General Physics I

Mechanics, optics, thermal dynamics, and other basic fundamental things.

TsungChe Liu

Lecture IV : Virial theorem

According the mechanical energy conservation $\int_{x_0}^{x} F(x)dx = \Delta E_k = T - T_0$

$\int_{x_0}^{x} F(x)dx$ is the work done on the particle by the impressed force F(x), thus work is equal to the change in the kinetic energy of particle.

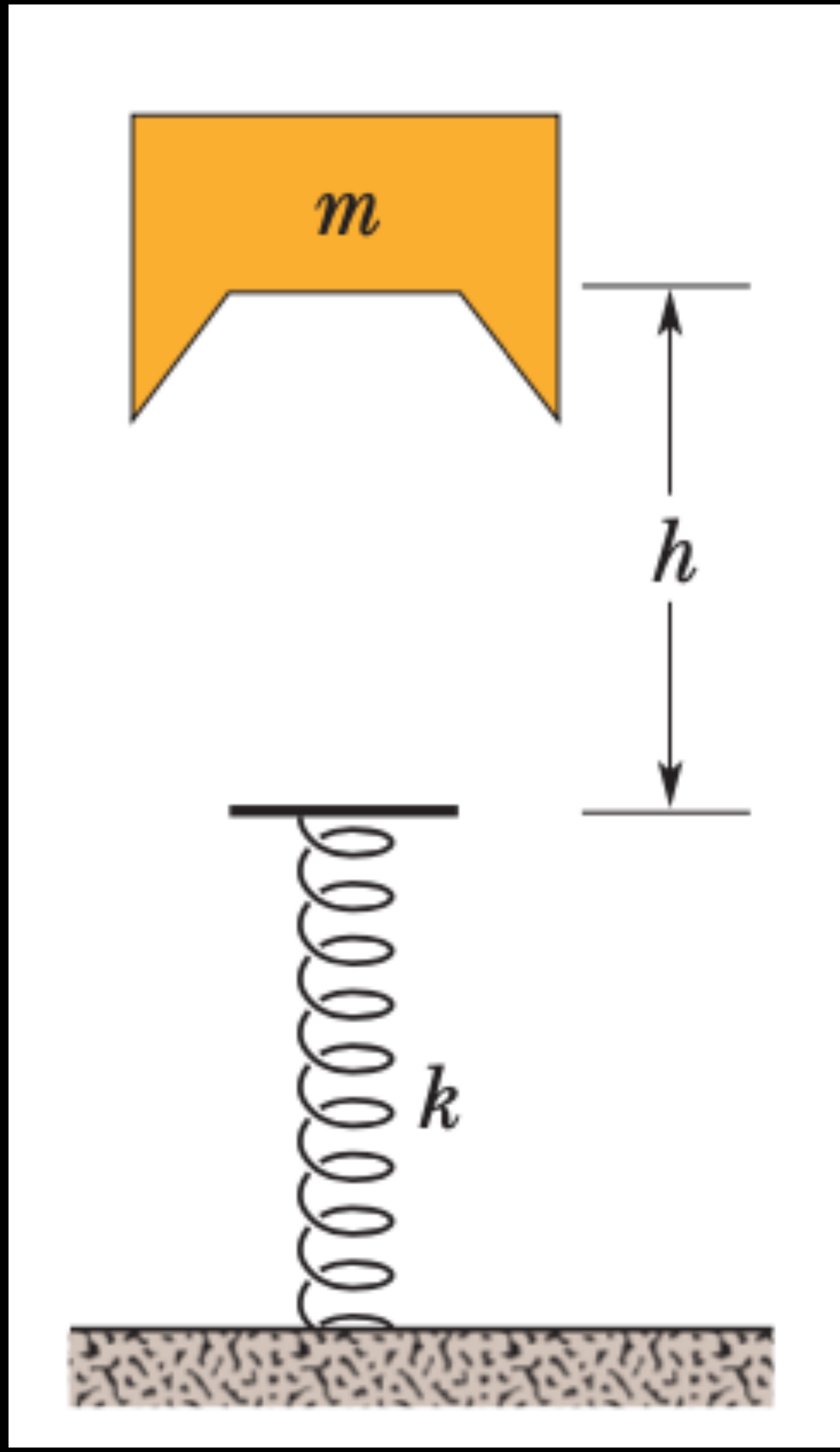
Hence we can define a function V(x) such that $-\frac{dV}{x} = F(x)$, then

$\int_{x_0}^{x} F(x)dx = -\int_{x_0}^{x} dV = T - T_0 = -V(x) + V(x_0)$ and find the function V(x) is the potential himself.

Then we can get the potential(potential energy function) from the force function. $-\frac{dV}{dx} = F(x)$

Lecture IV : Virial theorem $-\frac{dV}{dx} = F(x)$

1. Considering a spring with mass M, it force form is F=-kx, show the potential energy and the velocity.



2. Considering a body with mass M falling from height H to a spring.

Lecture IV : line integral

$$\int_c \overrightarrow{F}(\vec{r})d\vec{r} = \int_a^b (F_x dx + F_y dy + F_z dz) = \int_a^b (F_x x' + F_y y' + F_z z')dt$$

C is contour of the integral path from initial point a to point b, F_i is the component of $\overrightarrow{F}$, $\vec{r}$ is position vector.

Example: , , calculate the work from t=0 to t= 1:

$$\overrightarrow{F}(t) = x\hat{i} - 10\hat{j}$$

$$\vec{r}(t) = t\hat{i} + e^t\hat{j}$$

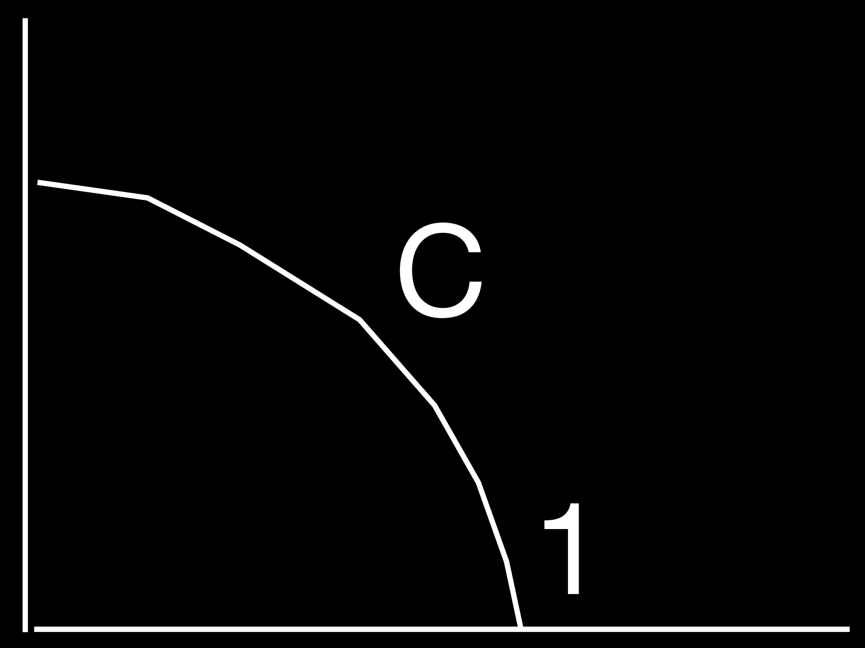
$$\overrightarrow{F}(\vec{r}(t)) = x(t)\hat{i} - 10\hat{j} = t\hat{i} - 10\hat{j} \qquad \vec{r}'(t) = r_x'\hat{i} + r_y'\hat{j} = \hat{i} + e^t\hat{j}$$

$$\int_c \overrightarrow{F}(\vec{r})d\vec{r} = \int_0^1 (t\hat{i} - 10\hat{j}) \cdot (\ \hat{i} + e^t\hat{j})dt = \int_0^1 (t - 10e^t)dt = \frac{1}{2} - 10e + 10$$

Lecture IV : line integral

$$\int_c \overrightarrow{F}(\vec{r})d\vec{r} = \int_a^b (F_x dx + F_y dy + F_z dz) = \int_a^b (F_x x' + F_y y' + F_z z')dt$$

Example: $\overrightarrow{F}(t) = -y\hat{i} + xy\hat{j}$ $\quad \vec{r}(t) = \cos t\hat{i} + \sin t\hat{j}$

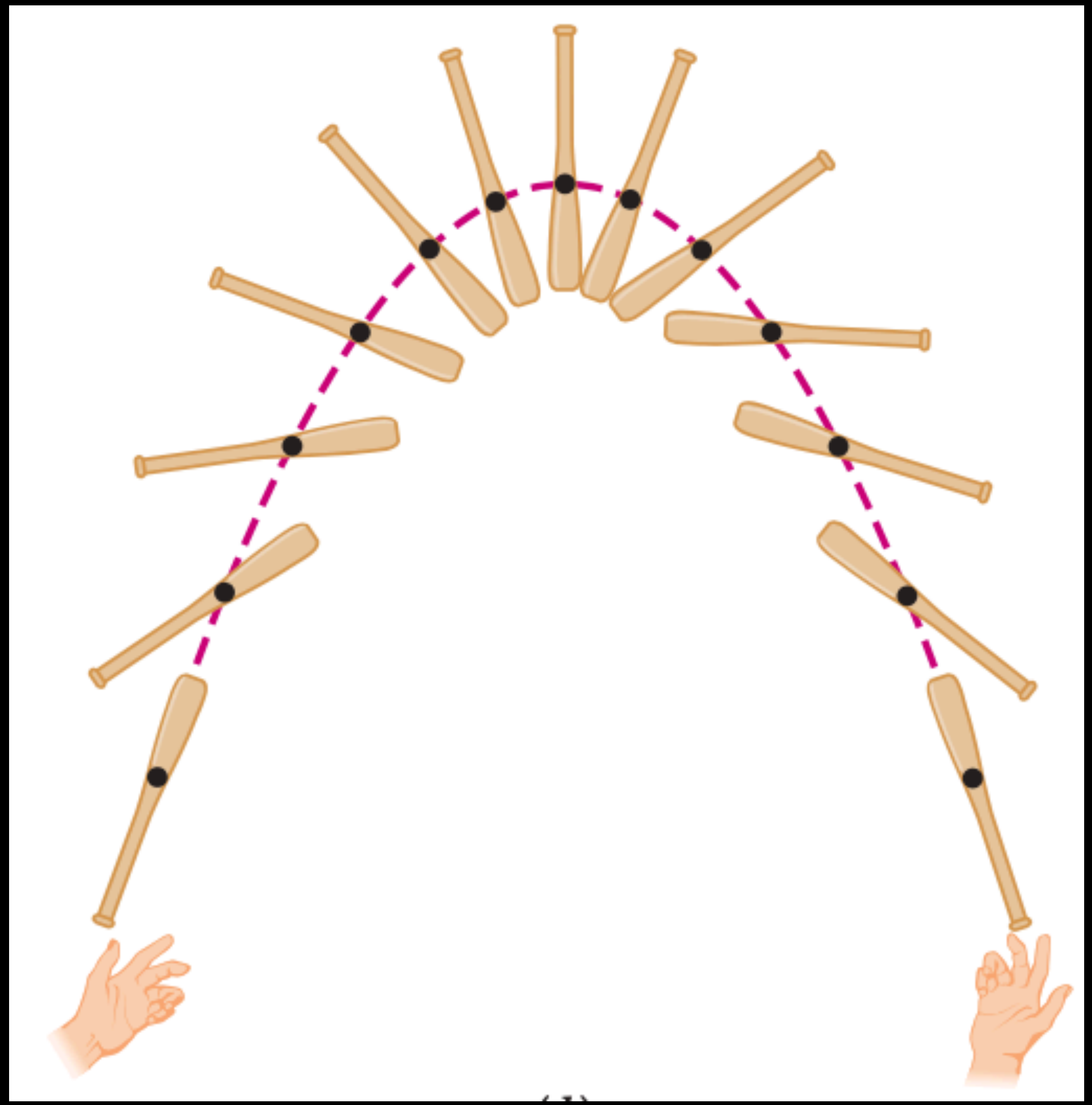


$$x = cost, y = sint$$

Lecture IV : Center of Mass

The center of mass of a system of particles is the point that moves as though

(1) all of the system's mass were concentrated there and
(2) all external forces were applied there.



Lecture IV : Center of Mass

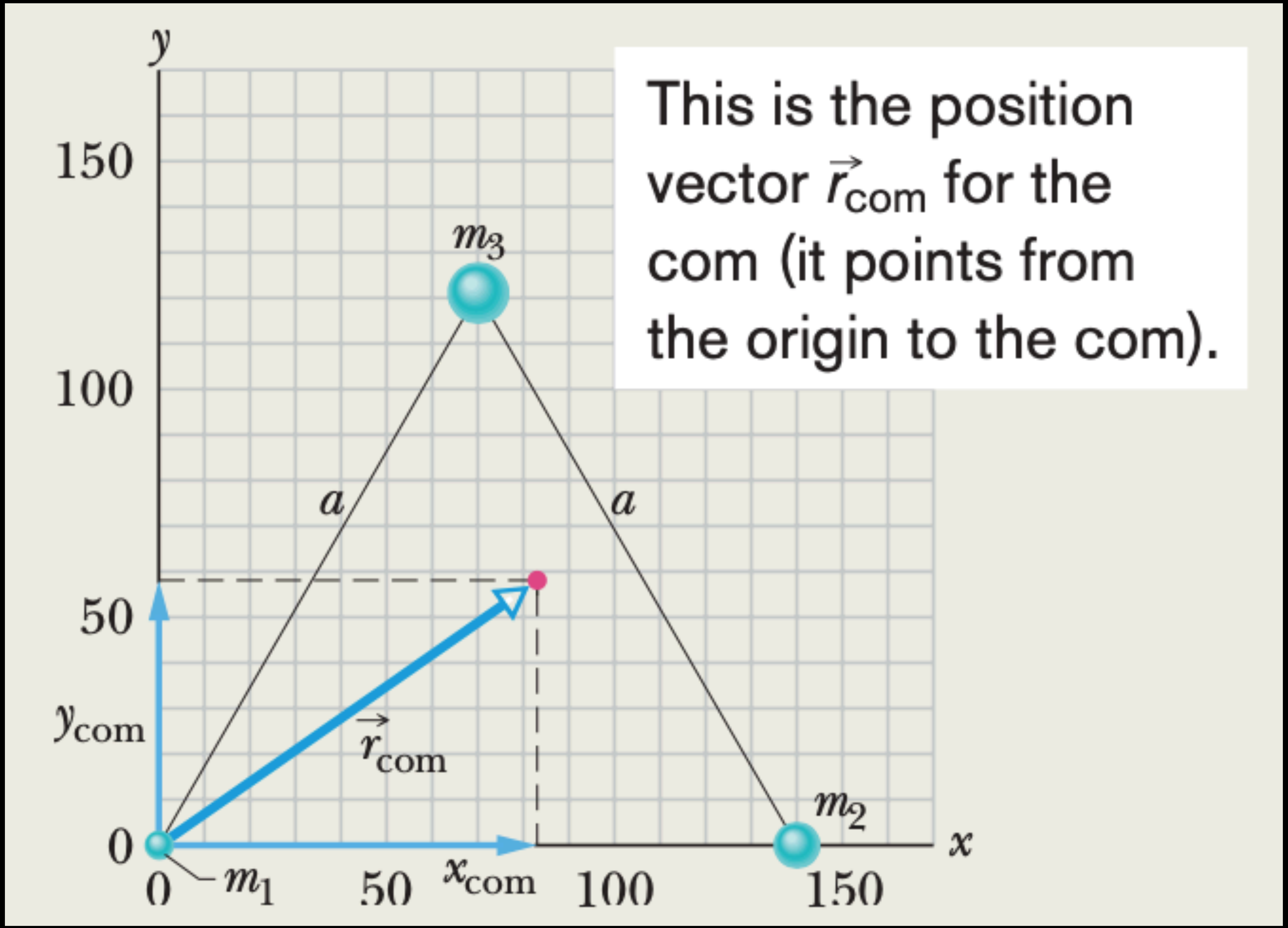
$$X_c = \frac{1}{M}\sum m_i x_i \qquad Y_c = \frac{1}{M}\sum m_i y_i \qquad Z_c = \frac{1}{M}\sum m_i z_i$$

$$\vec{r}_c = X_c\hat{i} + Y_c\hat{j} + Z_c\hat{j}$$

$$X_c = \frac{1}{M}\int x\,dm = \frac{1}{V}\int x\,dV$$

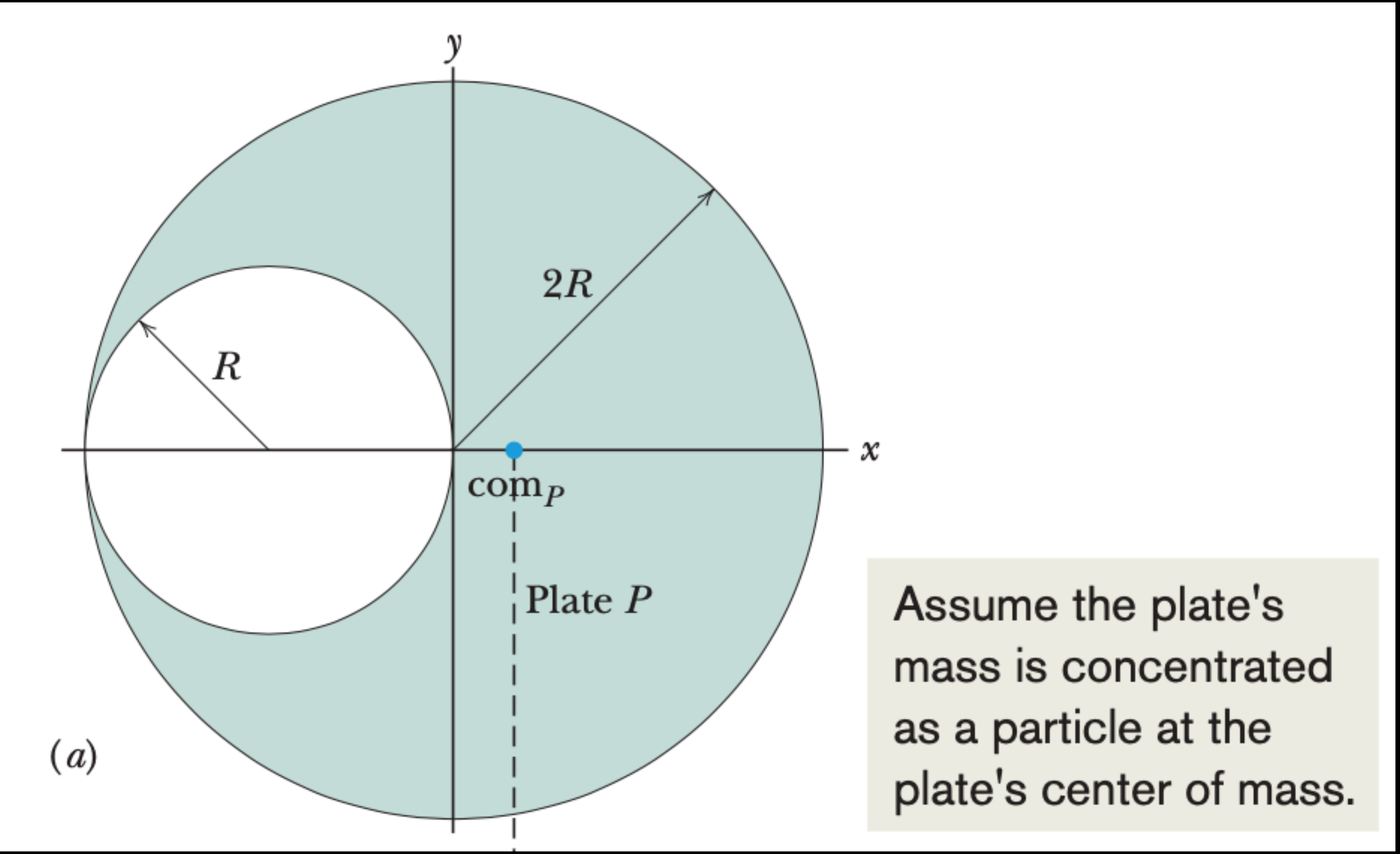
$$\rho = \frac{dm}{dV} = \frac{M}{V}$$

Lecture IV : CoM



Particle	Mass (kg)	x (cm)	y (cm)
1	1.2	0	0
2	2.5	140	0
3	3.4	70	120

Lecture IV : CoM



Lecture IV : Newton's second law for a system of particles

$$\overrightarrow{F}_{net} = M\overrightarrow{a}$$

$$\overrightarrow{F}_{net,x} = M\overrightarrow{a}_x$$

$$\overrightarrow{F}_{net,y} = M\overrightarrow{a}_y$$

$$\overrightarrow{F}_{net,z} = M\overrightarrow{a}_z$$

Lecture IV : Momentum

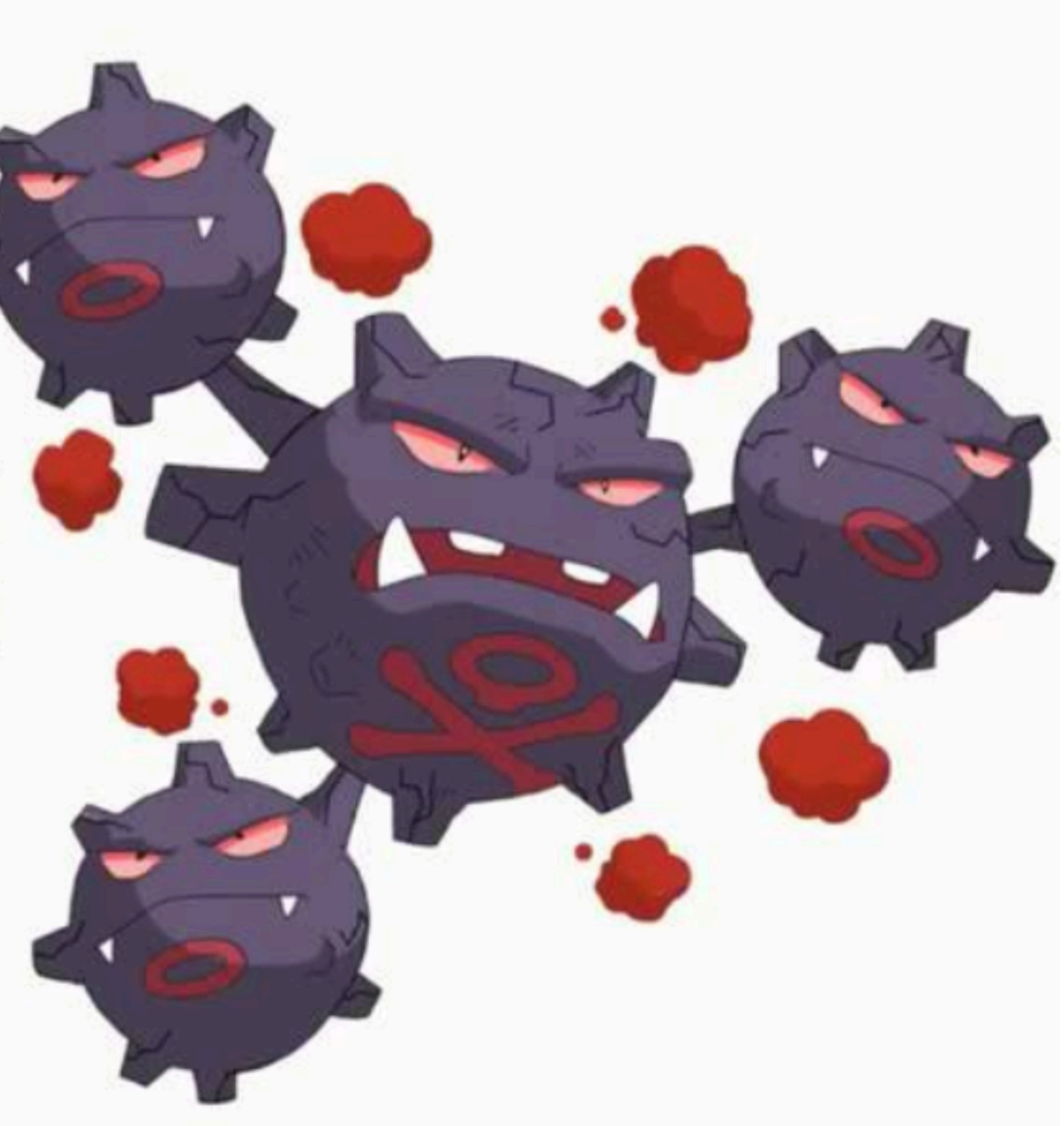
$$m\vec{v} \equiv \vec{P}$$

$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{dm\vec{v}}{dt}$$

The time rate of change of the momentum of a particle is equal to the net force acting on the particle and is in the direction of that force.

$$\vec{F} = \frac{d\vec{P}}{dt}$$

Lecture IV : The momentum of a system of particles



$$\vec{P}_{system} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \vec{p}_4 + \ldots$$

$$= m_1 \vec{v}_1 + m_2 \vec{v}_2 + m_3 \vec{v}_3 + \ldots$$

$$= M_{com} \vec{v}_{com}$$

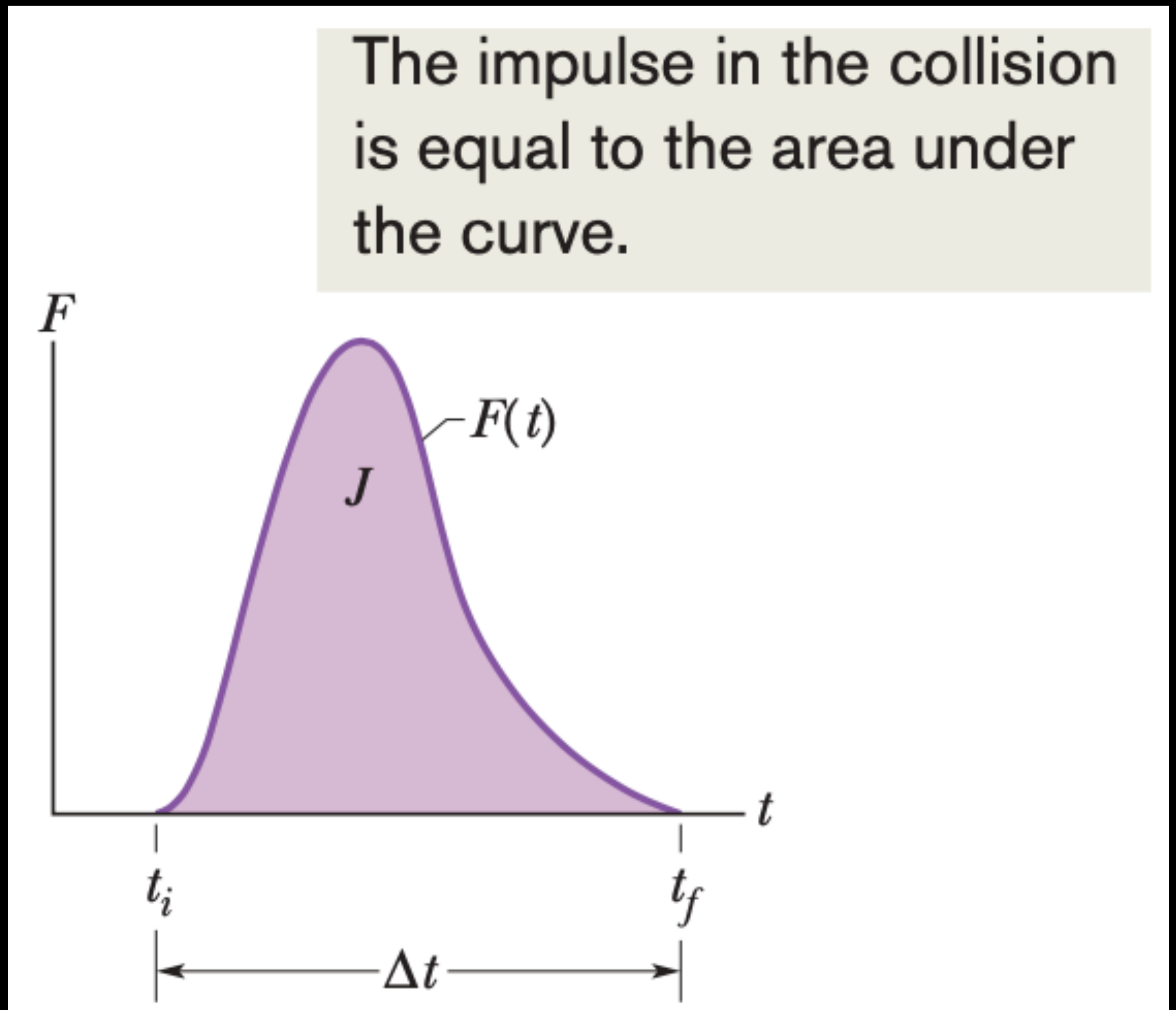
$$\frac{\vec{P}_{system}}{dt} = M \frac{\overrightarrow{v_{com}}}{dt} = M_{com} \vec{a}_{com}$$

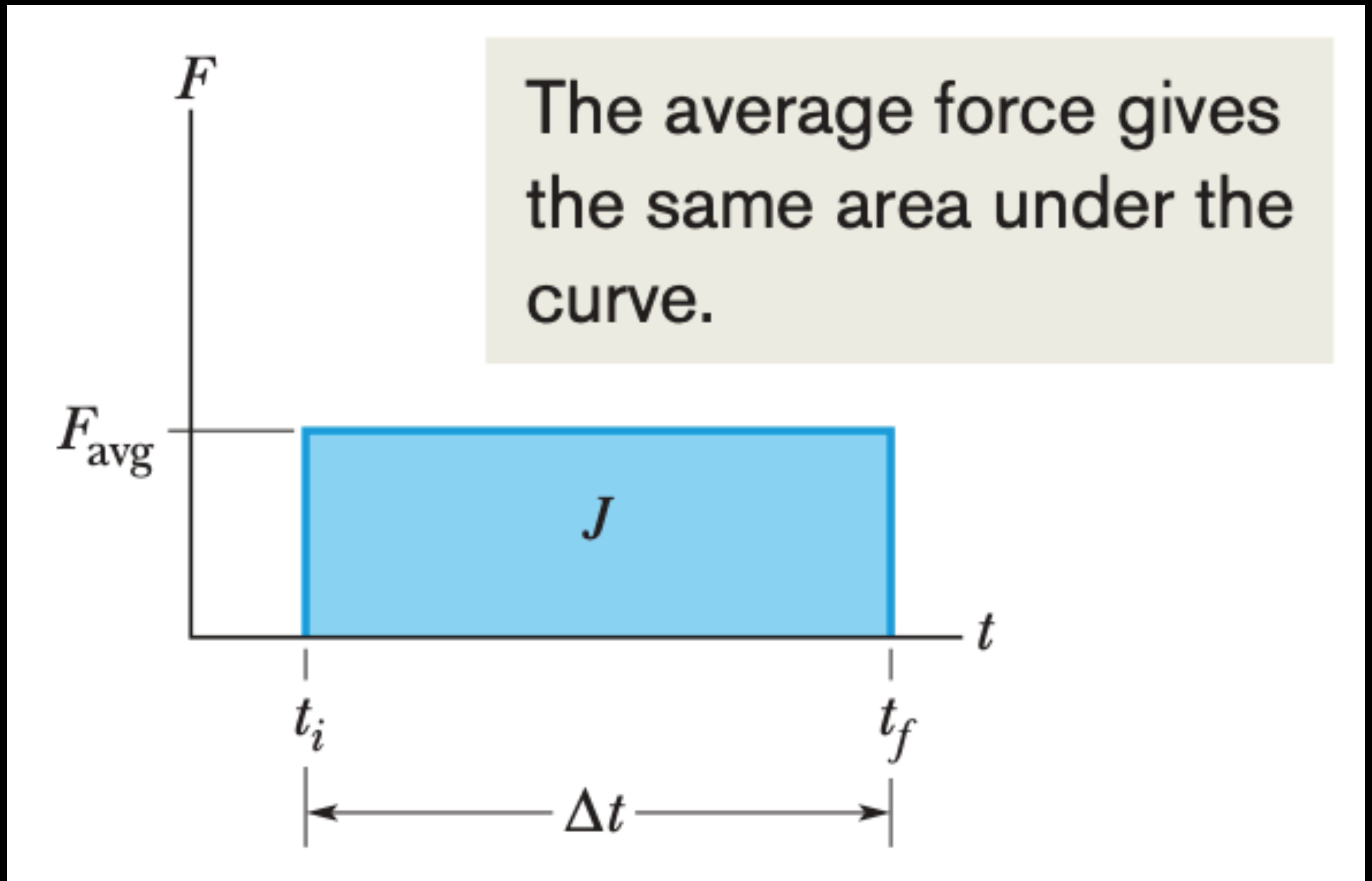
Lecture IV : Impulse (Integrating the Force.)

$$d\vec{p} = \vec{F}(t)dt$$

$$\vec{J} \equiv \int_{ti}^{tf} d\vec{p} = \int_{ti}^{tf} \vec{F}(t)dt = \Delta\vec{p}$$

$$\vec{J} = F_{avg}\Delta t$$





Lecture IV : Conservation of momentum

Suppose that the net external force $\overrightarrow{F}_{net}$ (and thus the net impulse : $\overrightarrow{J}$) acting on a net system of particles is zero (the system is isolated) we yields

$$\frac{d\overrightarrow{P}}{dt} = 0 \quad \text{Or} \quad \overrightarrow{P} = constant$$

If no net external force acts on a system of particles, the total linear momentum P of the system cannot change.

Lecture IV : Momentum & Kinetic Energy in Collisions

Considering the kinetic energy of a system of two colliding bodies.

If that total happens to be unchanged by the collision, then the kinetic energy of the system is conserved (it is the same before and after the collision). Such a collision is called an elastic collision.

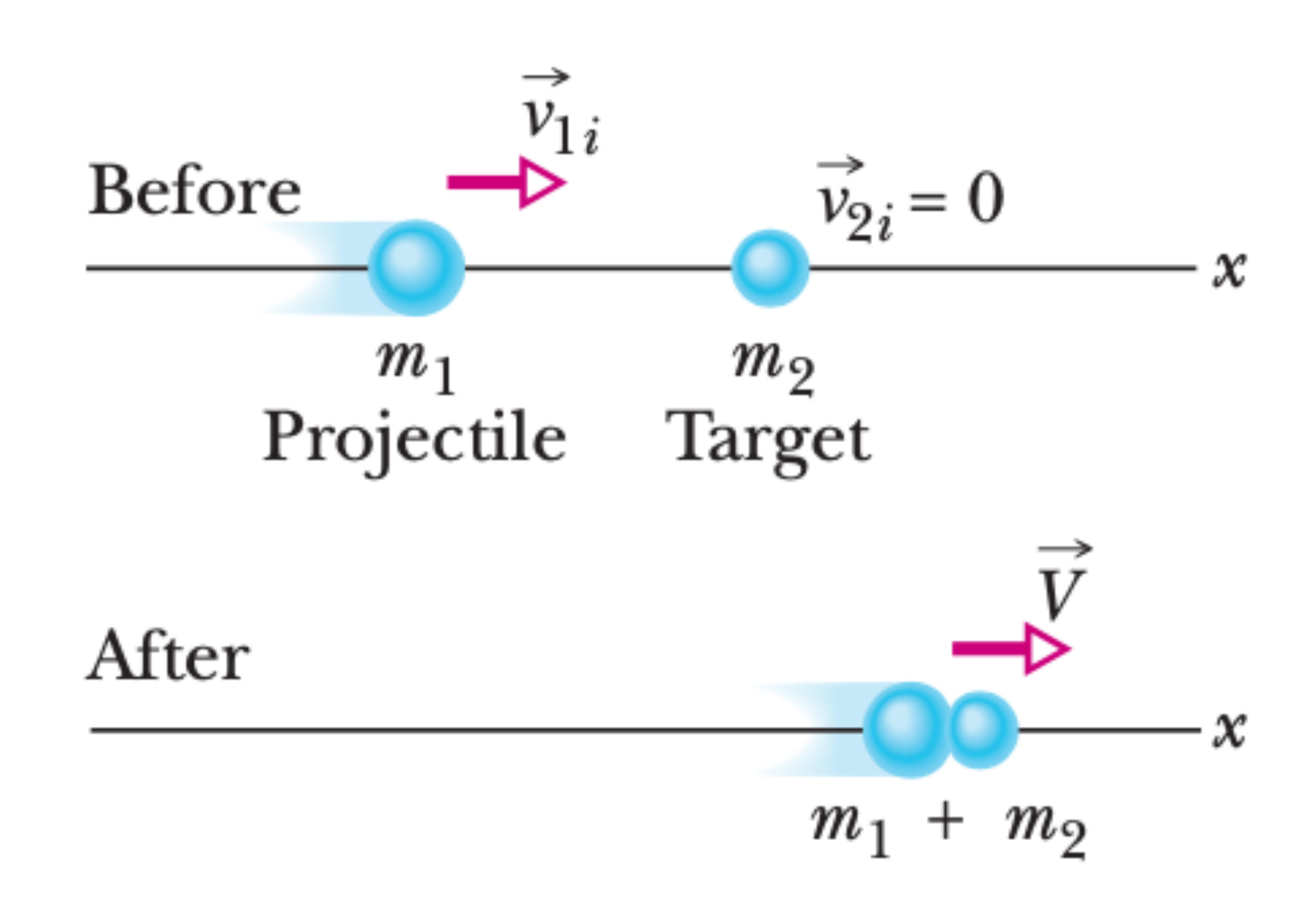
the kinetic energy of the system is not conserved. Such a collision is called an inelastic collision.

Lecture IV : Inelastic Collision in 1D

$$\overrightarrow{P} = constant$$

Total momentum $\overrightarrow{P}_i$ before the collision = Total momentum $\overrightarrow{P}_i$ after the collision

$$m_1 v_{1,i} + m_2 v_{2,i} = m_1 v_{1,f} + m_2 v_{2,f}$$



$$m_1 \overrightarrow{v}_{1,i} = (m_1 + m_2)\overrightarrow{V}$$

$$\overrightarrow{V} = \frac{m_1 \overrightarrow{v}_{1,i}}{(m_1 + m_2)}$$

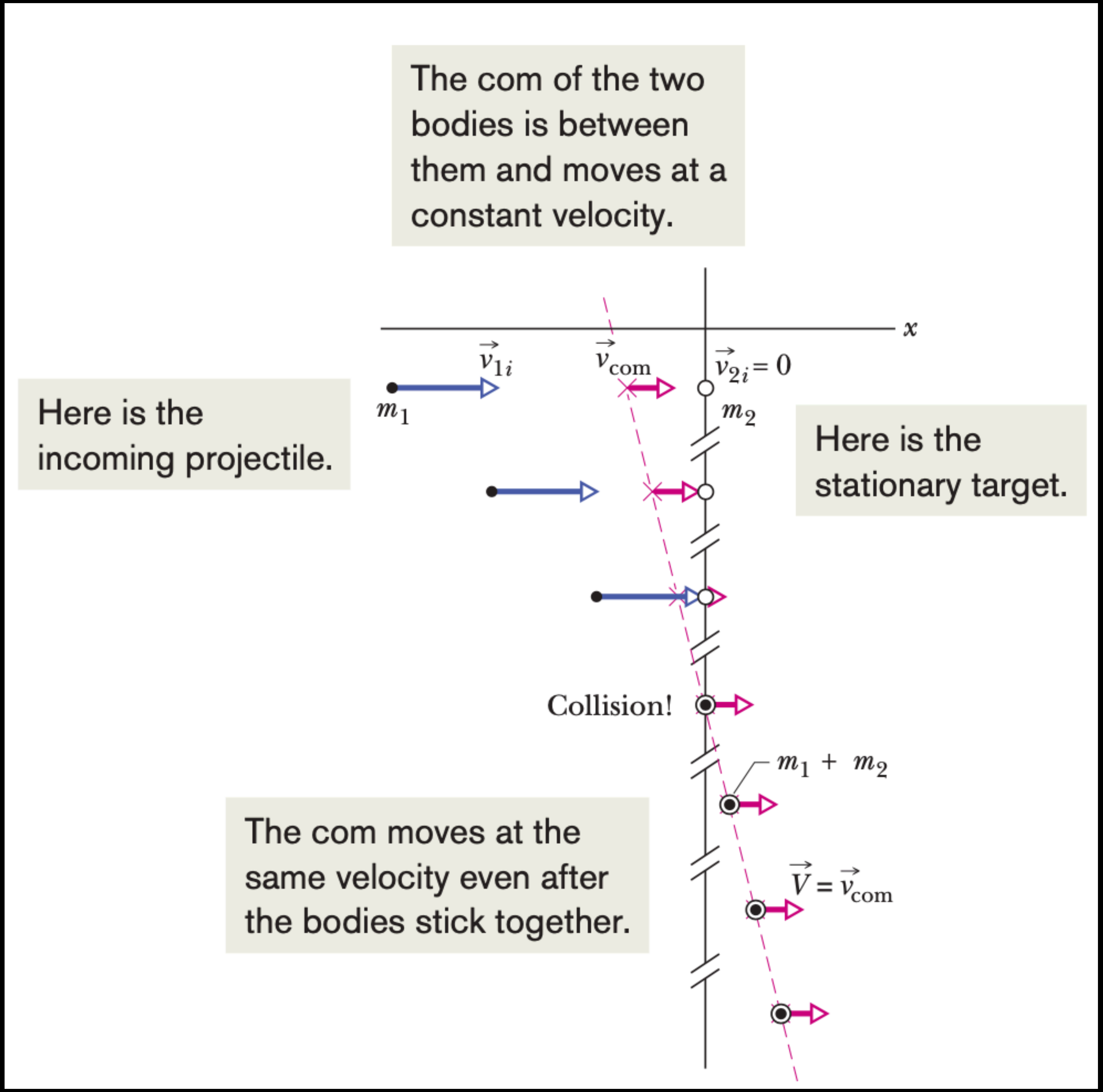
Lecture IV : Inelastic Collision in 1D

$$\vec{P} = M\vec{v}_{com}$$

$$\vec{P} = \vec{p}_{1,f} + \vec{p}_{2,f}$$

$$\vec{v}_{com} = \frac{\vec{P}}{M} = \frac{\vec{P}}{m_1 + m_2}$$

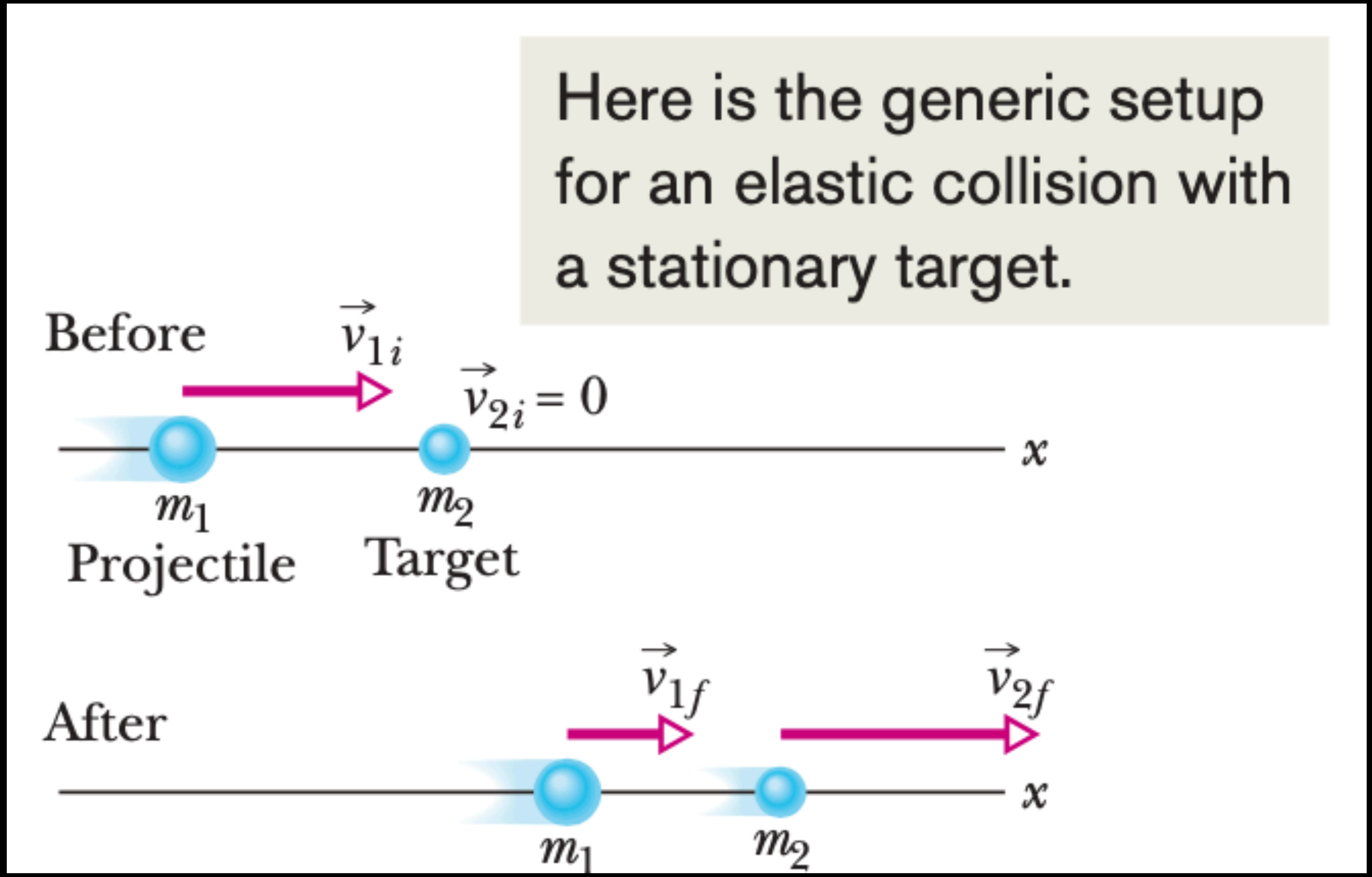
$$= \frac{\vec{p}_{1,i} + \vec{p}_{2,i}}{m_1 + m_2}$$



Lecture IV : Elastic Collision in 1D

the total kinetic energy of the colliding bodies is conserved and is not transferred to other forms of energy

Total kinetic energy $\overrightarrow{K}_i$ before the collision = Total momentum $\overrightarrow{K}_i$ after the collision



$$m_1 v_{1,i} + m_2 v_{2,i} = m_1 v_{1,f} + m_2 v_{2,f}$$

$$m_1 v_{1,i} = m_1 v_{1,f} + m_2 v_{2,f}$$

$$\frac{1}{2} m_1 v_{1,i}^2 + \frac{1}{2} m_2 v_{2,i}^2 = \frac{1}{2} m_1 v_{1,f}^2 + \frac{1}{2} m_2 v_{2,f}^2$$

$$\frac{1}{2} m_1 v_{1,i}^2 = \frac{1}{2} m_1 v_{1,f}^2 + \frac{1}{2} m_2 v_{2,f}^2$$

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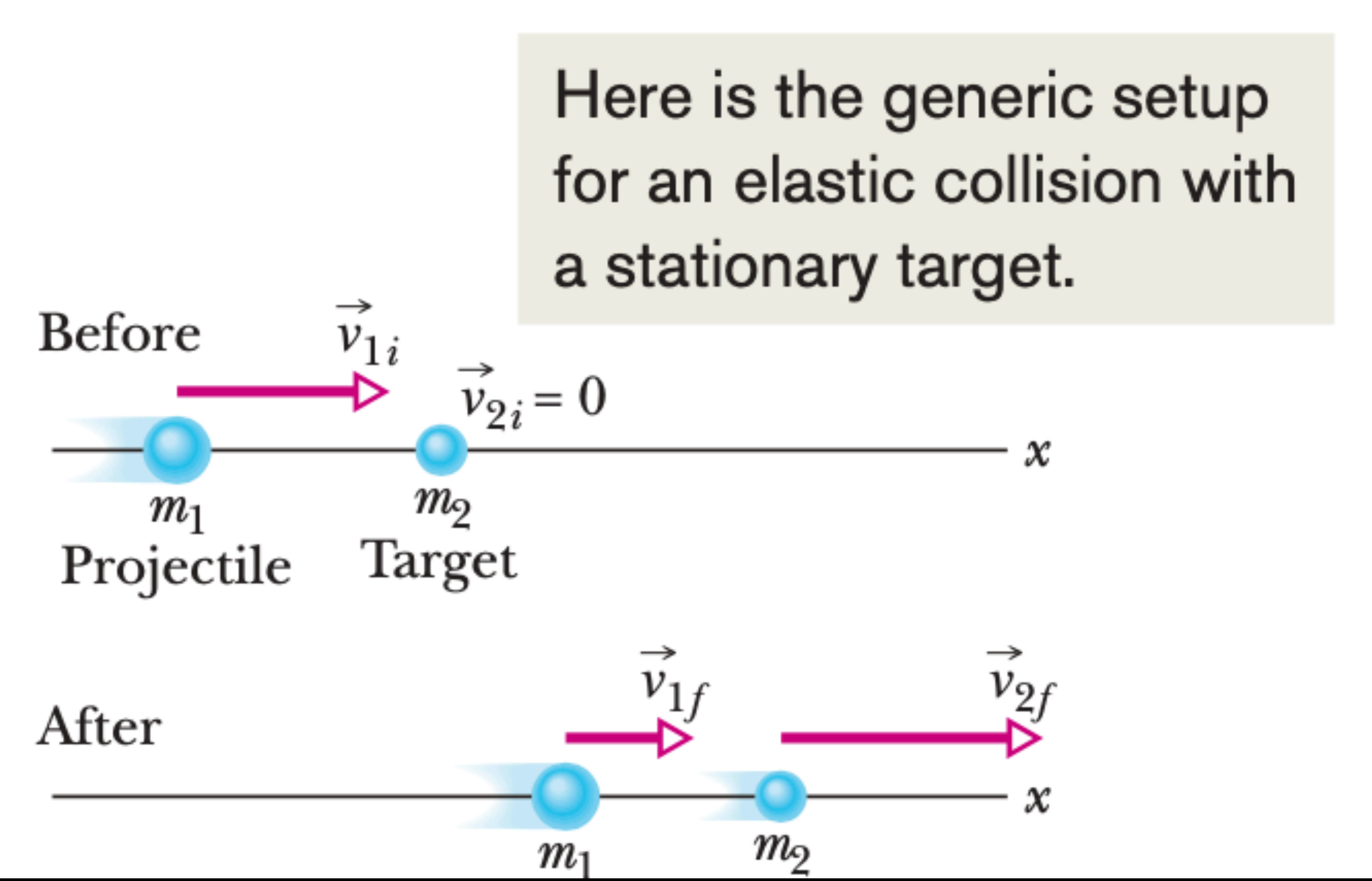

$$m_1 v_{1,i} = m_1 v_{1,f} + m_2 v_{2,f}$$

$$\frac{1}{2} m_1 v_{1,i}^2 = \frac{1}{2} m_1 v_{1,f}^2 + \frac{1}{2} m_2 v_{2,f}^2$$

Lecture IV : Elastic Collision in 1D

the total kinetic energy of the colliding bodies is conserved and is not transferred to other forms of energy

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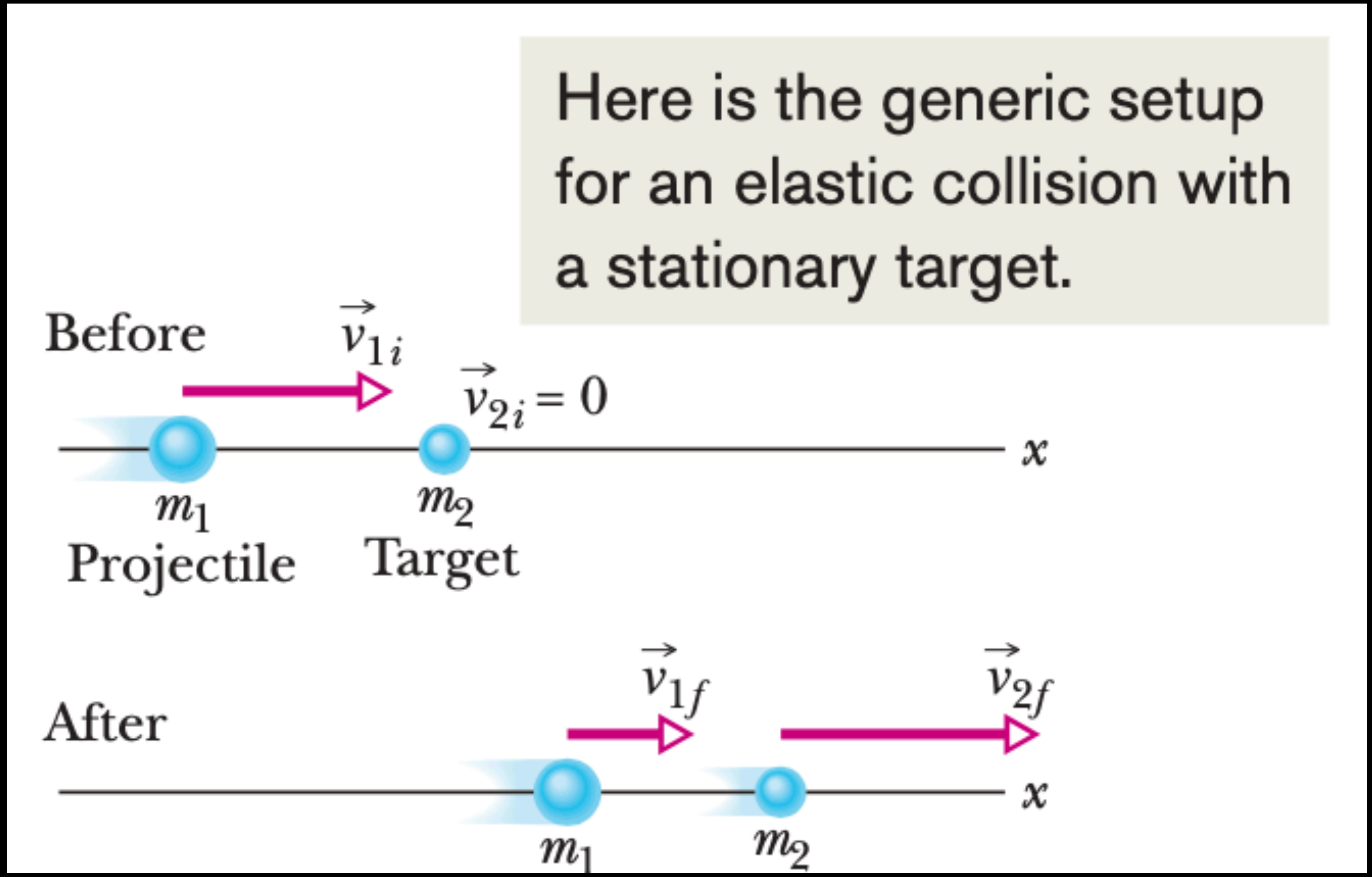


$$m_1 v_{1,i} = m_1 v_{1,f} + m_2 v_{2,f}$$

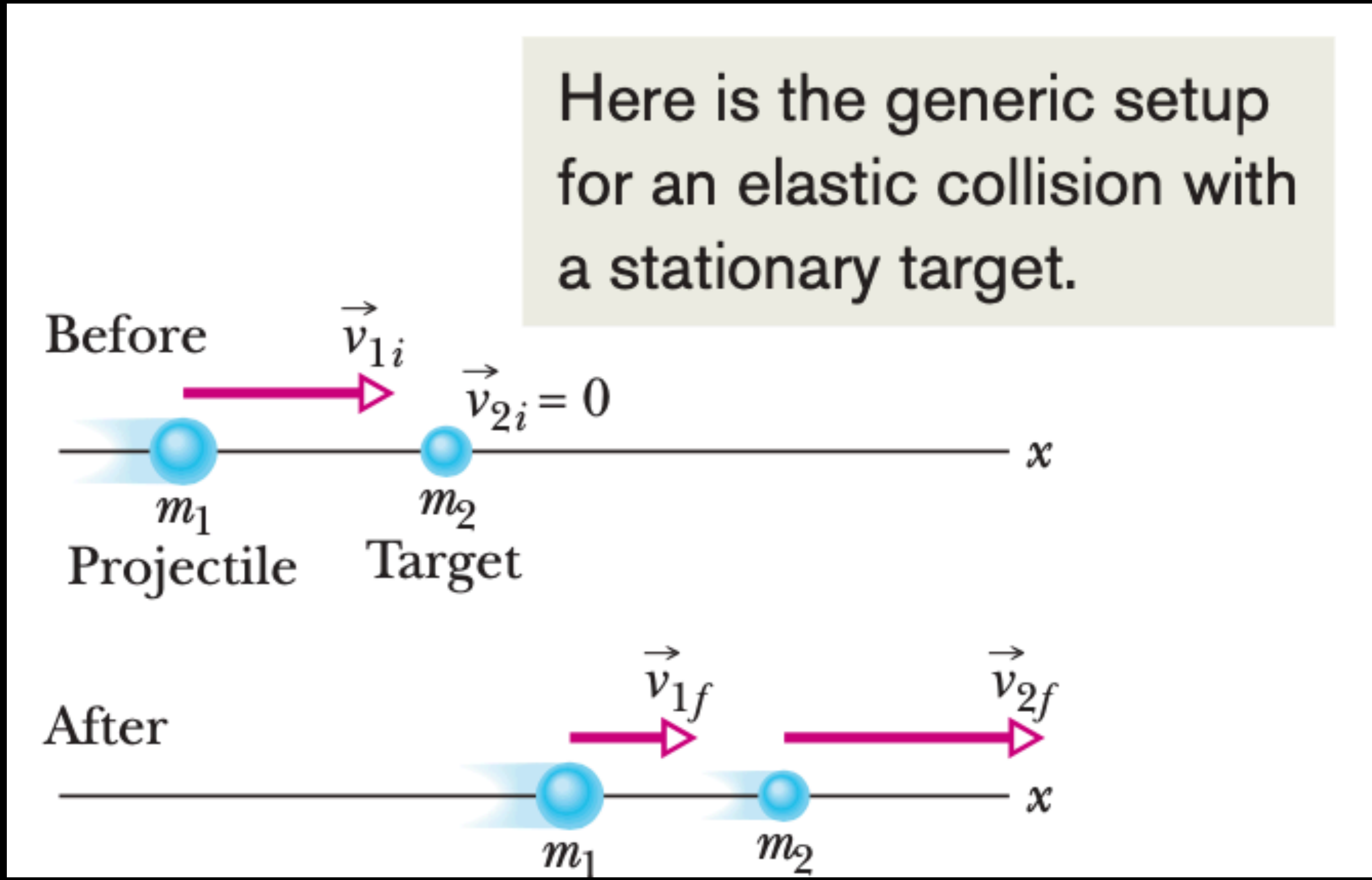
$$m_1 (v_{1,i} - v_{1,f}) = m_2 v_{2,f}$$

$$\frac{1}{2} m_1 v_{1,i}^2 = \frac{1}{2} m_1 v_{1,f}^2 + \frac{1}{2} m_2 v_{2,f}^2$$

Lecture IV : Elastic Collision in 1D

the total kinetic energy of the colliding bodies is conserved and is not transferred to other forms of energy

Total kinetic energy $\overrightarrow{K}_i$ before the collision = Total momentum $\overrightarrow{K}_i$ after the collision



$$m_1 v_{1,i} = m_1 v_{1,f} + m_2 v_{2,f}$$

$$m_1(v_{1,i} - v_{1,f}) = m_2 v_{2,f}$$

$$\frac{1}{2} m_1 v_{1,i}^2 = \frac{1}{2} m_1 v_{1,f}^2 + \frac{1}{2} m_2 v_{2,f}^2$$

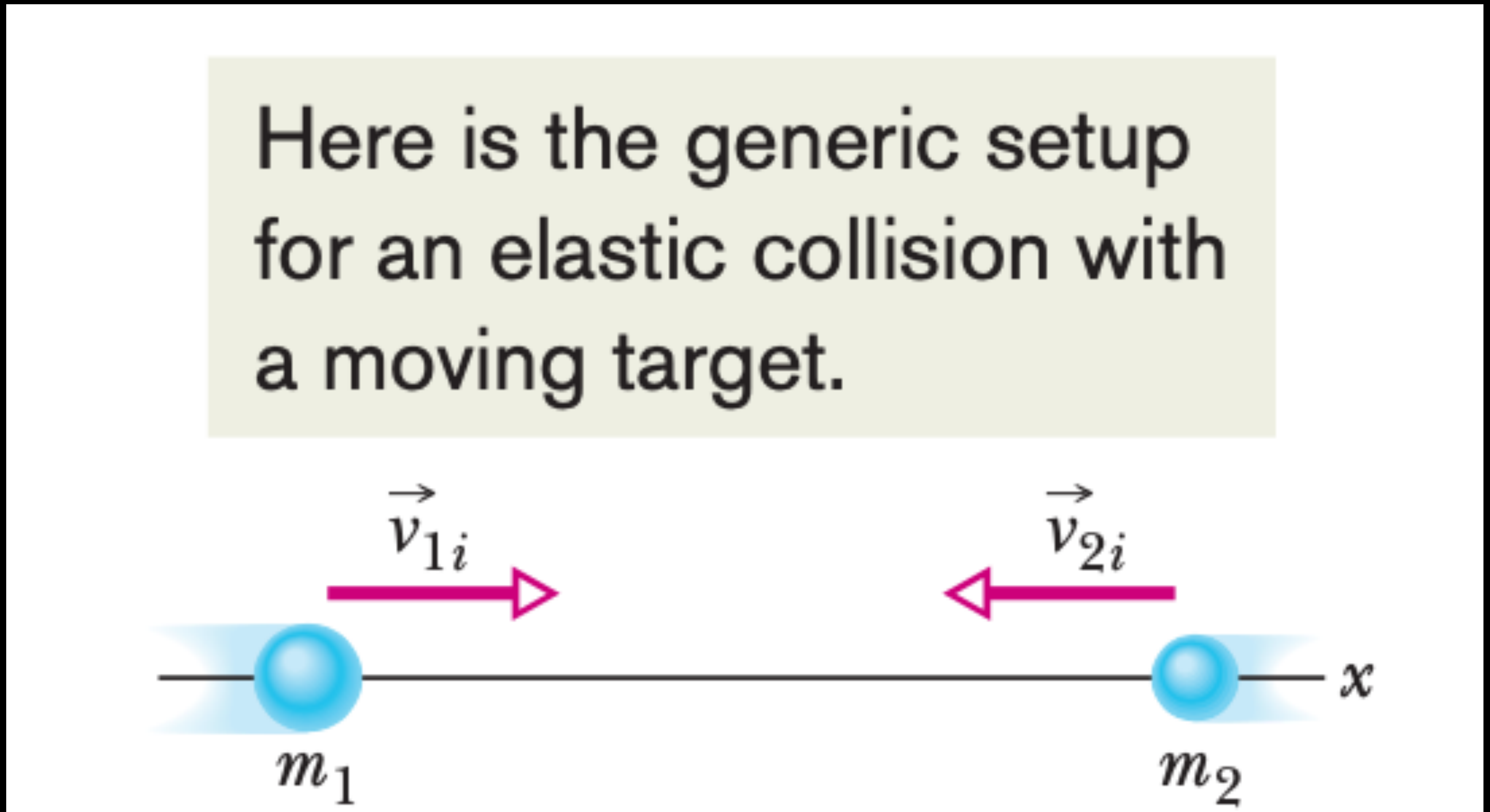
$$m_1 v_{1,i}^2 - m_1 v_{1,f}^2 = m_2 v_{2,f}^2$$

$$m_1(v_{1,i} - v_{1,f})(v_{1,i} + v_{1,f}) = m_2 v_{2,f}^2$$

Lecture IV : Elastic Collision in 1D

Total kinetic energy $\overrightarrow{K}_i$ before the collision = Total momentum $\overrightarrow{K}_i$ after the collision

$$m_1 v_{1,i} + m_2 v_{2,i} = m_1 v_{1,f} + m_2 v_{2,f}$$



$$m_1(v_{1,i} - v_{1,f}) = -m_2(v_{2,i} - v_{2,f})$$

$$\frac{1}{2}m_1 v_{1,i}^2 + \frac{1}{2}m_2 v_{2,i}^2 = \frac{1}{2}m_1 v_{1,f}^2 + \frac{1}{2}m_2 v_{2,f}^2$$

$$m_1(v_{1,i} - v_{1,f})(v_{1,i} + v_{1,f})$$
$$= -m_2(v_{2,i} - v_{2,f})(v_{2,i} + v_{2,f})$$

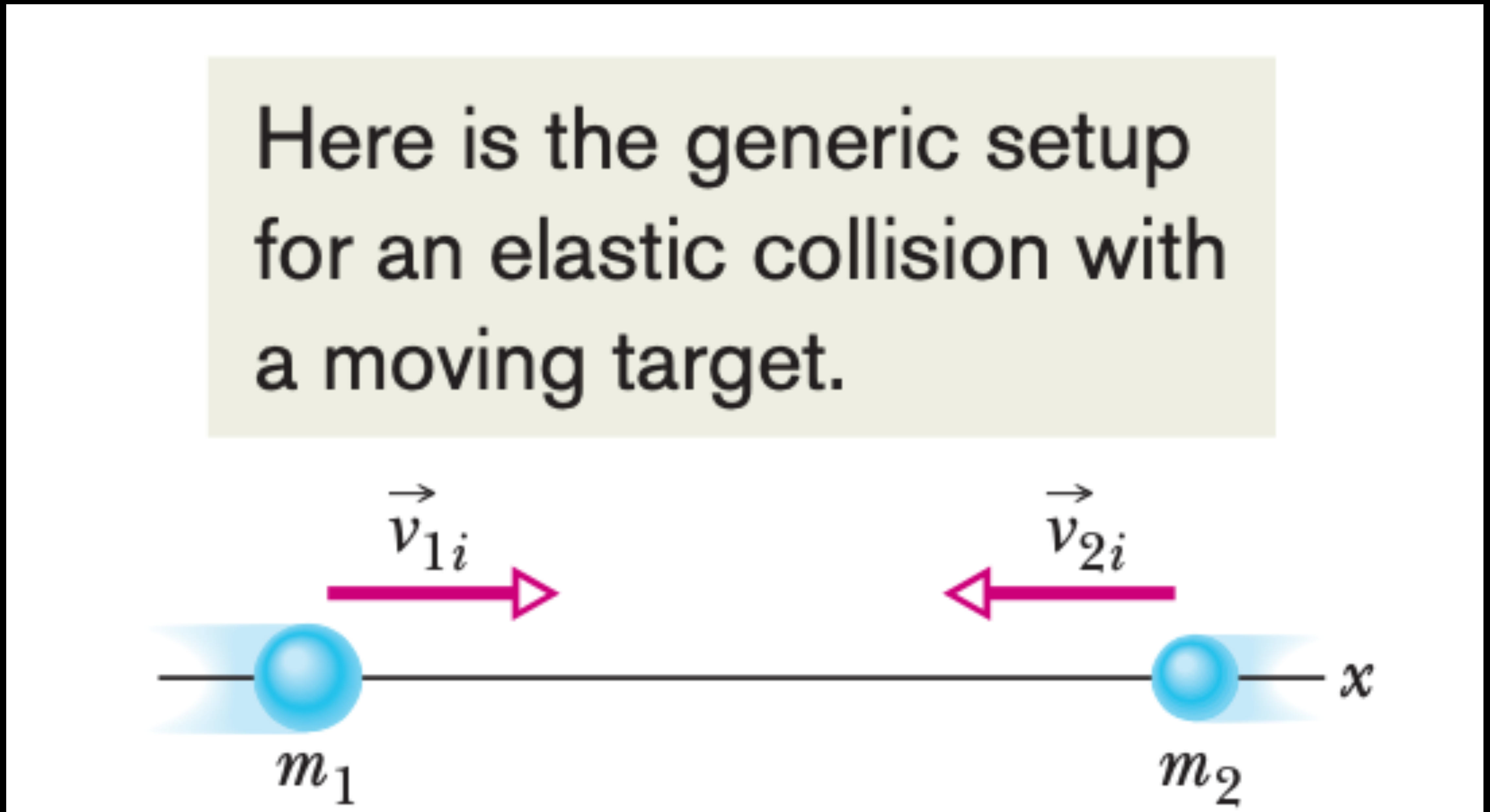
Lecture IV : Elastic Collision in 1D

Total kinetic energy $\overrightarrow{K}_i$ before the collision = Total momentum $\overrightarrow{K}_i$ after the collision

$$m_1(v_{1,i} - v_{1,f}) = -m_2(v_{2,i} - v_{2,f})$$

$$m_1(v_{1,i} - v_{1,f})(v_{1,i} + v_{1,f})$$

$$= -m_2(v_{2,i} - v_{2,f})(v_{2,i} + v_{2,f})$$



$$v_{1,f} = \frac{m_1 - m_2}{m_1 + m_2} v_{i,i} + \frac{2m_2}{m_1 + m_2} v_{2,i}$$

$$v_{2,f} = \frac{m_2 - m_1}{m_1 + m_2} v_{2,i} + \frac{2m_1}{m_1 + m_2} v_{1,i}$$

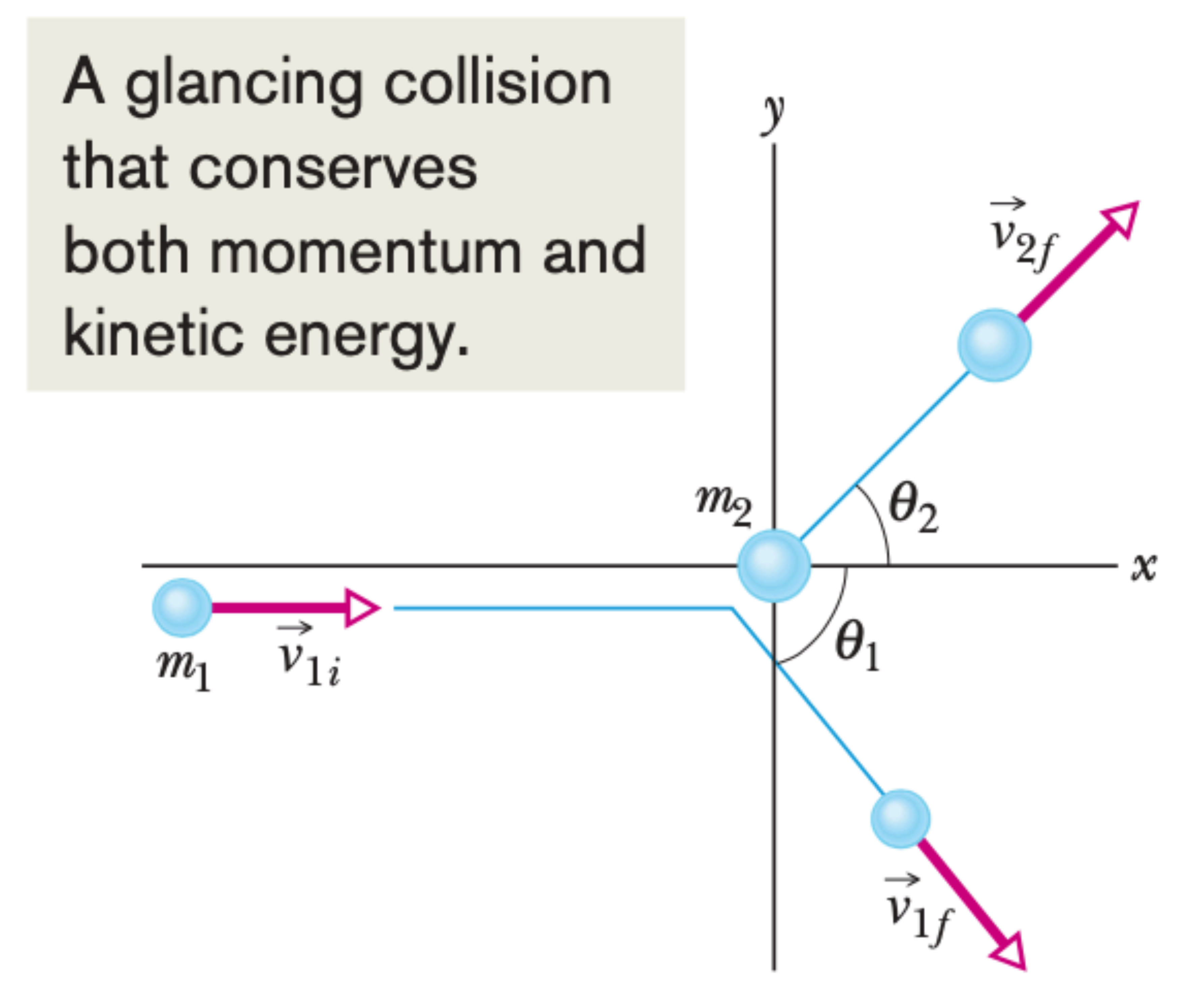
Lecture IV : Elastic Collision in 2D

Total kinetic energy $\vec{K}_i$ before the collision = Total momentum $\vec{K}_i$ after the collision

$$m_1 v_{1,i} + m_2 v_{2,i} = m_1 v_{1,f} + m_2 v_{2,f}$$

X-axis $$m_1 v_{1,i} = m_1 v_{1,f} \cos\theta_1 + m_2 v_{2,f} \cos\theta_2$$

Y-axis $$0 = -m_1 v_{1,f} \sin\theta_1 + m_2 v_{2,f} \sin\theta_2$$



$$\frac{1}{2}m_1 v_{1,i}^2 + \frac{1}{2}m_2 v_{2,i}^2 = \frac{1}{2}m_1 v_{1,f}^2 + \frac{1}{2}m_2 v_{2,f}^2$$

$$\frac{1}{2}m_1 v_{1,i}^2 = \frac{1}{2}m_1 v_{1,f}^2 + \frac{1}{2}m_2 v_{2,f}^2$$