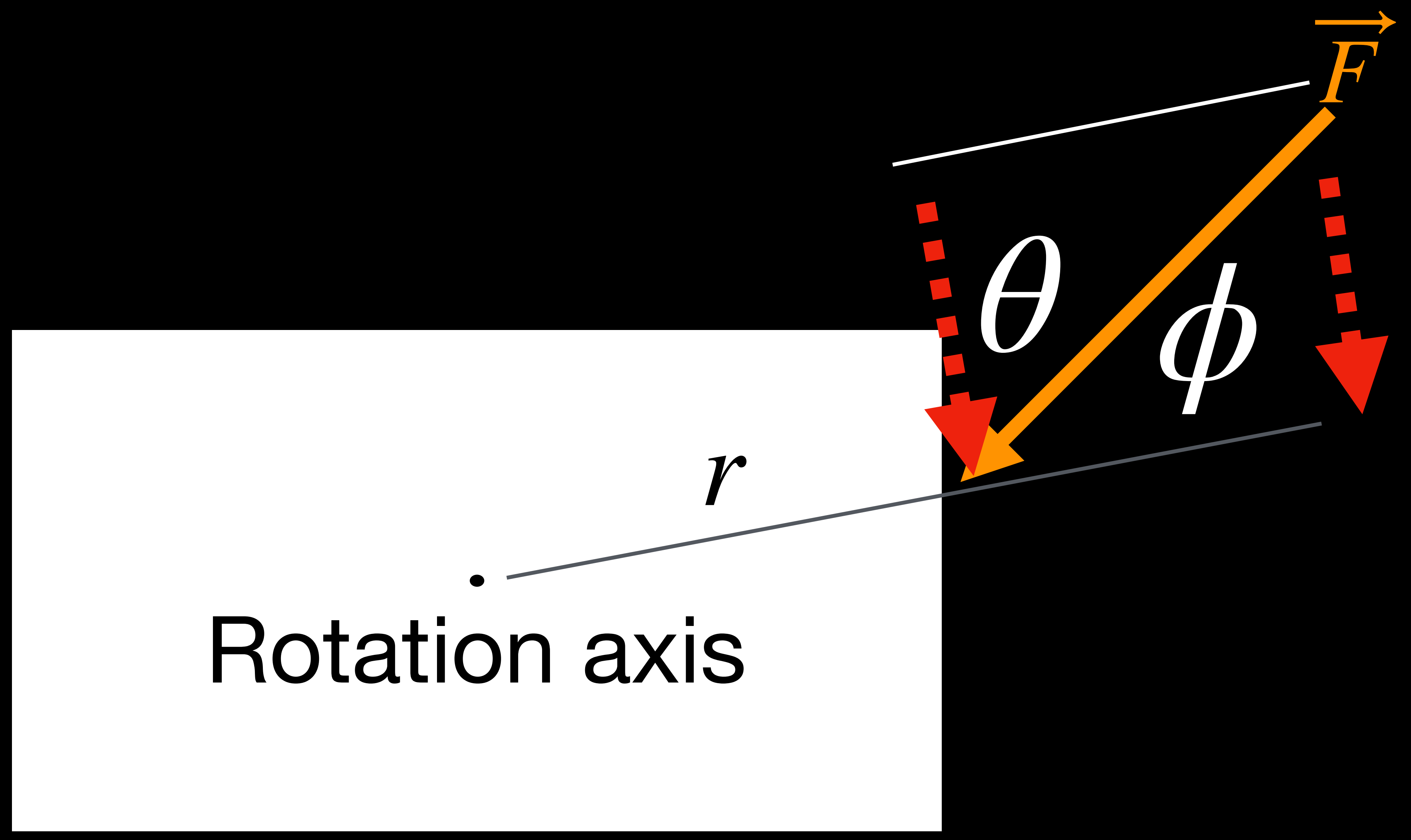


Lecture V : Torque

Torque is a ability to rotate the body

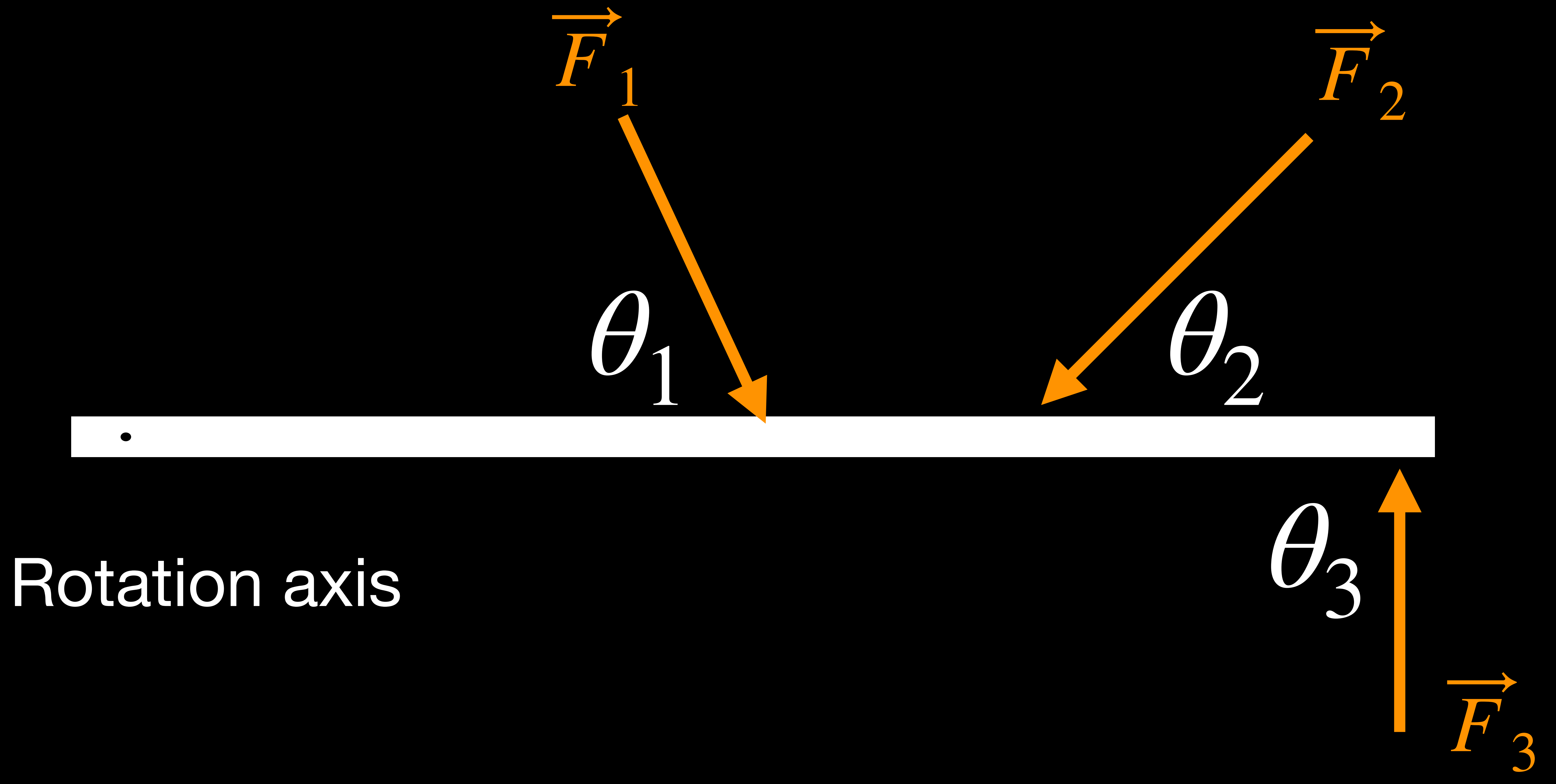
$$\tau = \vec{r} \times \overrightarrow{F} = rF\sin\phi = rF\cos\theta$$





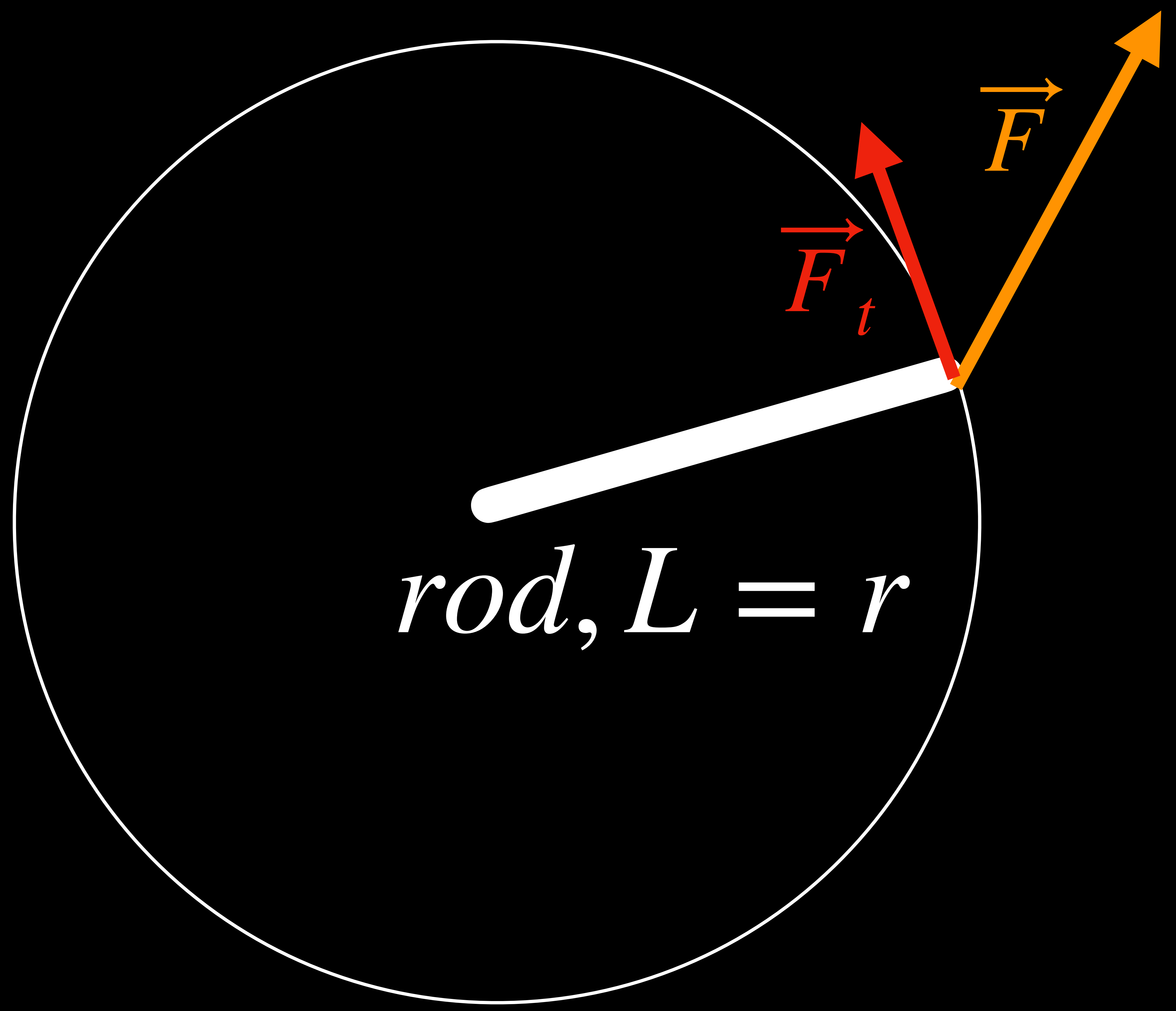
Lecture V : Net Torque

$$\tau_{net} = r_1 \overrightarrow{F}_1 \sin\theta_1 + r_2 \overrightarrow{F}_2 \sin\theta_2 + r_3 \overrightarrow{F}_3 \sin\theta_3$$



Lecture V : Newton's second law for rotation

Start from Newton's second law: $F_{net} = ma$



$$F_t = ma_t$$

$$F_t r = ma_t r$$

$$\tau = ma_t r = m(\alpha r)r$$

$$\tau = I\alpha$$

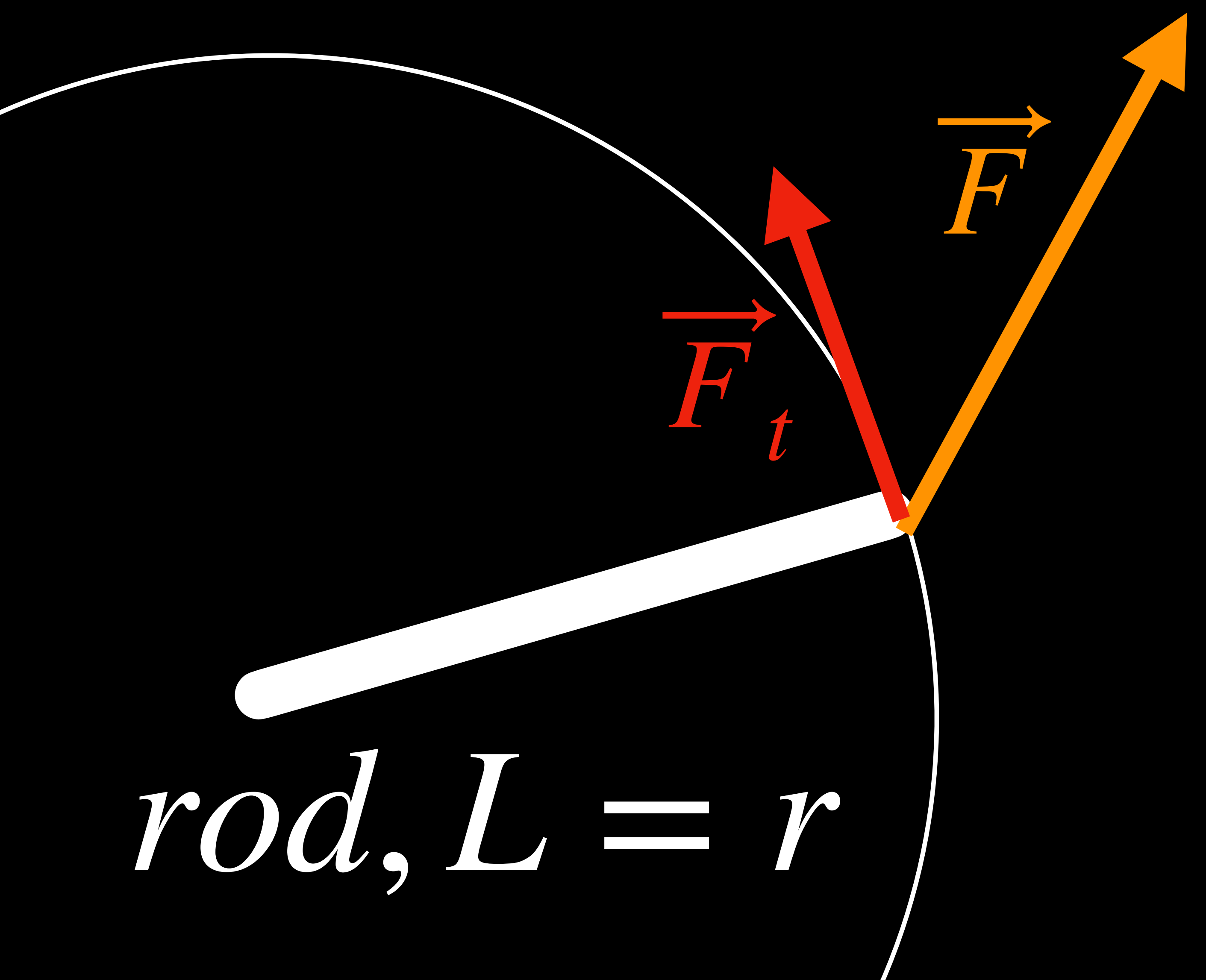
Lecture V : Work and rotational kinetic energy

$$\Delta K = K_f - K_i = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2 = W$$

$$W = \int \vec{F} dx = \int \vec{F}_t r d\theta = \int \tau d\theta$$

$$= \tau(d\theta_f - d\theta_i)$$

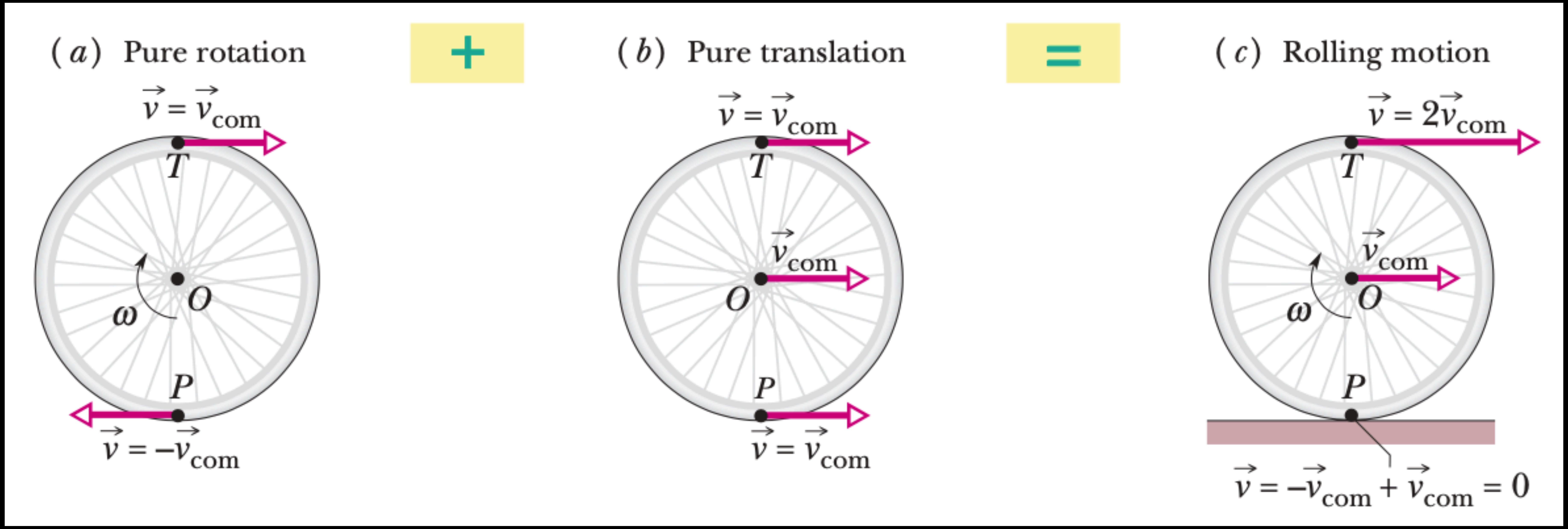
$$P = \frac{dW}{dt} = \tau(d\theta_f - d\theta_i)/dt = \tau\omega$$

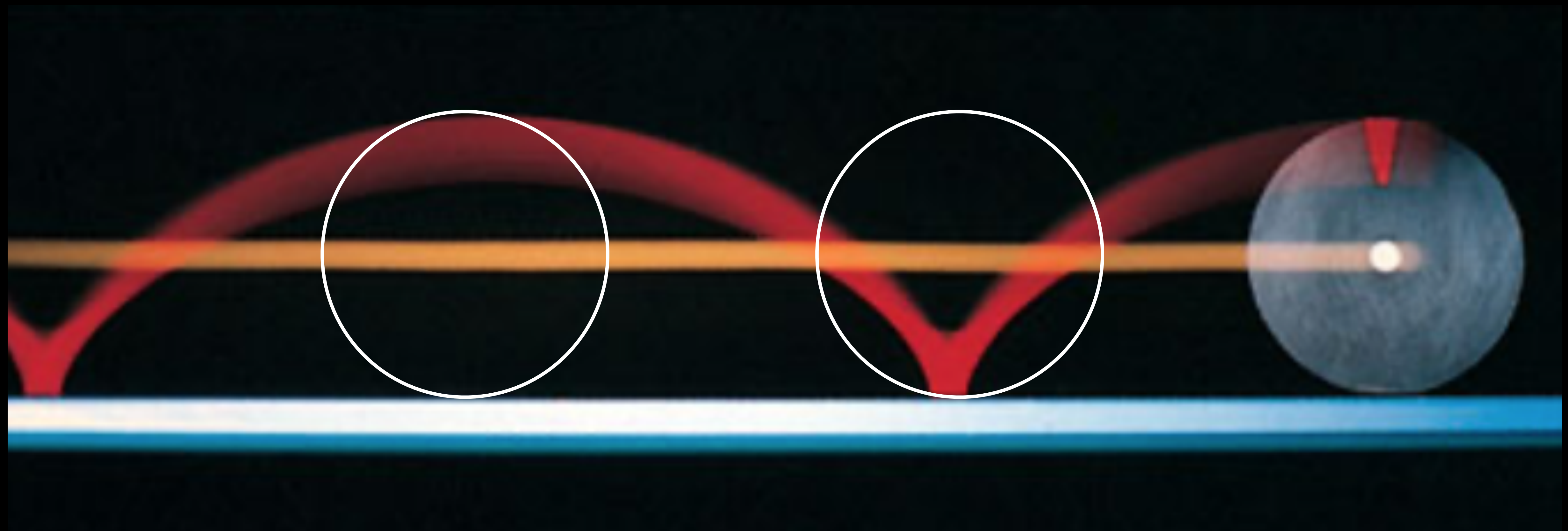


Lecture V : Rolling

$$s = \Delta\theta r$$

$$v_{com} = \omega r$$

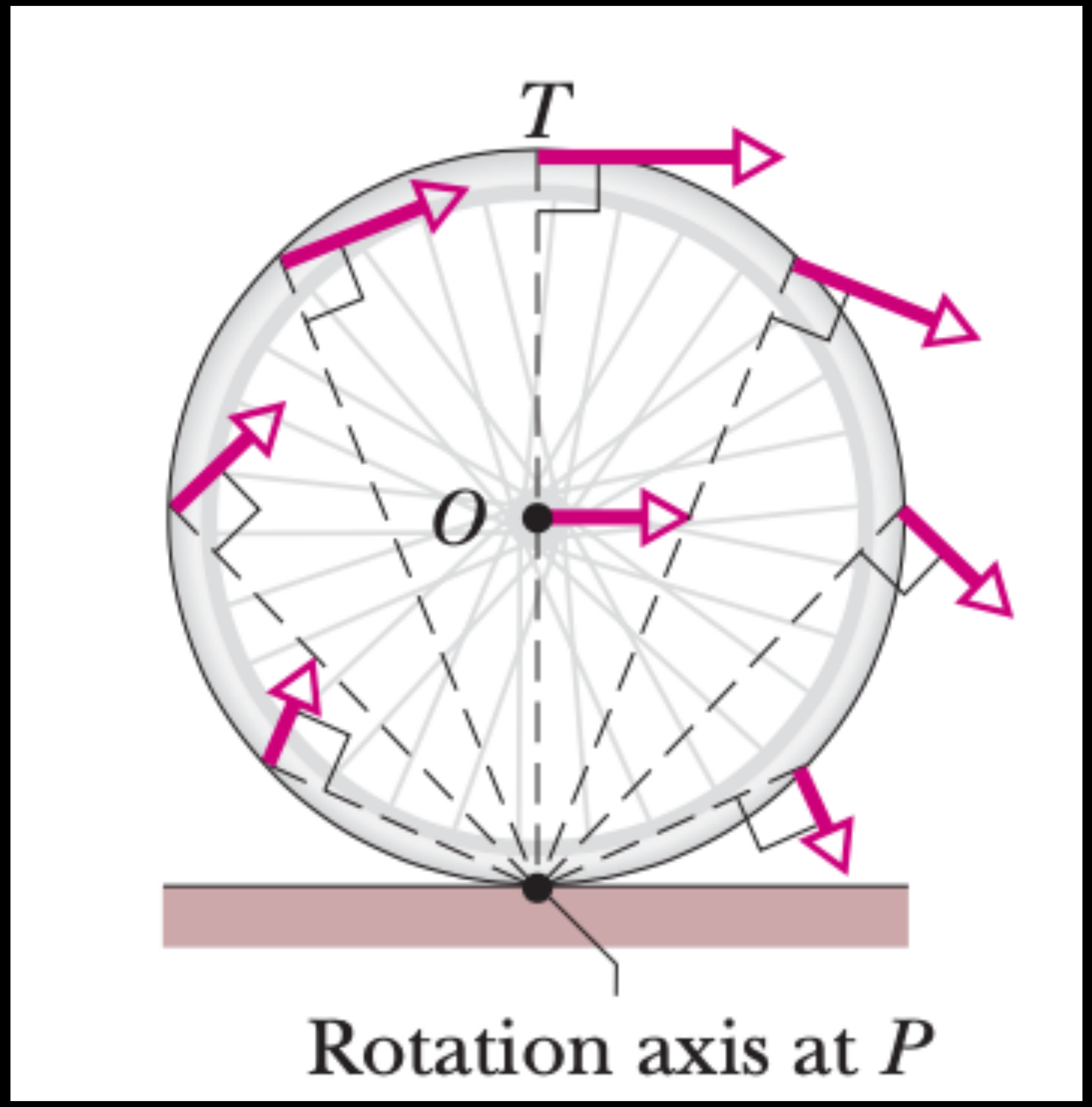




Lecture V : Kinetic Energy of Rolling

$$s = \Delta\theta r$$

$$v_{com} = \omega r$$



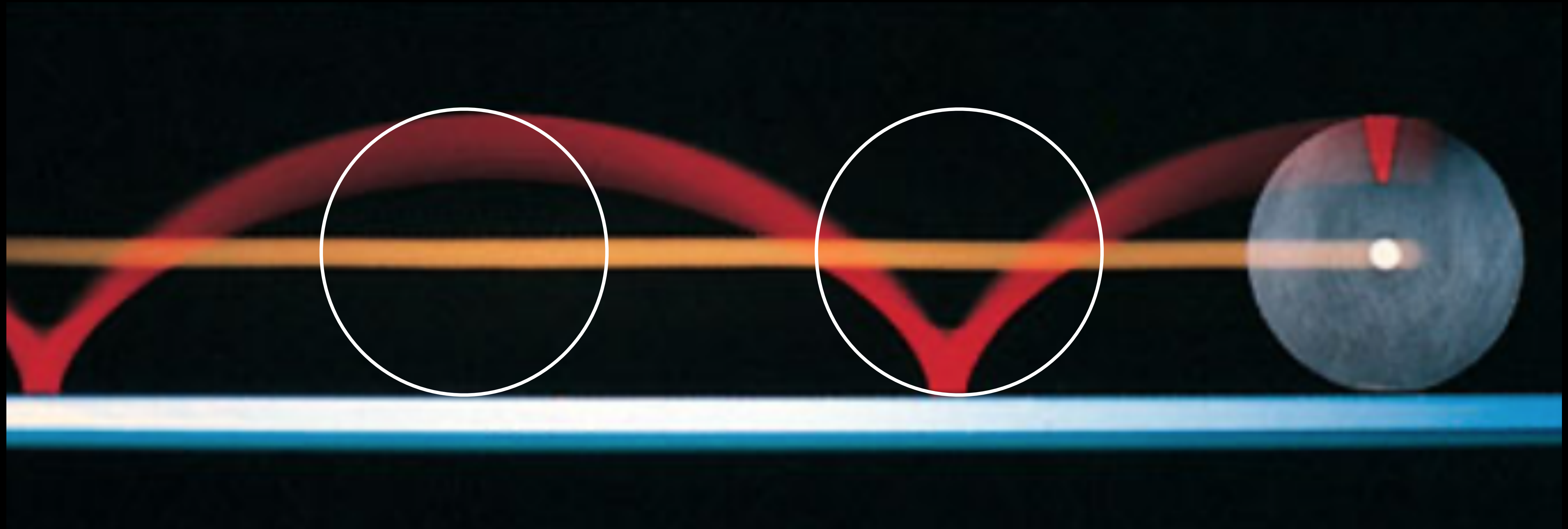
$$I = I_{com} + Mh^2$$

$$h = R$$

$$K = \frac{1}{2} I_P \omega^2$$

$$= \frac{1}{2}(I_{com} + MR^2)\omega^2$$

$$= \frac{1}{2}(I_{com}\omega^2 + Mv^2)$$



Lecture V : Torque & Angular momentum

$$l \equiv \vec{r} \times \vec{p}$$

$$= m(\vec{r} \times \vec{v})$$

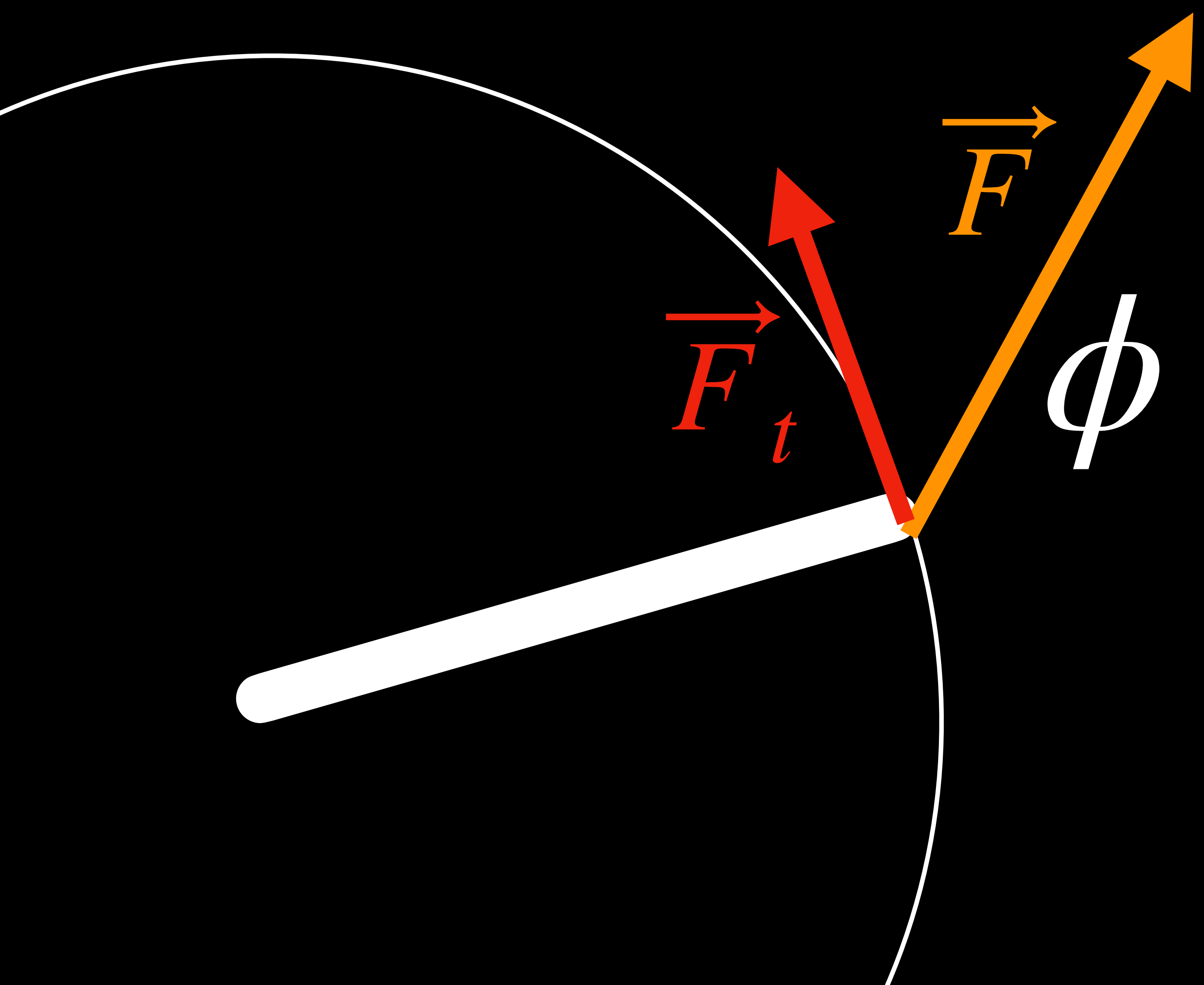
$$= mrv \sin \phi$$

$$\tau = \vec{r} \times \vec{F} = \vec{r} \times m\vec{a}$$

$$= \vec{r} \times m\vec{a} + \underbrace{\vec{v} \times m\vec{v}}_{=0}$$

$$= \vec{r} \times m\frac{d\vec{v}}{dt} + \frac{d\vec{r}}{dt} \times m\vec{v}$$

$$= \frac{d}{dt}(\ \vec{r} \times \vec{p}\ ) \equiv \frac{d}{dt}(\ l\ )$$



Lecture V : Torque & Angular momentum

$$l \equiv \vec{r} \times \vec{p}$$

$$= m(\vec{r} \times \vec{v})$$

$$= mrv \sin\phi$$

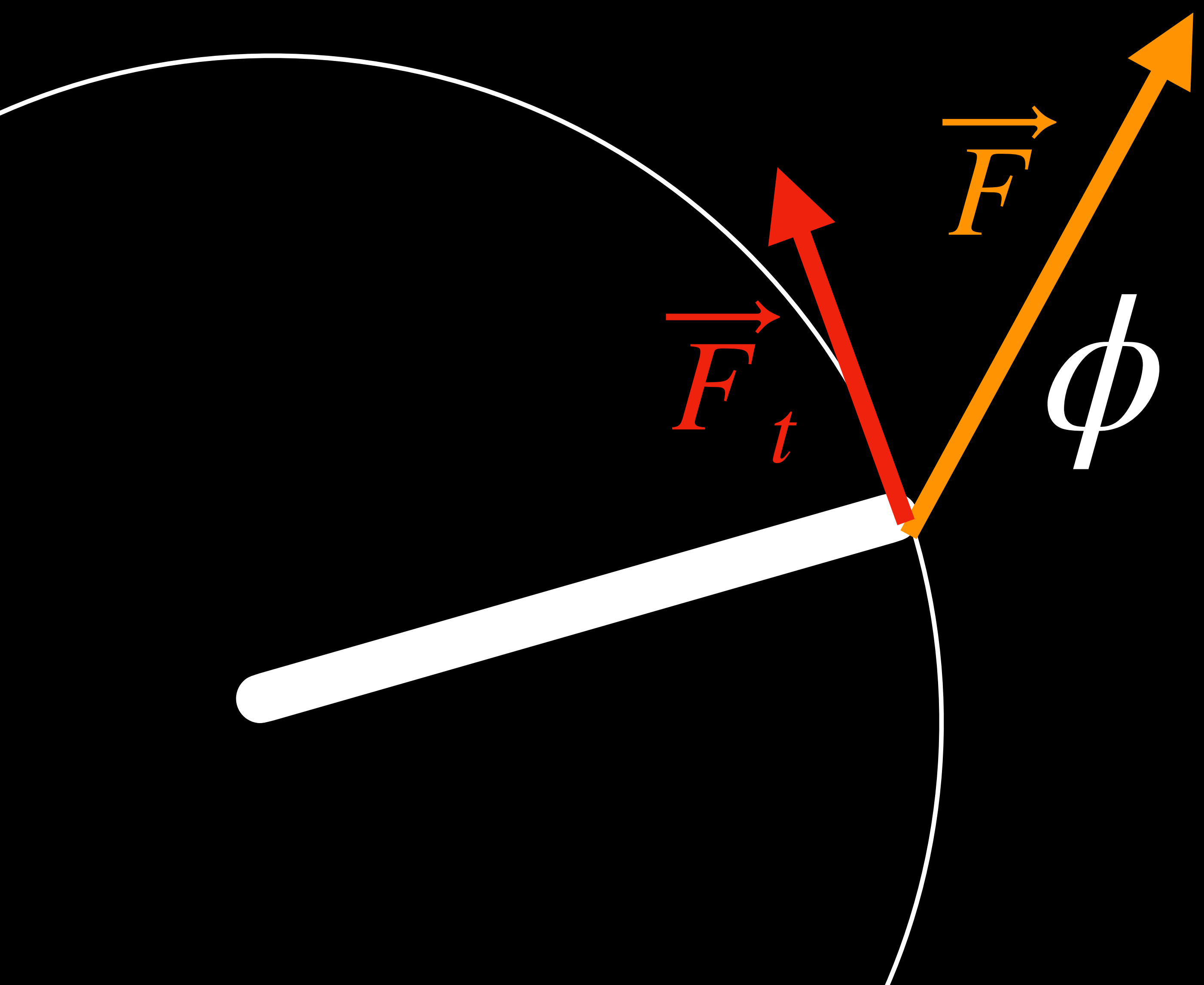
$$\tau_{net} = \sum \vec{r}_i \times \vec{F}_i = m \sum \vec{r}_i \times \vec{a}_i$$

$$= m \sum \vec{r}_i \times \frac{d\vec{v}_i}{dt} + \underline{\vec{v}_i \times \vec{v}_i} = 0$$

$$= m \sum \vec{r}_i \times \frac{d\vec{v}_i}{dt} + \frac{d\vec{r}_i}{dt} \times \vec{v}$$

$$= \vec{r} \times \frac{dM\vec{v}}{dt} + \frac{d\vec{r}}{dt} \times M\vec{v}$$

$$= \frac{d}{dt}(\ \vec{r}_i \times \vec{p}_i\ ) \equiv \frac{d}{dt}(\ L\ )$$



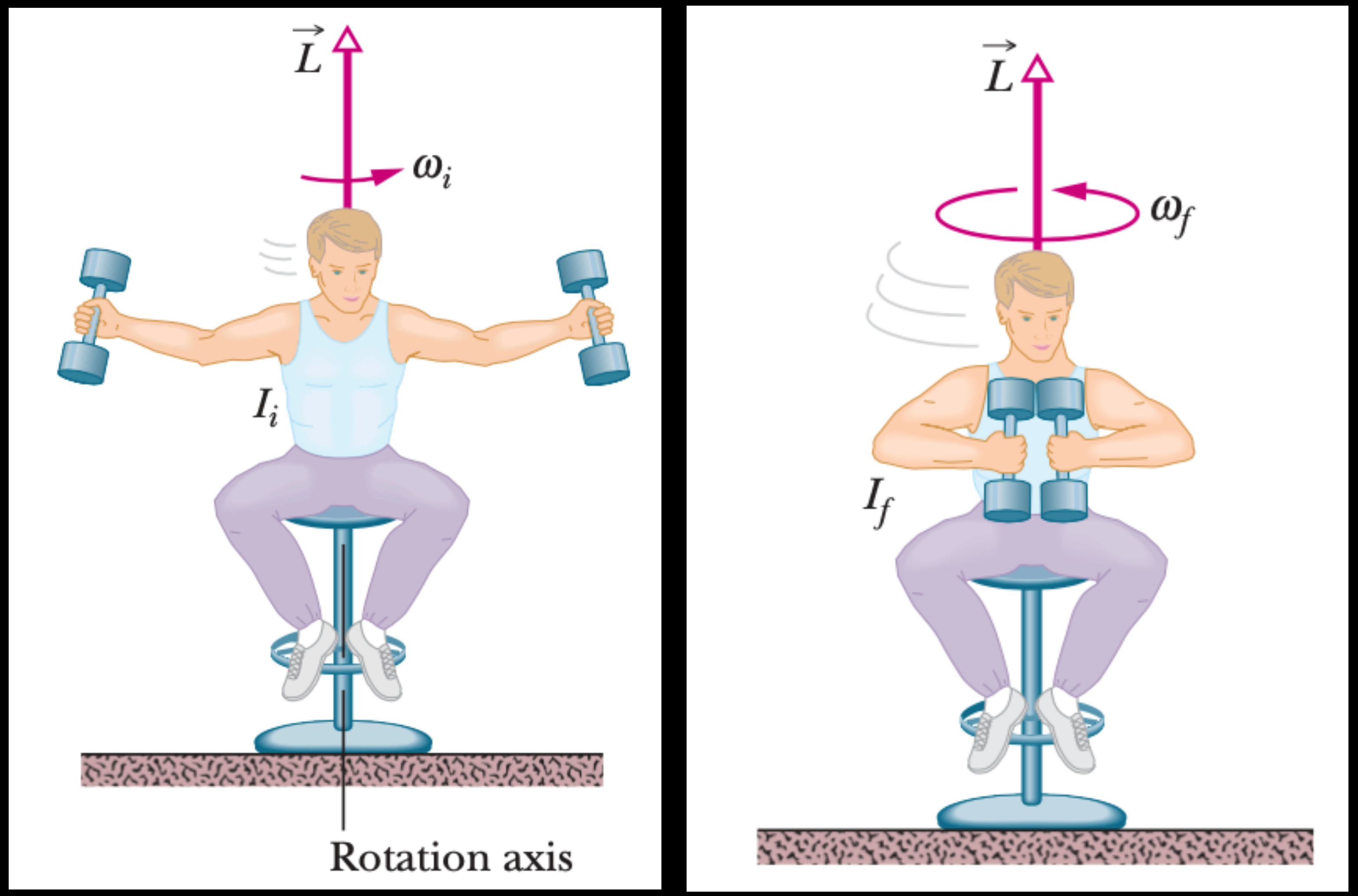
Lecture V : Conservation of Momentum

If no net external torque acts on the system :

$$\frac{d}{dt}(\vec{L}) = 0, \quad L = \text{constant}$$

Net angular momentum $\vec{L}_i$ at initial time

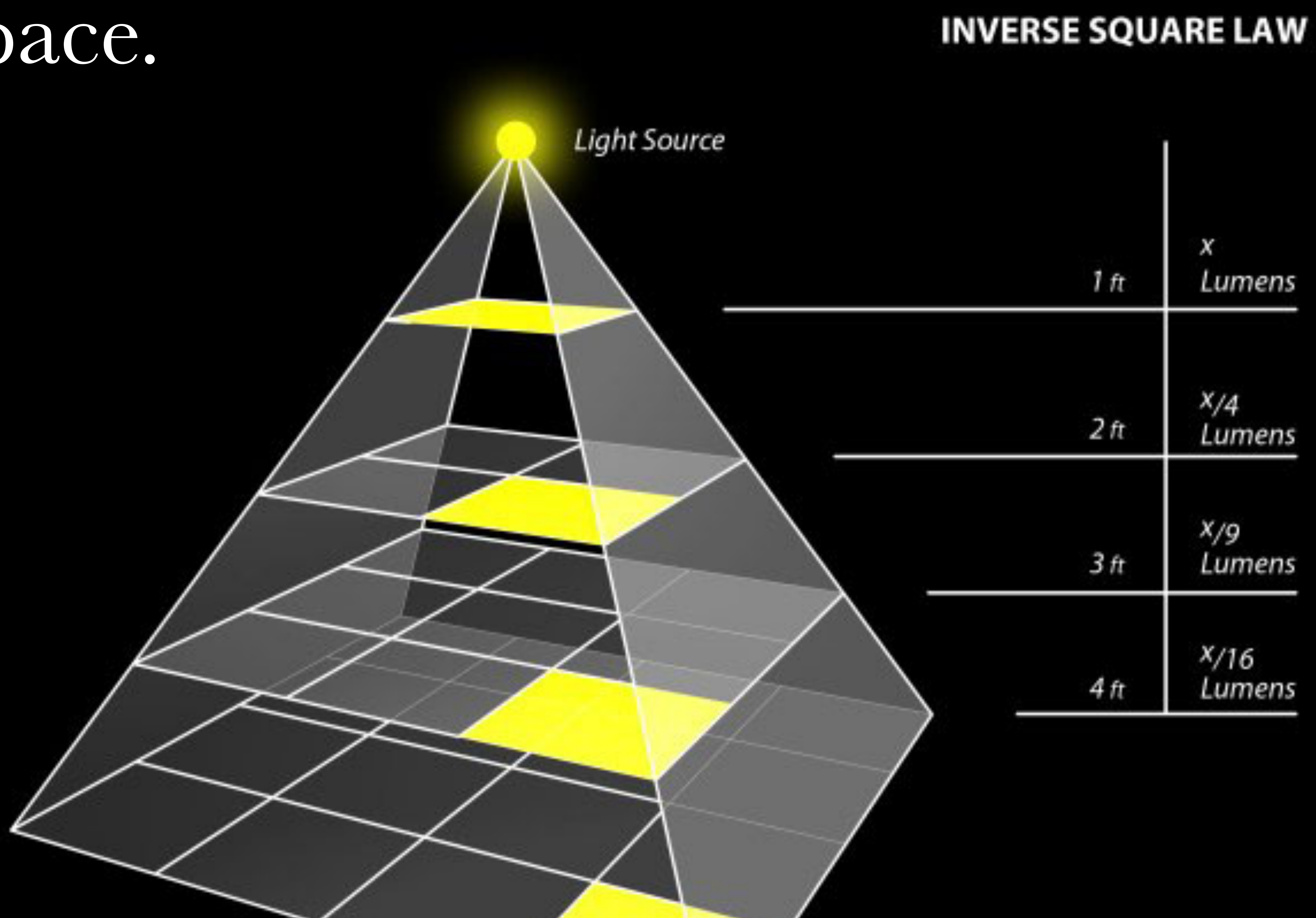
= net angular momentum $\vec{L}_f$ at later time



$$\vec{L}_i = \vec{L}_f \quad \text{(isolated system)}$$

Lecture VI : Gravitation

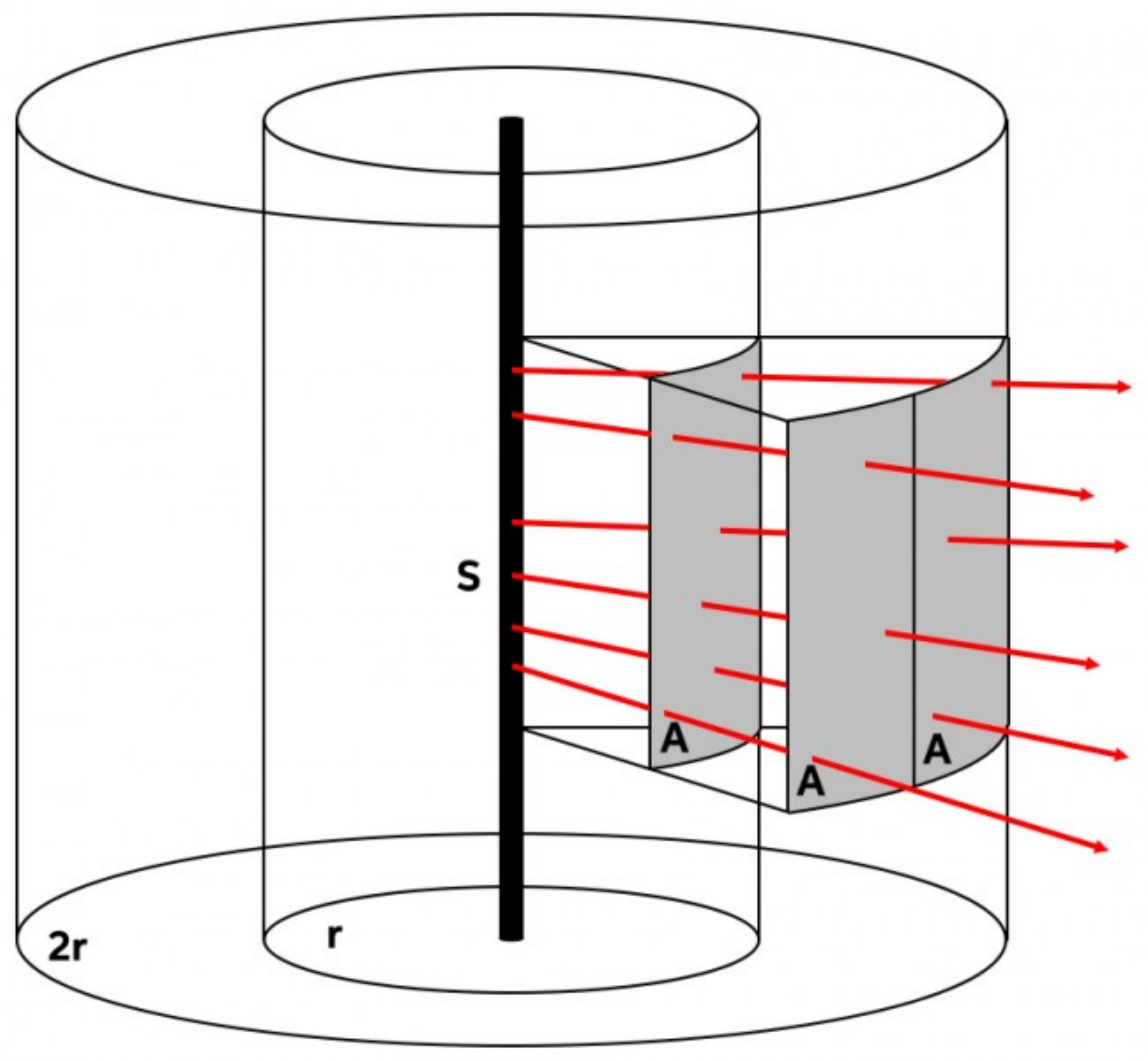
Inverse square law: point-source radiation into three-dimensional space.



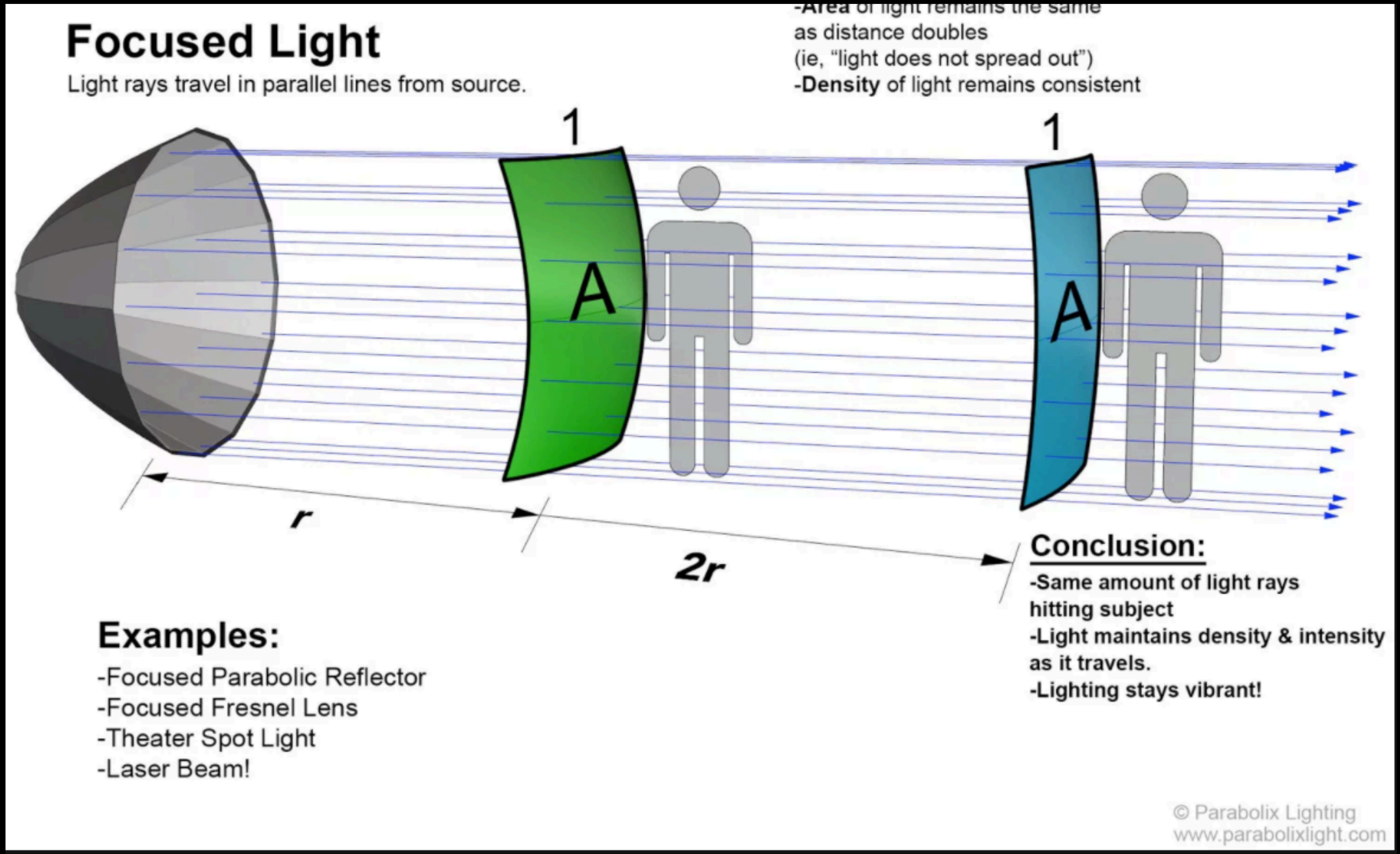
point source case

Lecture VI : Gravitation

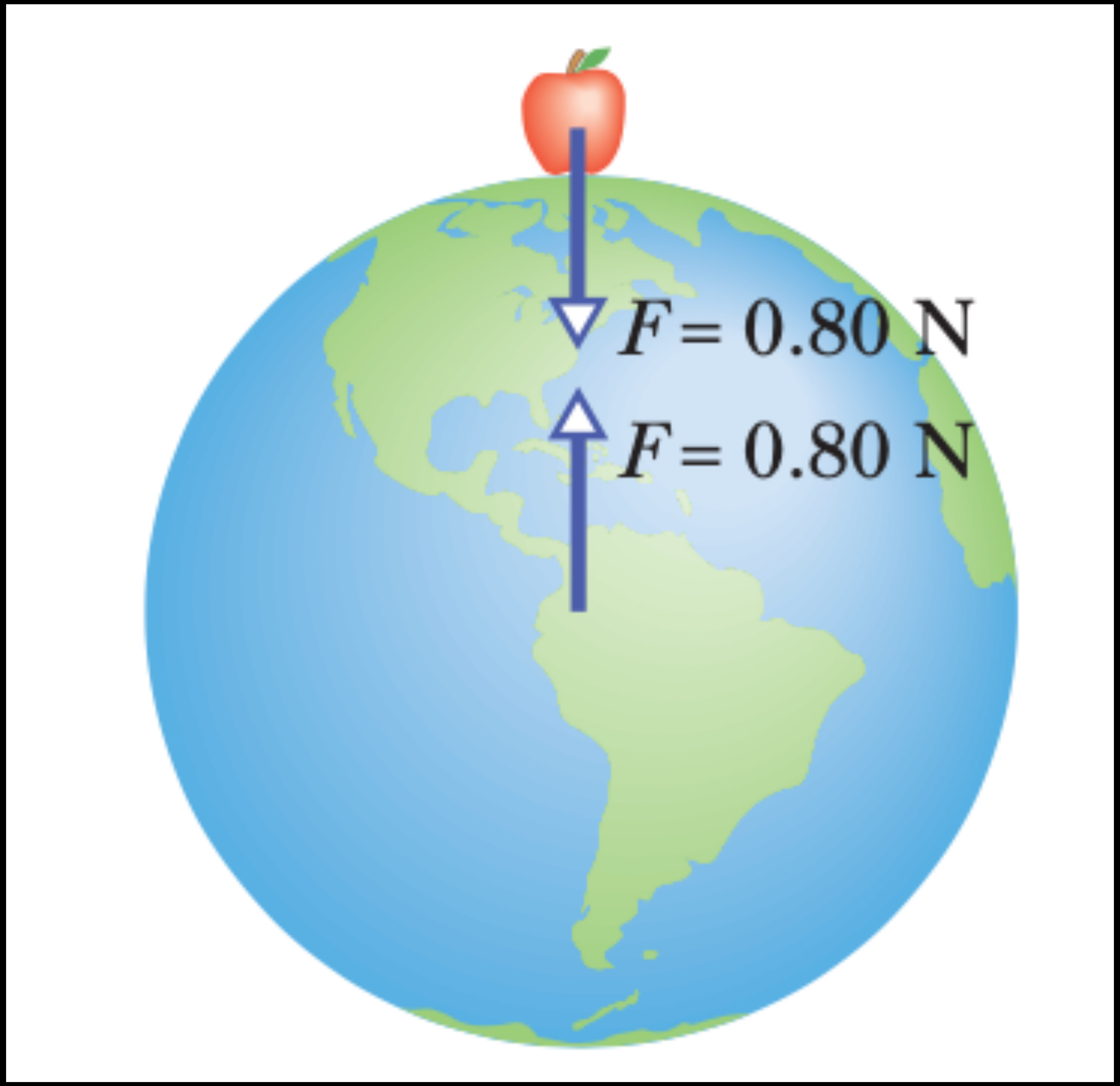
Line source case



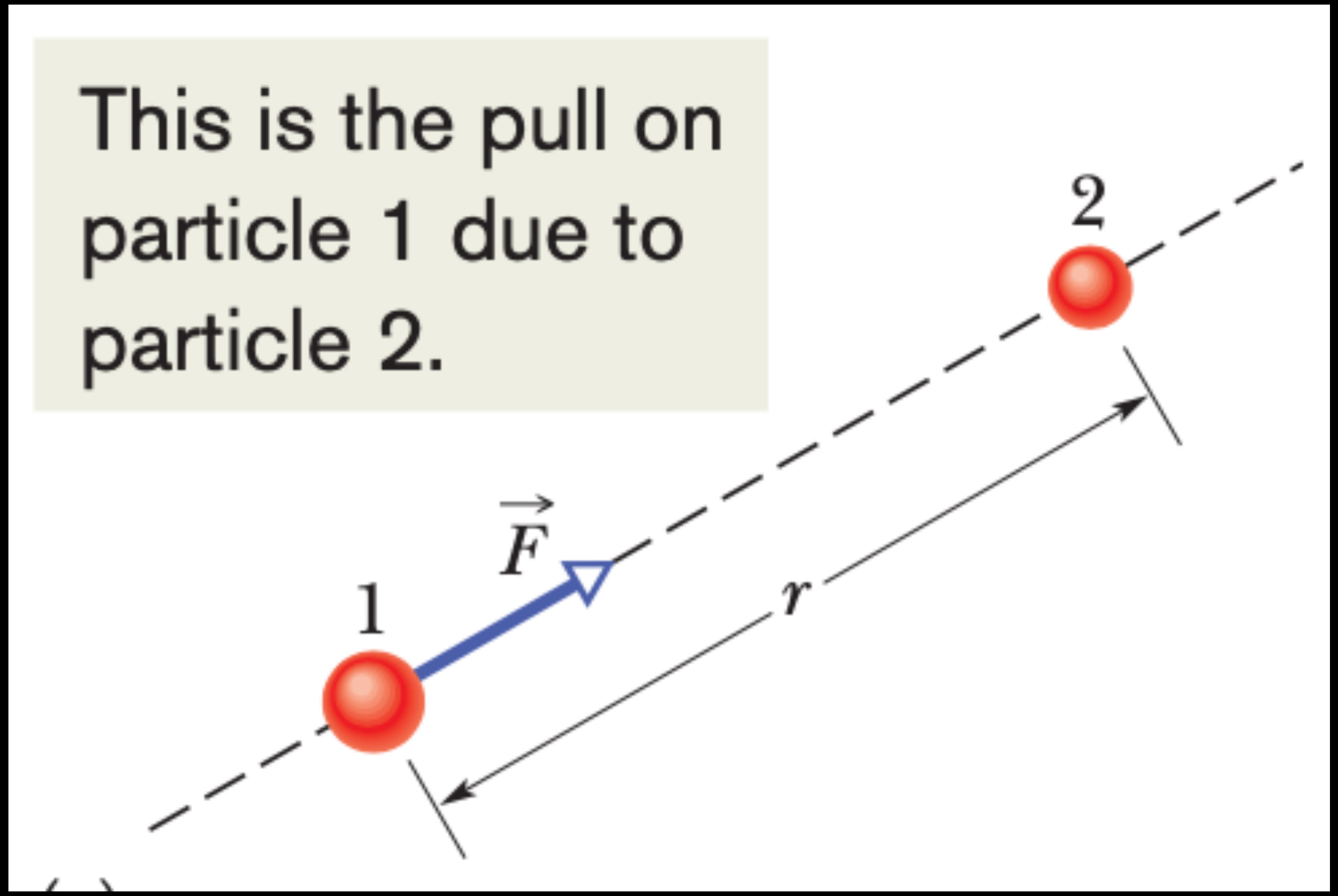
Plane source case



Lecture VI : Gravitation



$$F = \frac{Gm_1m_2}{r^2}\hat{r}$$



Lecture VI : Gravitation

Altitude (km)	a_g (m/s^2)	Altitude Example
0	9.83	Mean Earth surface
8.8	9.80	Mt. Everest
36.6	9.71	Highest crewed balloon
400	8.70	Space shuttle orbit
35 700	0.225	Communications satellite

$$F = \frac{Gm_1m_2}{r^2}\hat{r}$$

Lecture VI : Gravitation

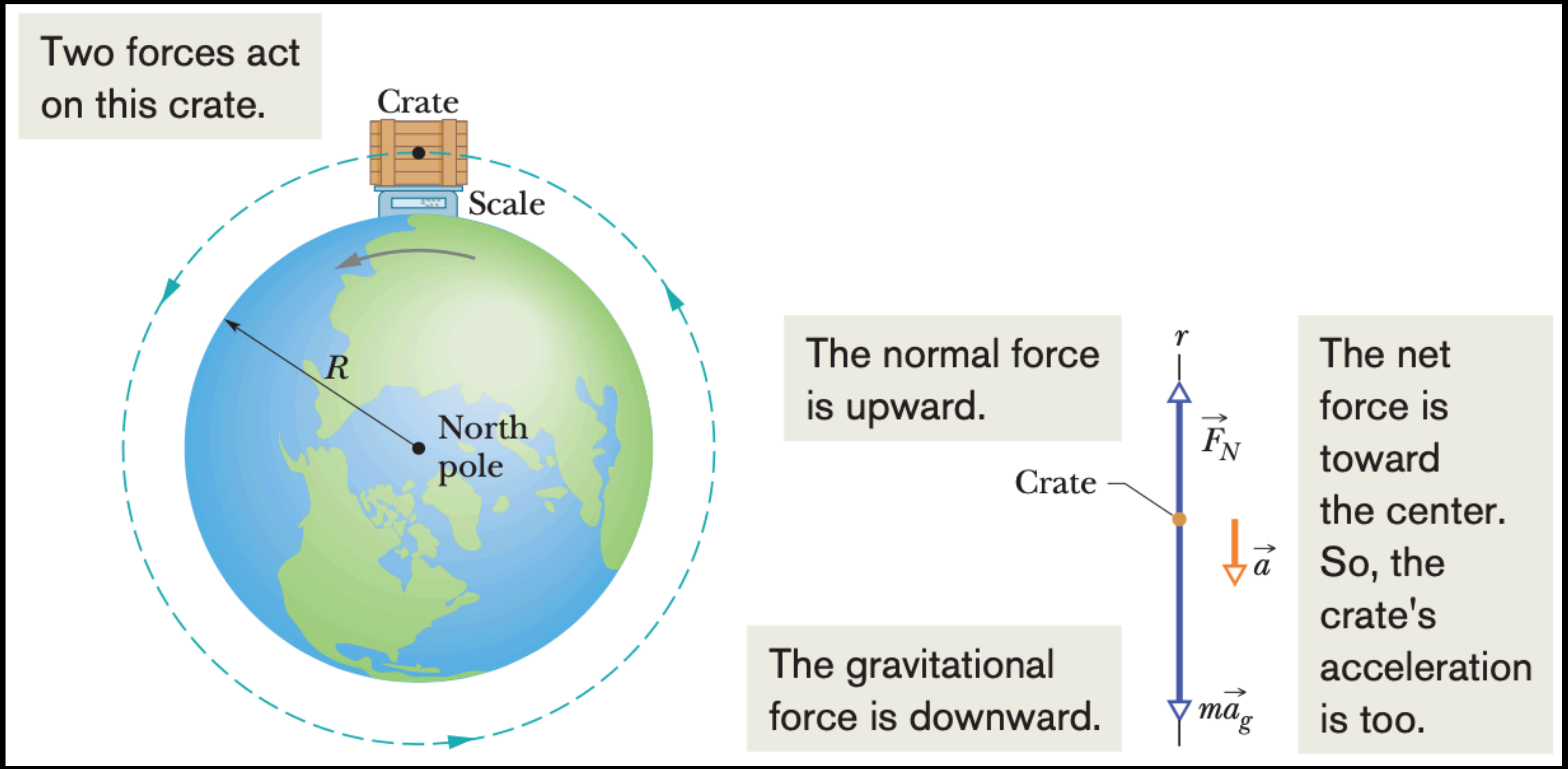
$$F_N - ma_g = m(-\omega R^2)$$

$$mg = ma_g + m(-\omega R^2)$$

Weight

Gradational force

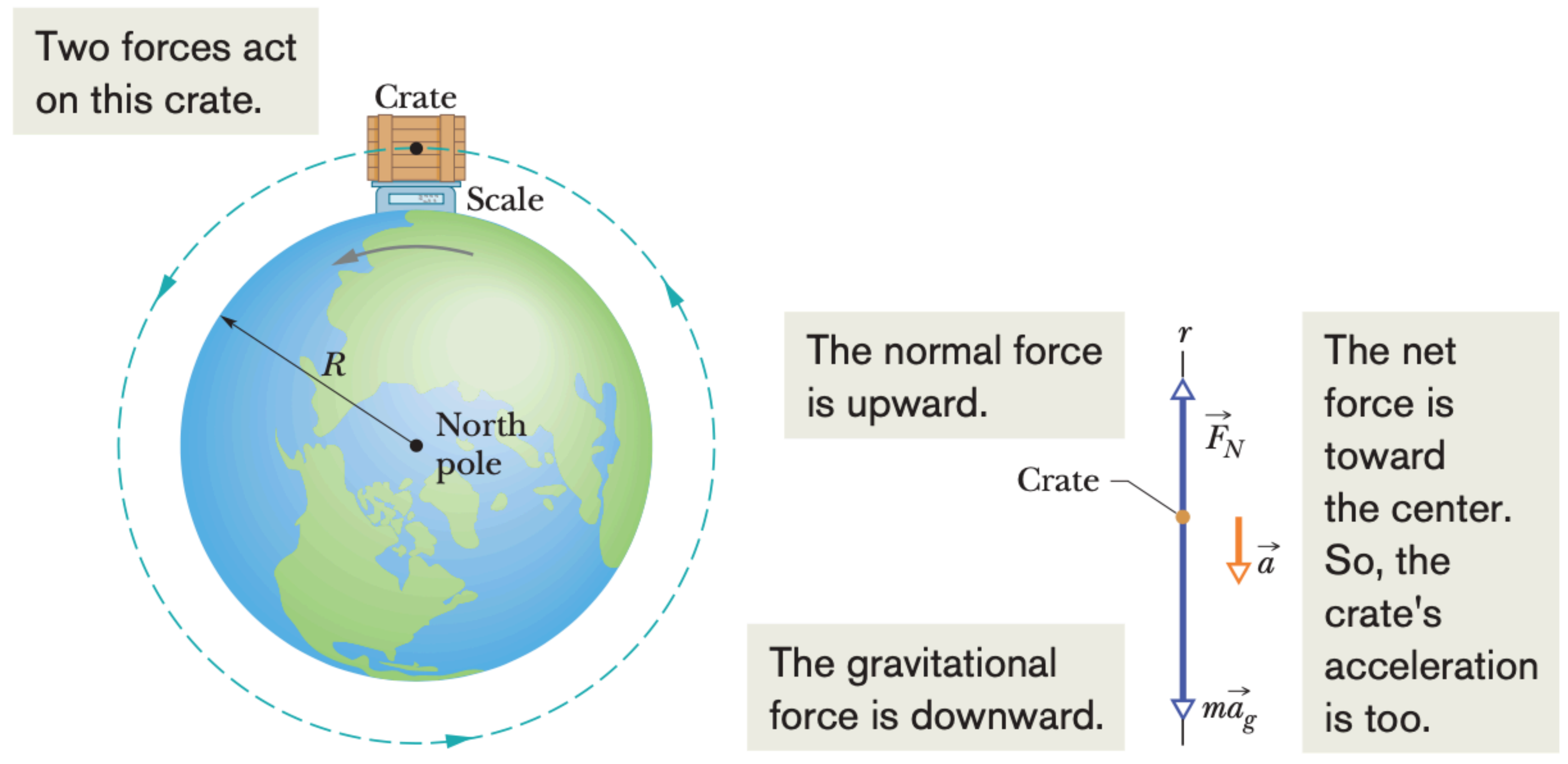
Centripetal force



Lecture VI : Escape Speed

$$U = -\frac{GMm}{r}$$

$$\frac{1}{2}mv^2 = \frac{GMm}{r}$$



Body	Mass (kg)	Radius (m)	Escape Speed (km/s)
Ceres[a]	1.17×10^{21}	3.8×10^5	0.64
Earth's moon[a]	7.36×10^{22}	1.74×10^6	2.38
Earth	5.98×10^{24}	6.37×10^6	11.2
Jupiter	1.90×10^{27}	7.15×10^7	59.5
Sun	1.99×10^{30}	6.96×10^8	618
Sirius B[b]	2×10^{30}	1×10^7	5200
Neutron star[c]	2×10^{30}	1×10^4	2×10^5

Lecture VI : Kepler's laws

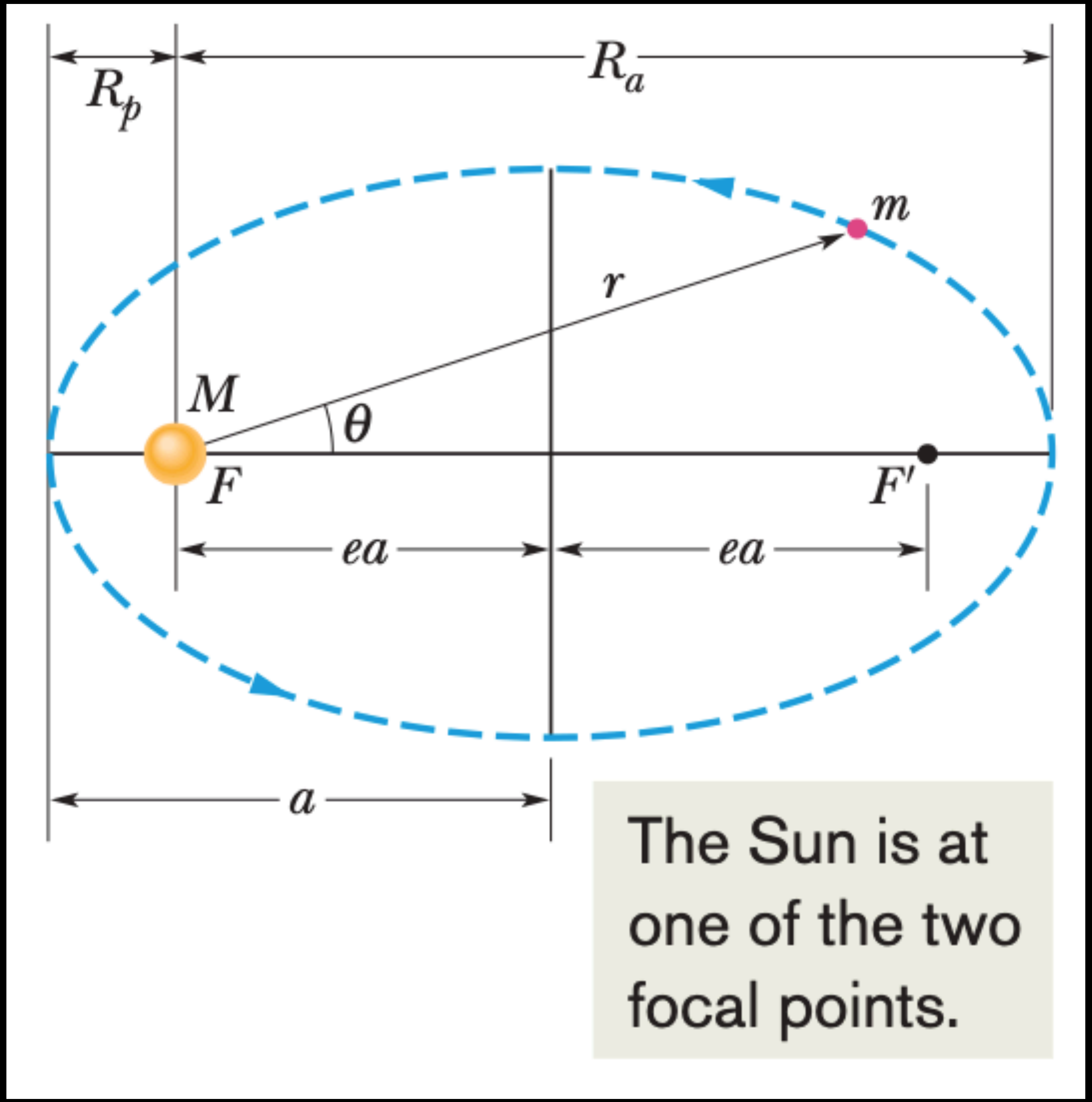
1. THE LAW OF ORBITS: All planets move in elliptical orbits, with the Sun at one focus.

2. THE LAW OF AREAS: A line that connects a planet to the Sun sweeps out equal areas in the plane of the planet's orbit in equal time intervals; that is, the rate dA/dt at which it sweeps out area A is constant.

3. The law of periods: The square of the period of any planet is proportional to the cube of the semi-major axis of its orbit.

Lecture VI : Kepler's laws

1. THE LAW OF ORBITS: All planets move in elliptical orbits, with the Sun at one focus.



Lecture VI : Kepler's laws

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