General Physics I

Mechanics, optics, thermal dynamics, and other basic fundamental things.

TsungChe Liu

Reminder of the Lecture II:

Keyword:

- vector:
  - Unit vector
  - Position vector
  - adding, scalar product, vector product
- Newton’s 1st & 2nd law
- 1D & 2D motion

Lecture III : Force & Energy Again

Newton's first law

If no force acts on a body, the body's velocity cannot change; that is, the body cannot accelerate.

Newton's second law

The net force on a body is equal to the product of the body's mass and its acceleration.

Lecture III : Force & Energy again

Newton's first law

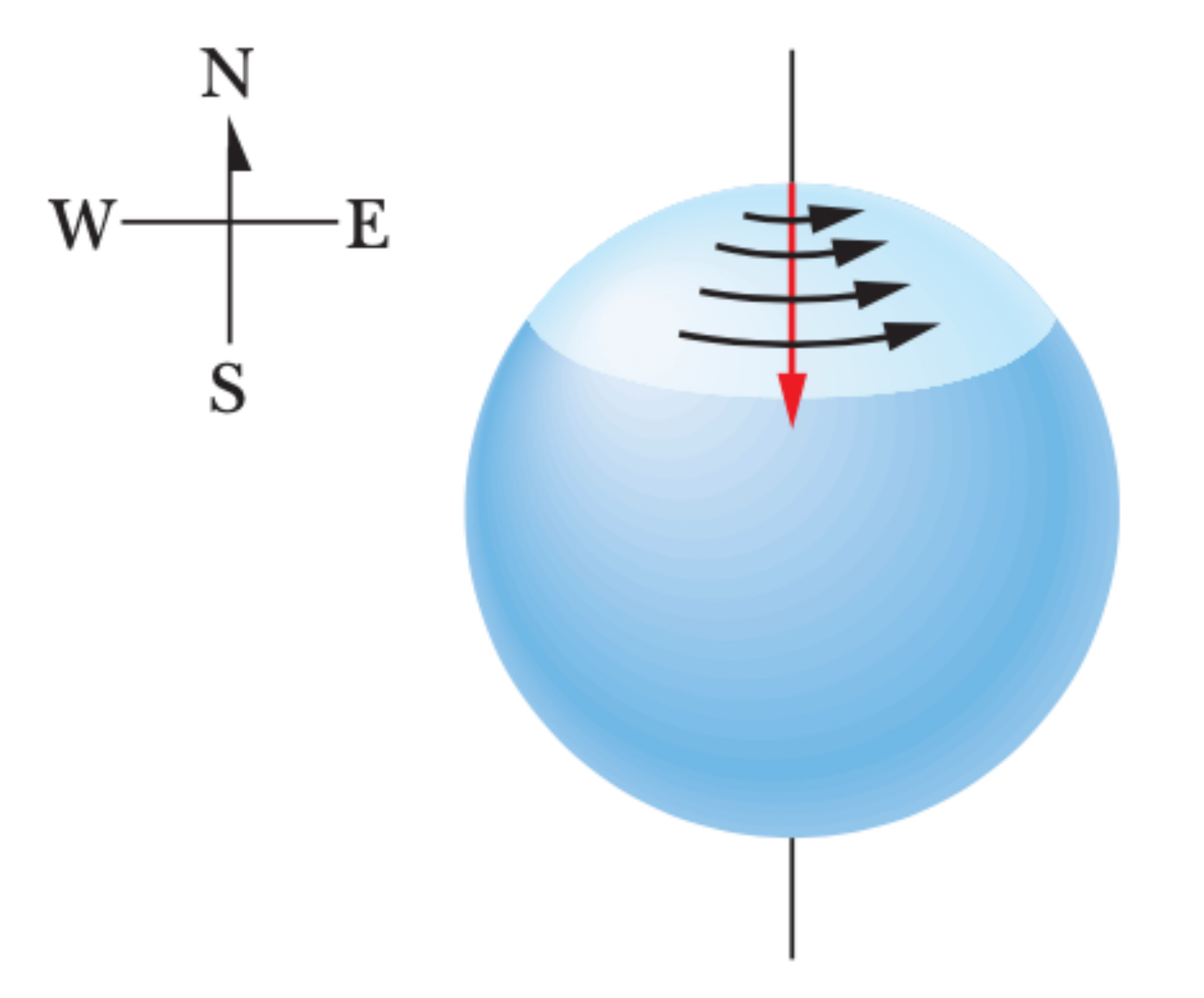
If no force acts on a body, the body's velocity cannot change; that is, the body cannot accelerate.

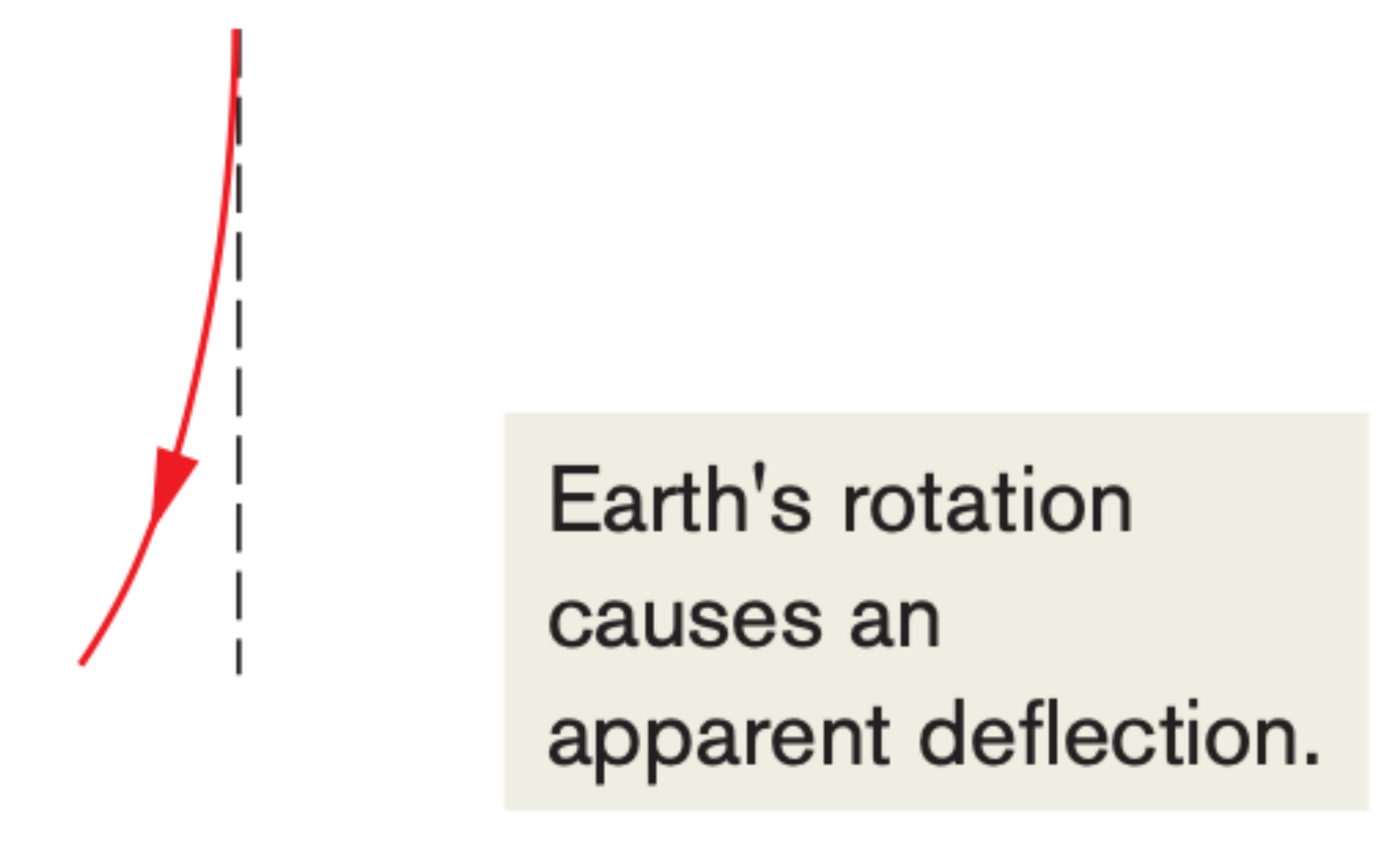
If no net force acts on a body ($\vec{F}_{net} = 0, \sum_i \vec{F}_i = 0,$), the body's velocity cannot change; that is, the body cannot accelerate.

Newton's first law is not true in all reference frames, but we can always find reference frames in which it (as well as the rest of Newtonian mechanics) is true. Such special frames are referred to as inertial reference frames, or simply inertial frames.

Lecture III : Force & Energy again

Example of non inertial frame:





Lecture III : Force & Energy again

Newton's second law

The net force on a body is equal to the product of the body's mass and its acceleration.

$$\vec{F}_{net} = m\vec{a}$$

System	Force	Mass	Acceleration
SI	newton(N)	kilogram(kg)	m/s^2
CGS	dyne	gram(g)	cm/s^2

International System of Units (SI)

Lecture III : Force & Energy again

Some forces

The gravitational force: $\vec{F}_g = m\vec{g}$

Weight: W

The normal force: N

Friction force

Tension

International System of Units (SI)

Lecture III : Force & Energy again

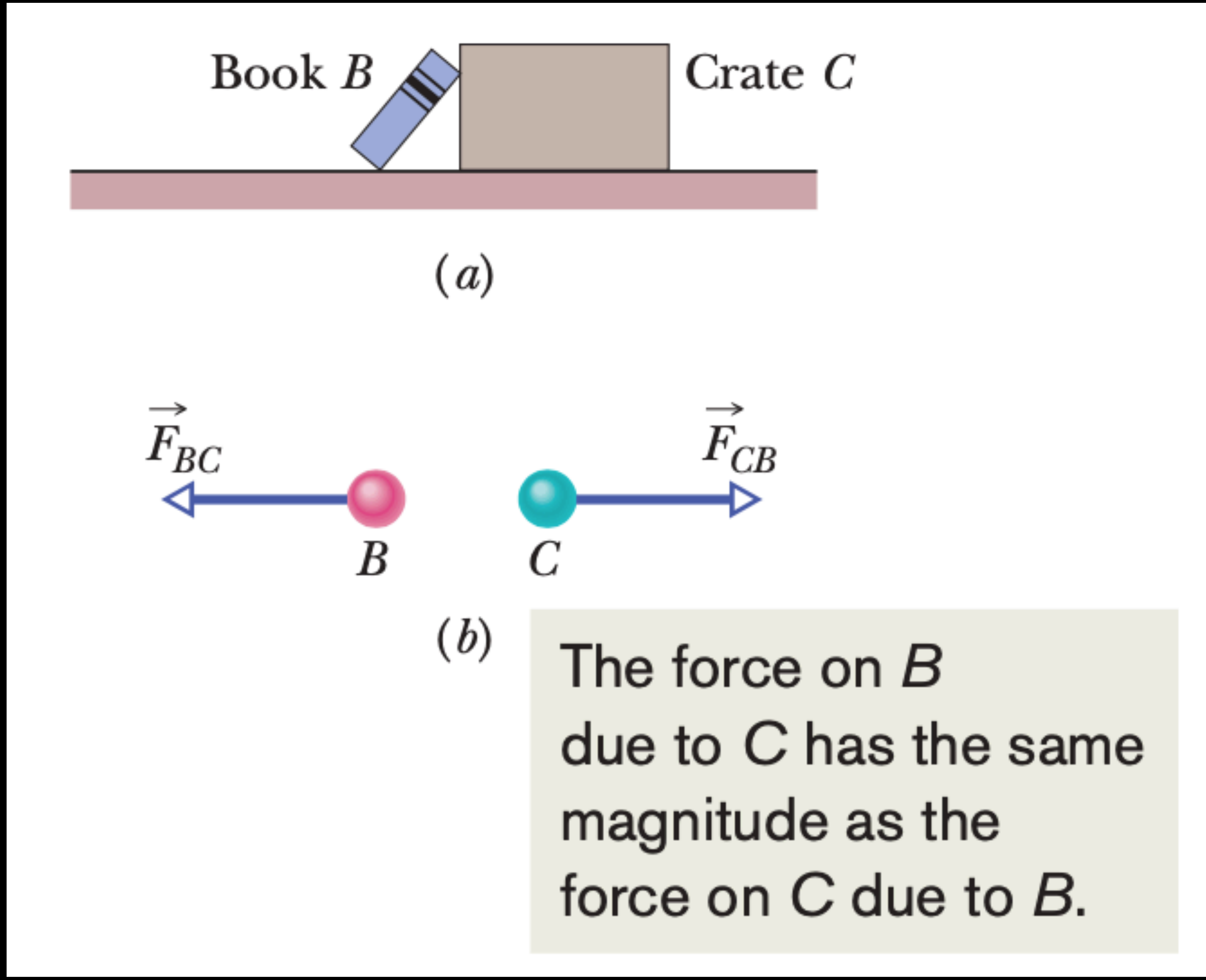
Newton’s third law

When two bodies interact, the forces on the bodies from each other are always equal in magnitude and opposite in direction.

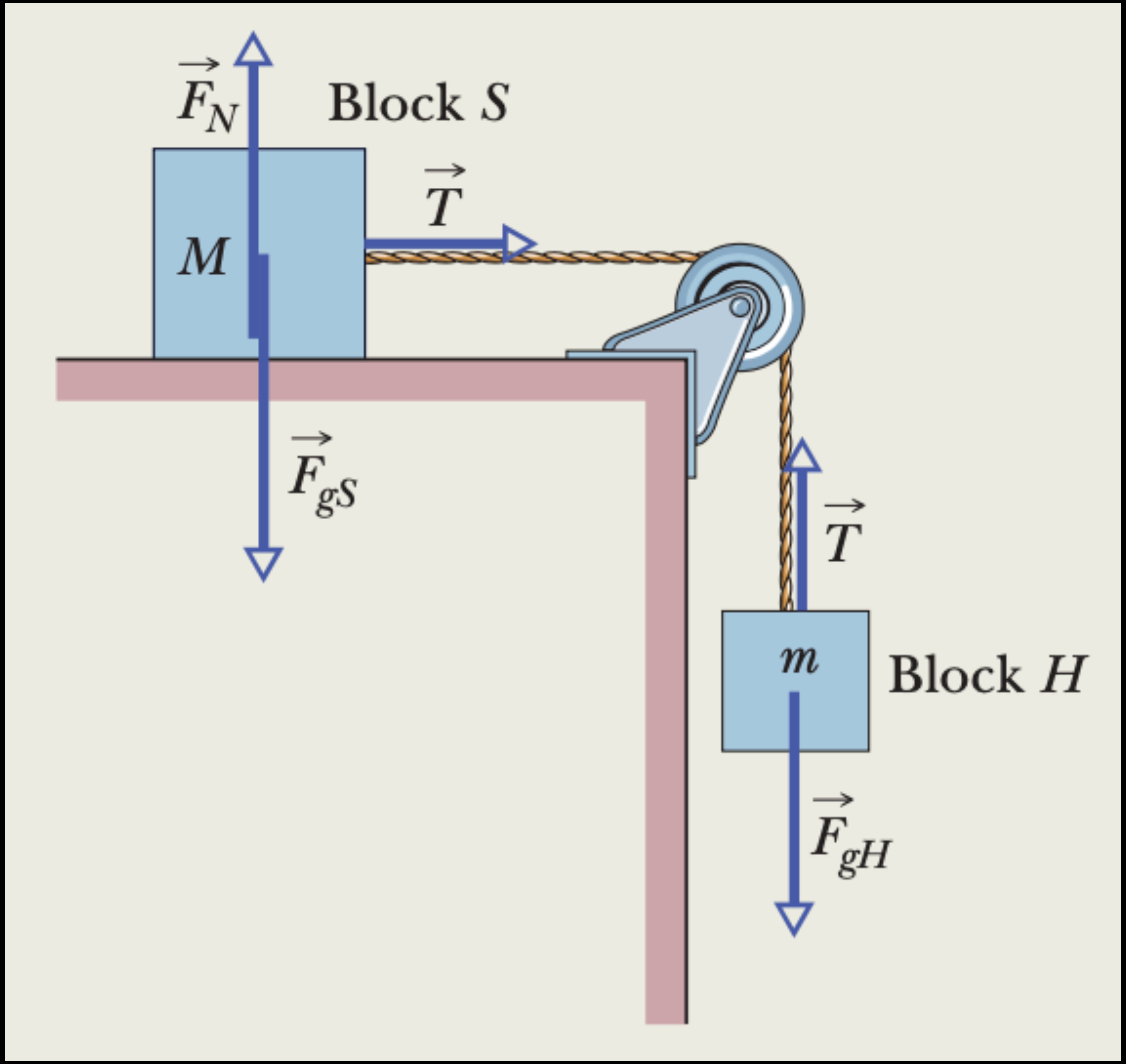
$$F_{BC} = F_{CB}$$

$$\vec{F}_{BC} = -\vec{F}_{CB}$$

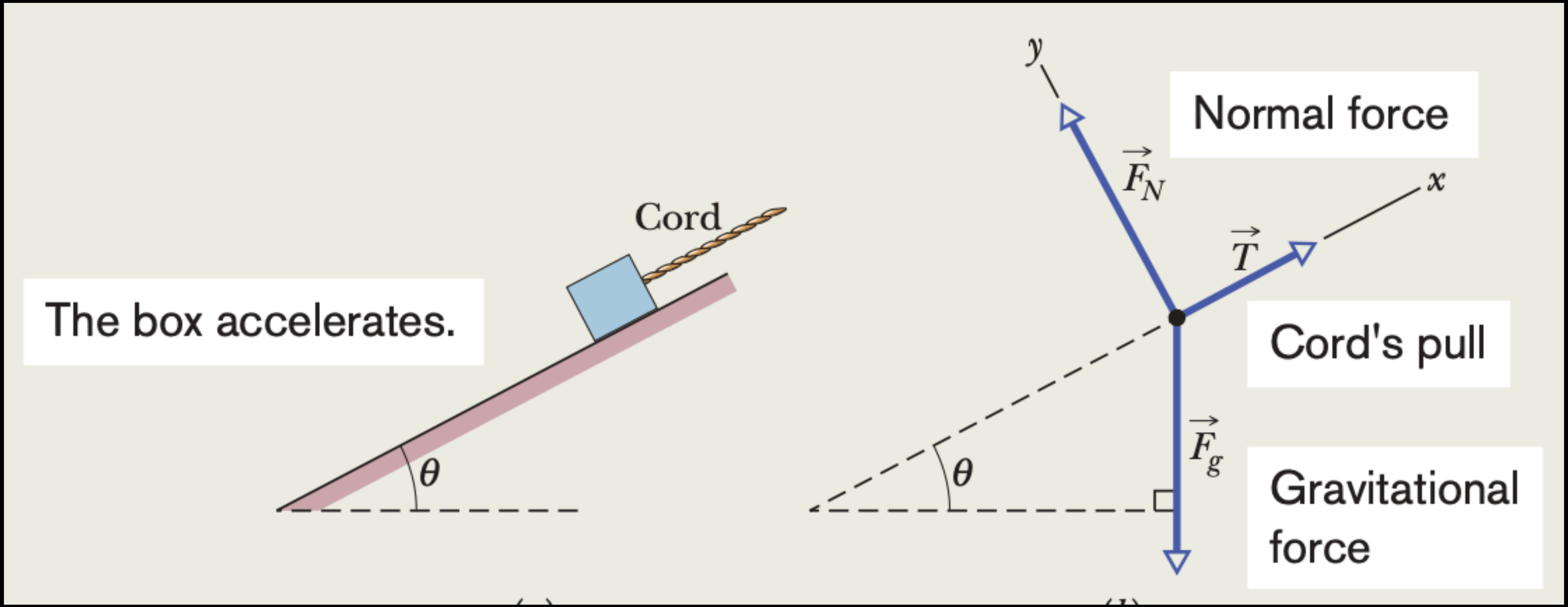
third-law force pair



Lecture III : Applying Newton's Laws



Lecture III : Applying Newton's Laws



Lecture III : Applying Newton's Laws

Newtonian Mechanics The velocity of an object can change (the object can accelerate) when the object is acted on by one or more **forces** (pushes or pulls) from other objects. *Newtonian mechanics* relates accelerations and forces.

Force Forces are vector quantities. Their magnitudes are defined in terms of the acceleration they would give the standard kilogram. A force that accelerates that standard body by exactly 1 m/s^2 is defined to have a magnitude of 1 N. The direction of a force is the direction of the acceleration it causes. Forces are combined according to the rules of vector algebra. The **net force** on a body is the vector sum of all the forces acting on the body.

Newton's First Law If there is no net force on a body, the body remains at rest if it is initially at rest or moves in a straight line at constant speed if it is in motion.

Inertial Reference Frames Reference frames in which Newtonian mechanics holds are called *inertial reference frames* or *inertial frames.* Reference frames in which Newtonian mechanics does not hold are called *noninertial reference frames* or *noninertial frames.*

Mass The **mass** of a body is the characteristic of that body that relates the body's acceleration to the net force causing the acceleration. Masses are scalar quantities.

Newton's Second Law The net force $\vec{F}_{\text{net}}$ on a body with mass m is related to the body's acceleration $\vec{a}$ by

$$\vec{F}_{\text{net}} = m\vec{a}, \tag{5-1}$$

which may be written in the component versions

$$F_{\text{net},x} = ma_x \qquad F_{\text{net},y} = ma_y \qquad \text{and} \qquad F_{\text{net},z} = ma_z. \tag{5-2}$$

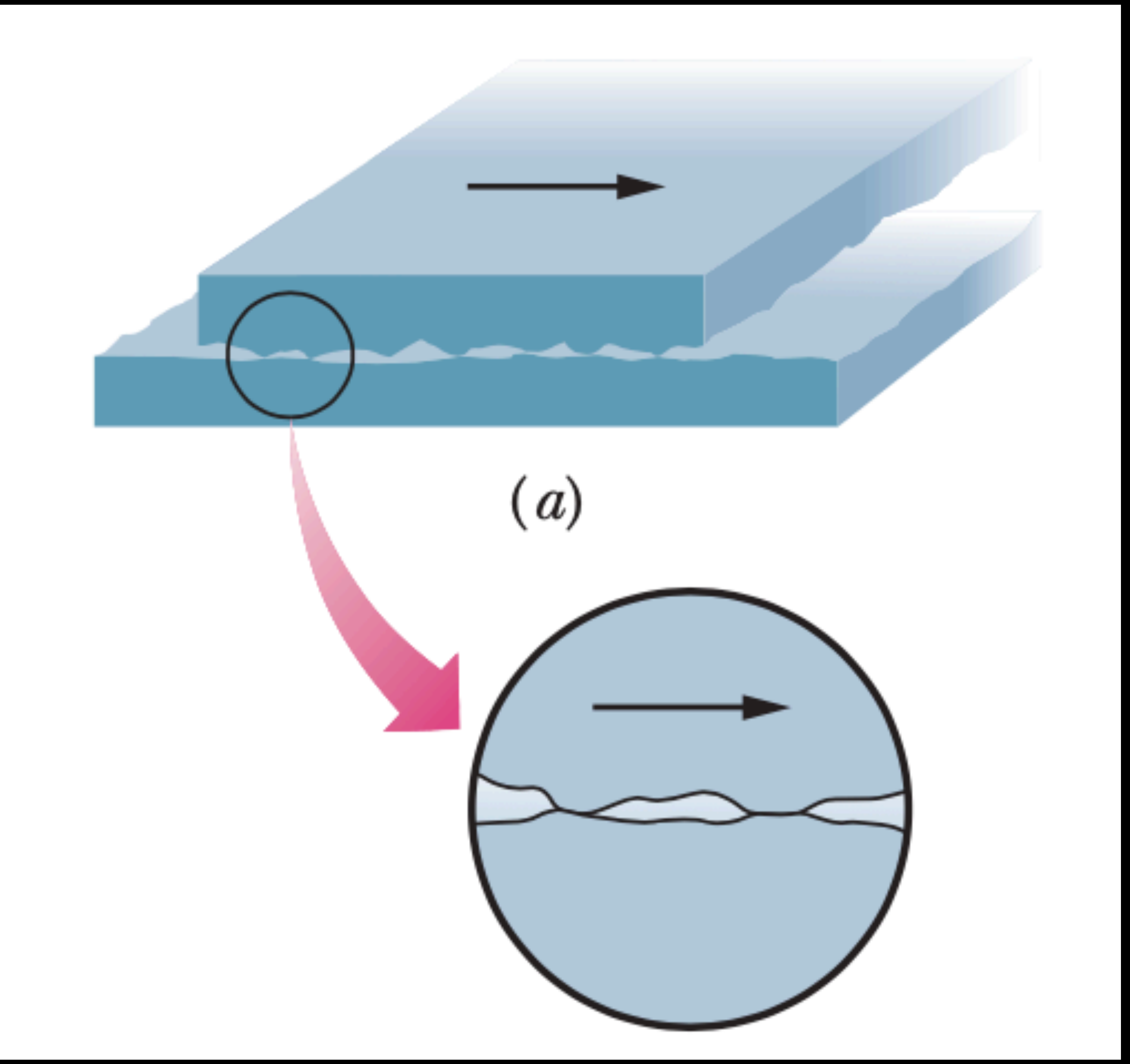
The second law indicates that in SI units

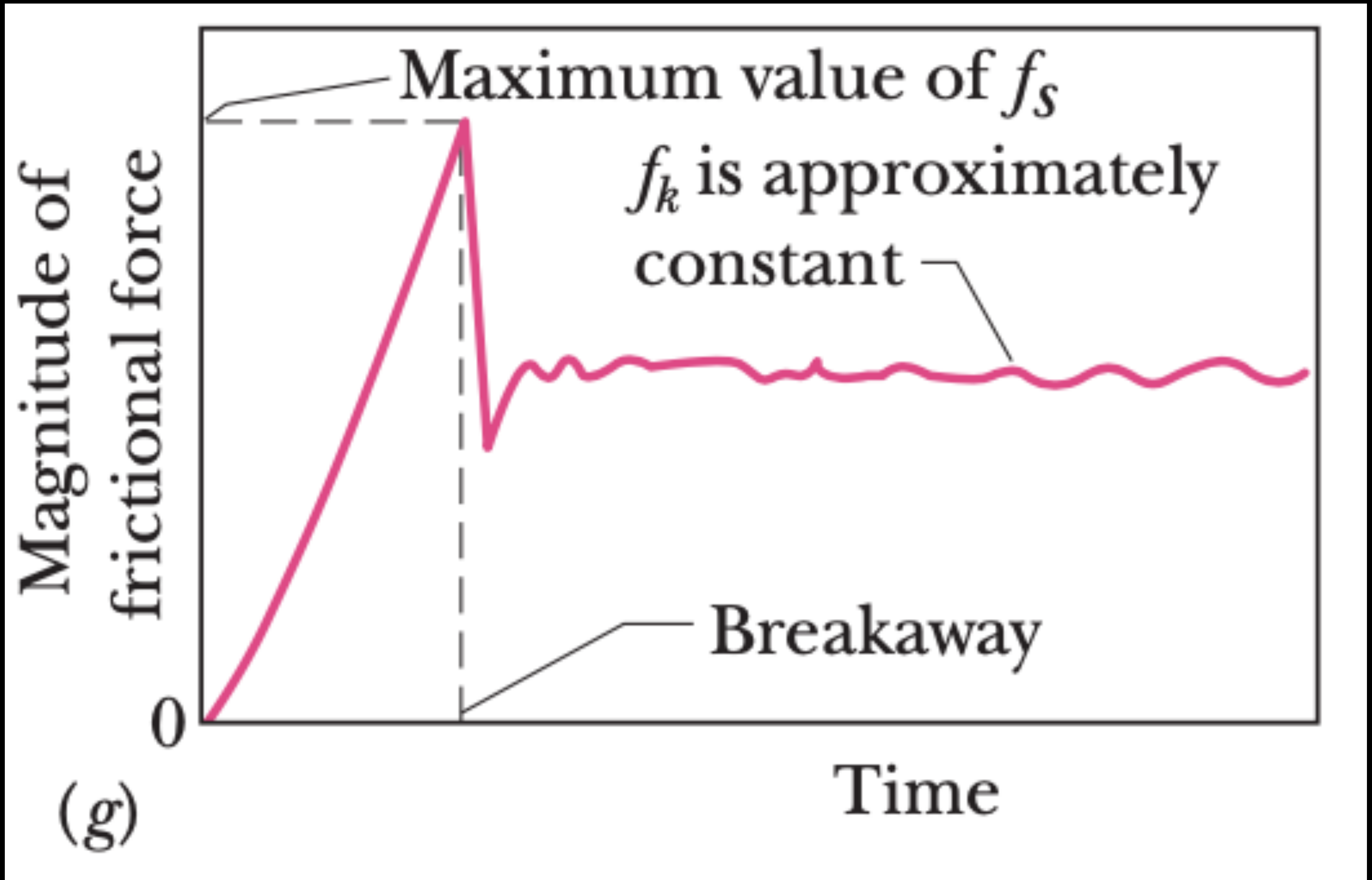
$$1\ \text{N} = 1\ \text{kg} \cdot \text{m/s}^2. \tag{5-3}$$

Lecture III : Force & Energy again

Properties of Friction

1. Static frictional force
2. Kinetic frictional force



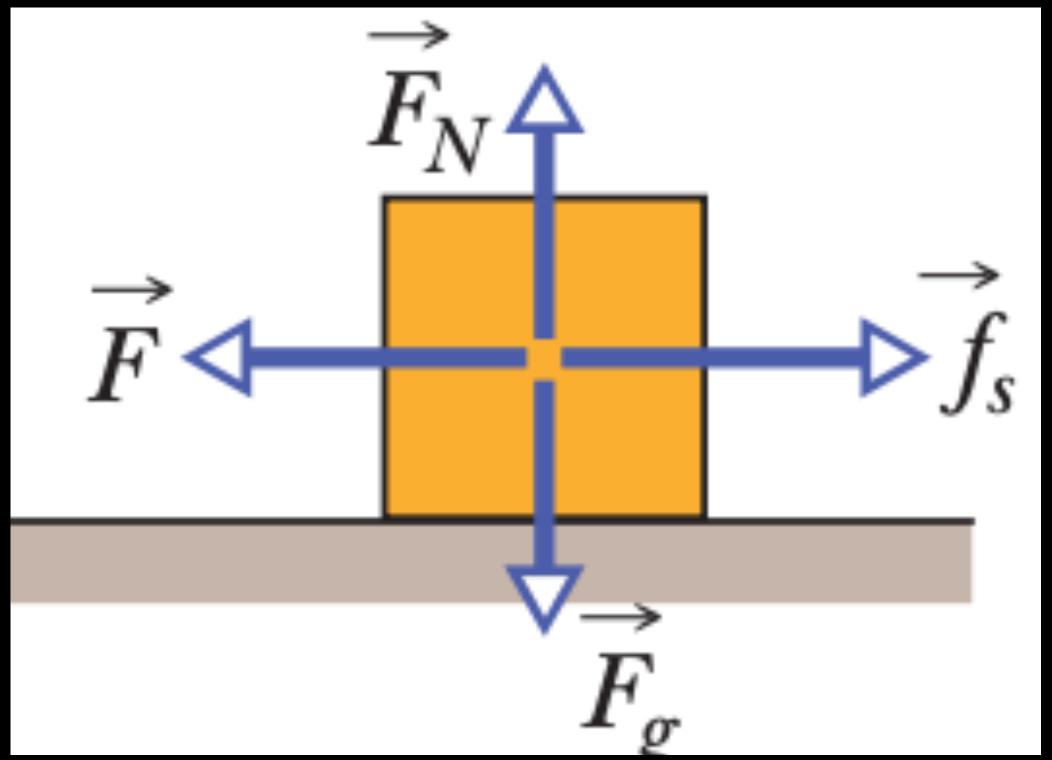


Lecture III : Force & Energy again

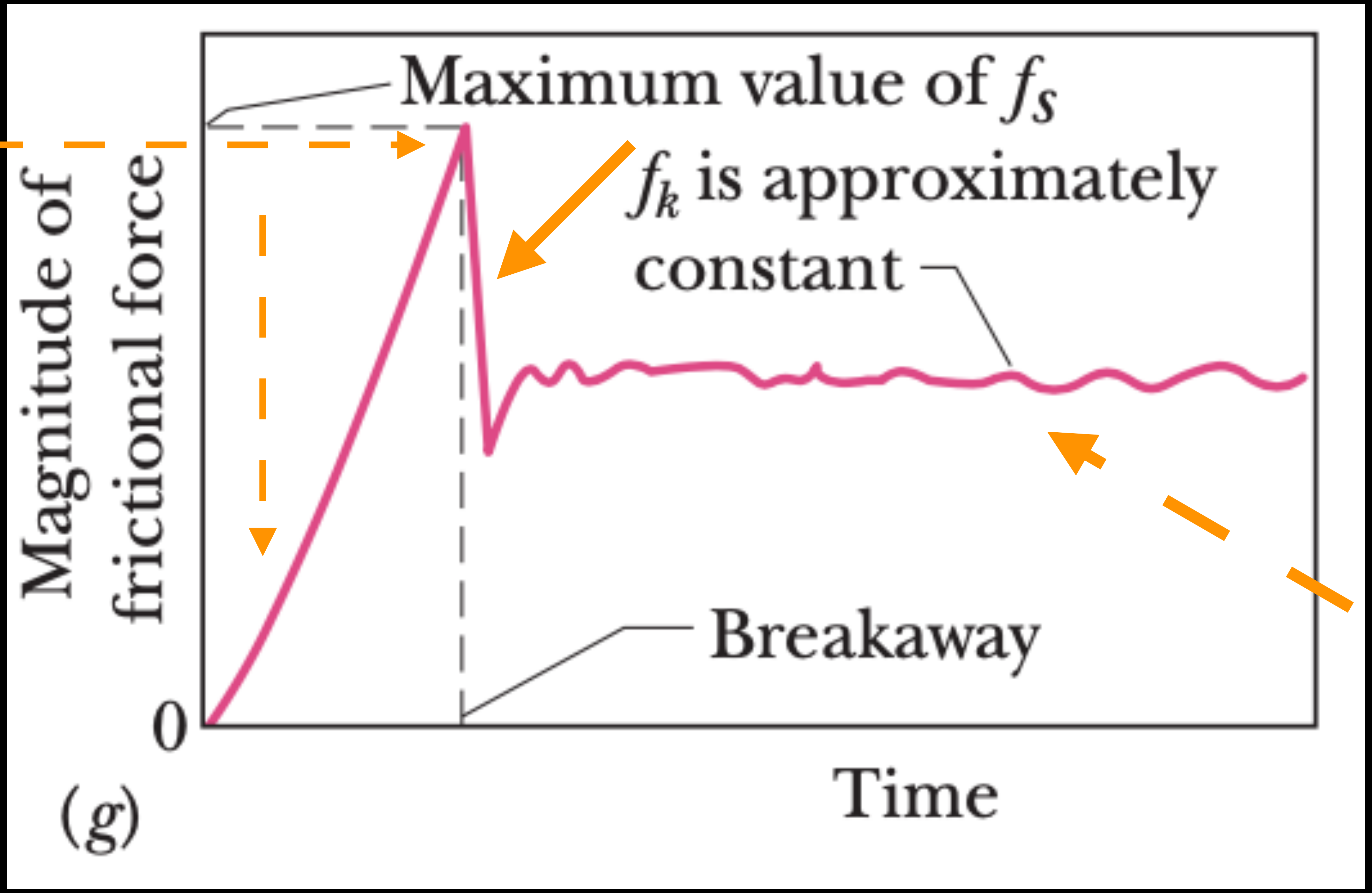
Properties of Friction

Lecture III : Force & Energy again

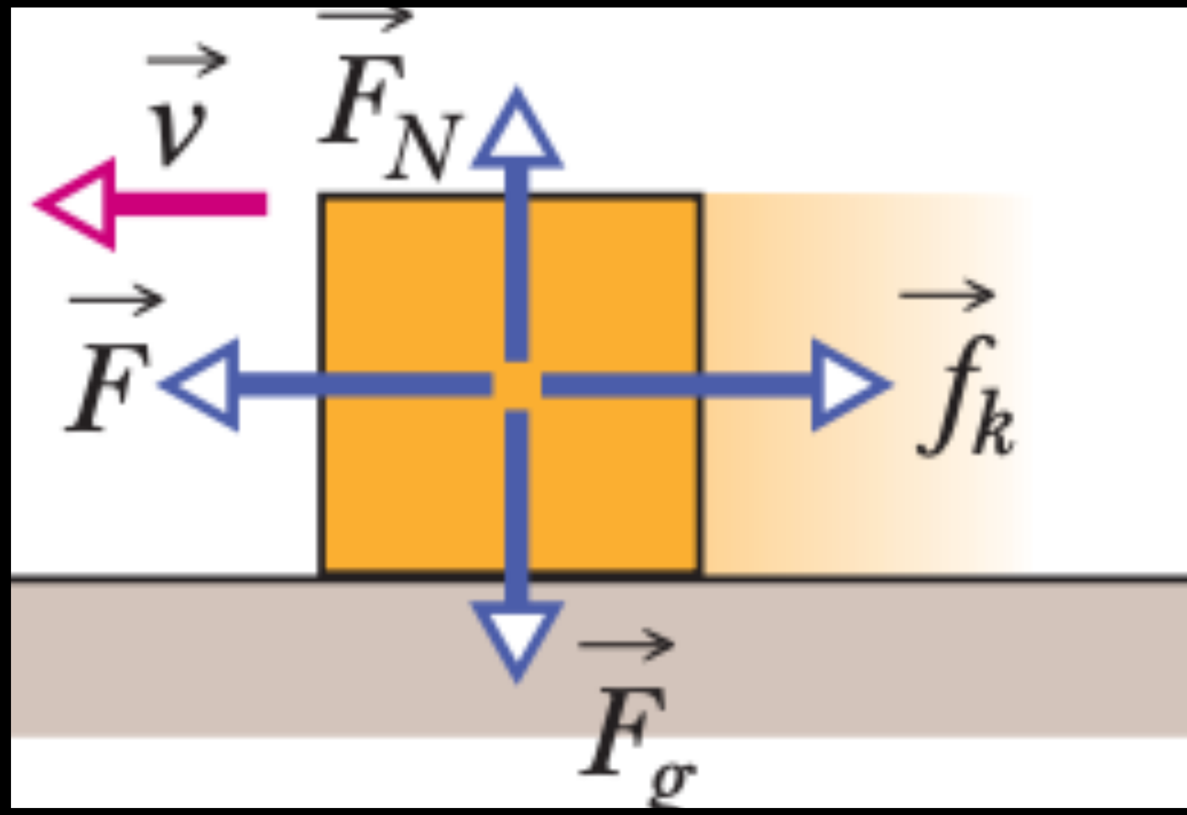
Properties of Friction



$$F_s = \mu_s F_N$$



$$F_k = \mu_k F_N$$



Lecture III : Force & Energy again

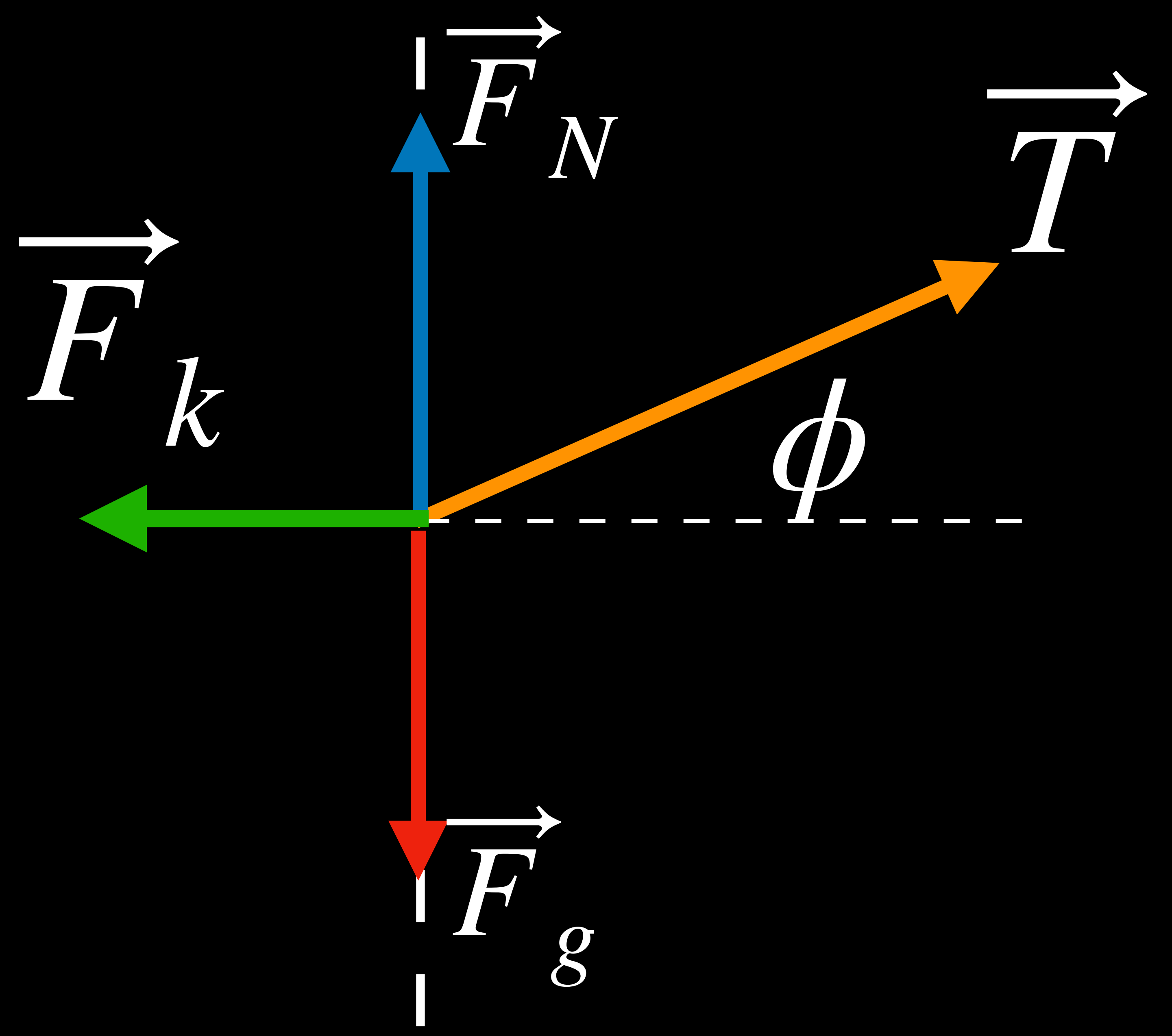
Properties of Friction

$$\vec{T} + \vec{F}_N + \vec{F}_g + \vec{F}_k = 0$$

$$\left.\begin{array}{l} T_x + 0 + 0 - F_k = 0 \\ T\cos\phi - \mu_k F_N = 0 \end{array}\right\} \text{X-axis}$$

$$\left.\begin{array}{l} T_y + F_N - F_g + 0 = 0 \\ T\sin\phi + F_N - mg = 0 \end{array}\right\} \text{y-axis}$$

$$T = \frac{\mu_k mg}{\cos\phi + \mu_k \sin\phi}$$



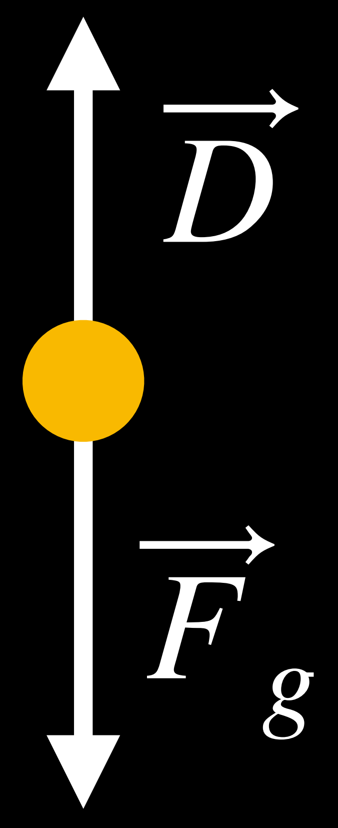
Lecture III : Force & Energy again

Drag force

drag coefficient C

A fluid is anything that can flow—generally either a gas or a liquid.

When there is a relative velocity between a fluid and a body (either because the body moves through the fluid or because the fluid moves past the body), the body experiences a drag force $\vec{D}$ that opposes the relative motion and points in the direction in which the fluid flows relative to the body.



Lecture III : Force & Energy again

Drag force

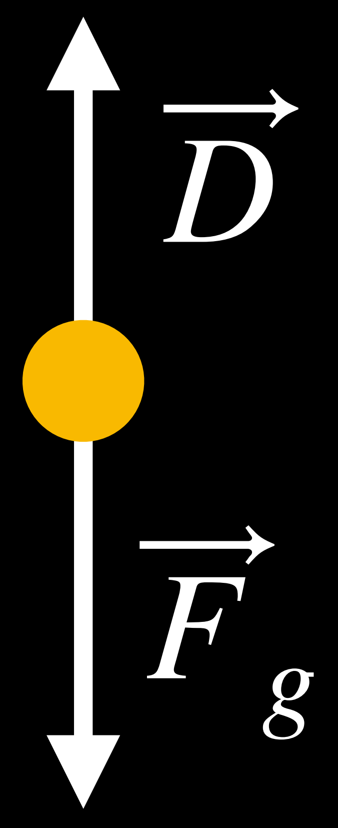
drag coefficient C

Terminal speed

the body's speed no longer increases. The body then reaches at a constant speed, called the terminal speed V_t

$$D - F_g = ma$$

Object	Terminal Speed (m/s)	95% Distance[a] (m)
Shot (from shot put)	145	2500
Sky diver (typical)	60	430
Baseball	42	210
Tennis ball	31	115
Basketball	20	47
Ping-Pong ball	9	10
Raindrop (radius = 1.5 mm)	7	6
Parachutist (typical)	5	3



Circular motion

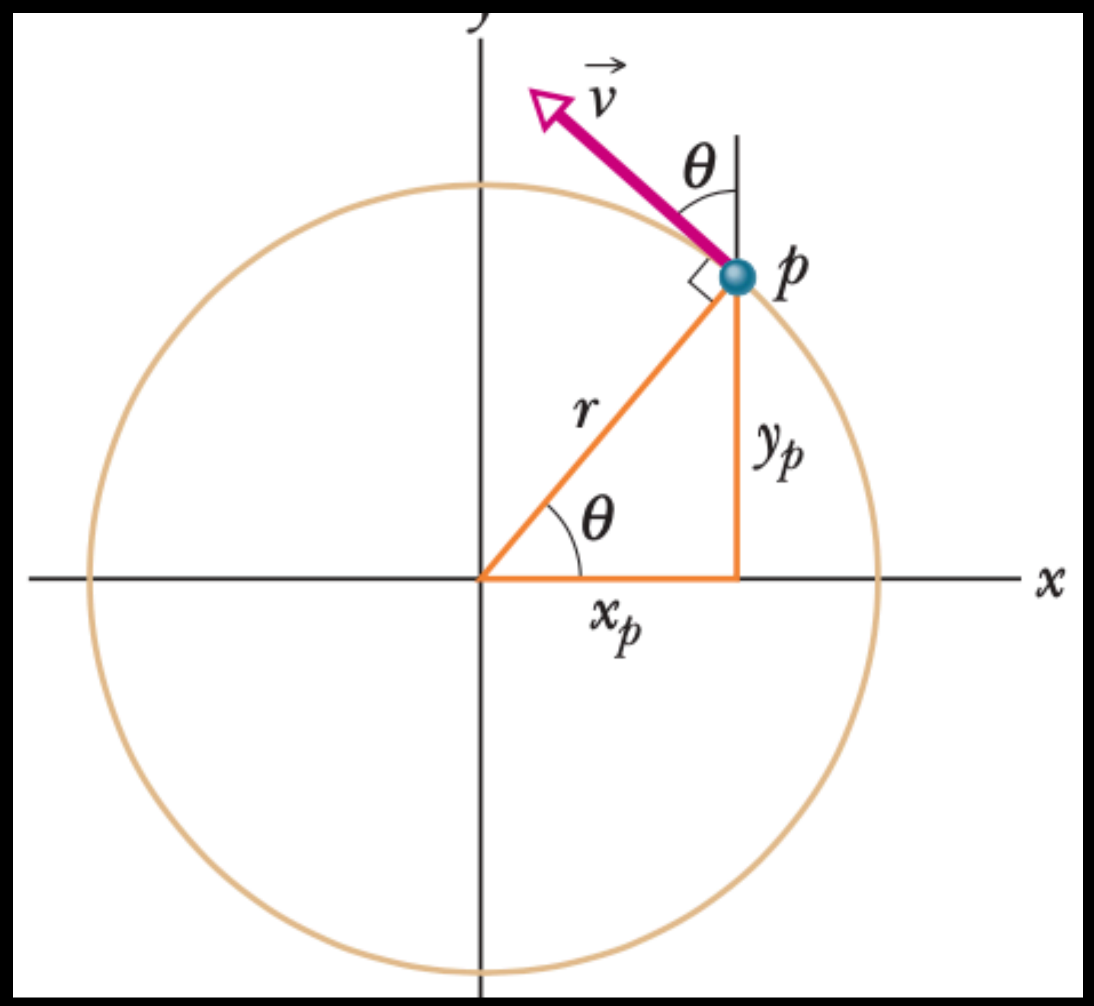
$$\vec{v} = v_x\hat{i} + v_y\hat{j} = (-v\sin\theta)\hat{i} + (v\cos\theta)\hat{j}$$

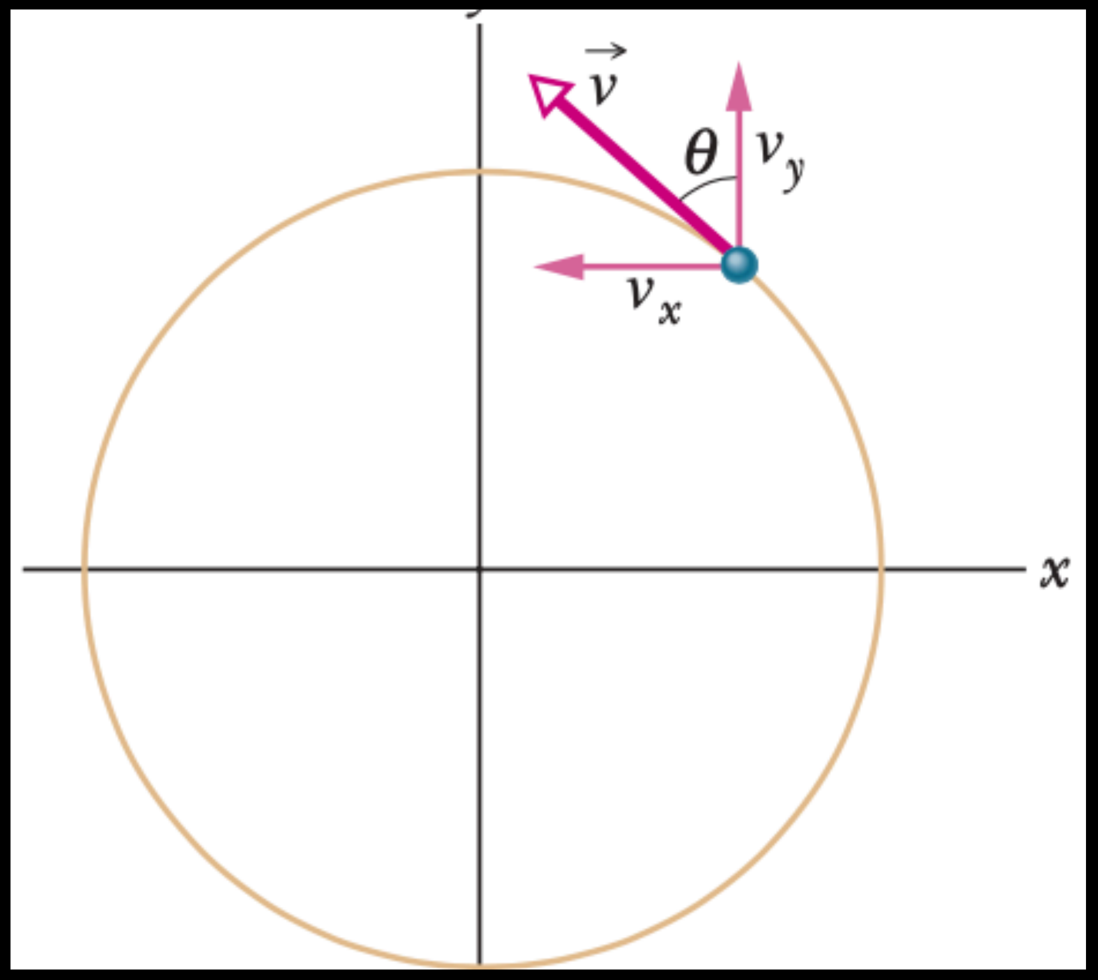
$$\sin\theta = \frac{y_p}{r} \quad \cos\theta = \frac{x_p}{r}$$

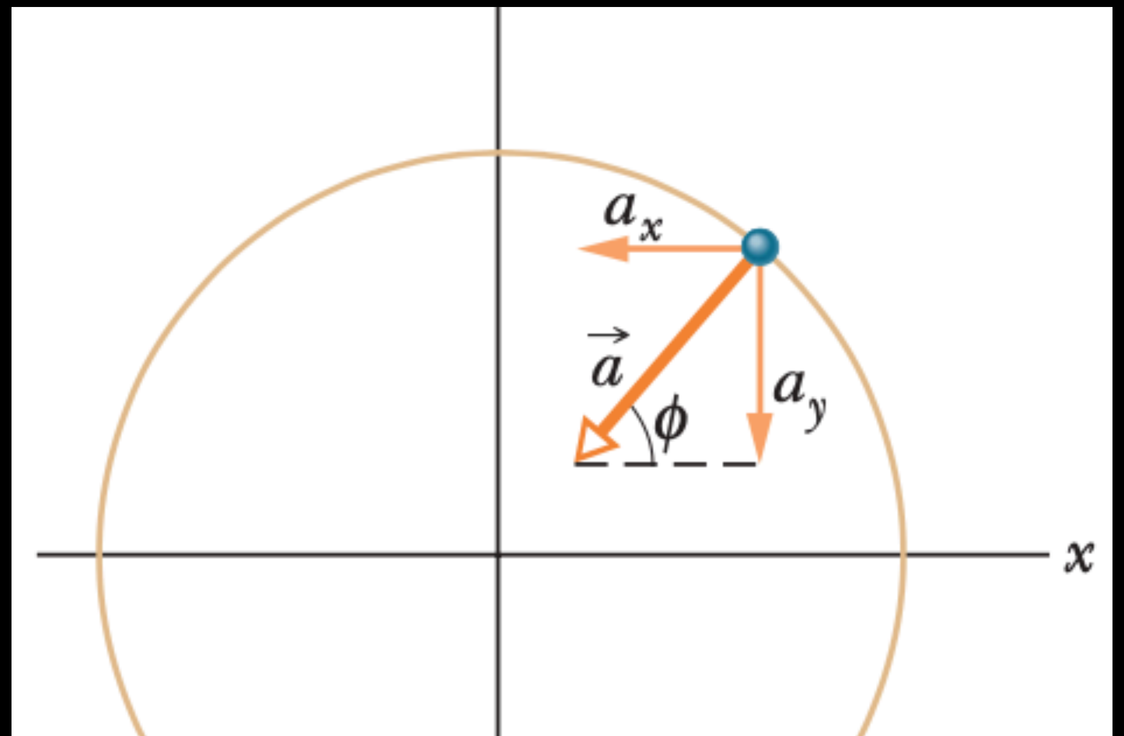
$$\vec{a} = \frac{d\vec{v}}{dt} = (-\frac{v}{r}\frac{dy_p}{dt})\hat{i} + (\frac{v}{r}\frac{dx_p}{dt})\hat{j}$$

$$\vec{a} = (-\frac{v^2}{r}\cos\theta)\hat{i} + (-\frac{v^2}{r}\sin\theta)\hat{j}$$

$$a = \sqrt{a_x^2 + a_y^2} \qquad F = ma = m\frac{v^2}{r}$$







Lecture II : Energy

Energy can be transformed from one type to another and transferred from one object to another, but the total amount is always the same

unit: joule

Lecture II : Work

Work W is energy transferred to or from an object by means of a force acting on the object. Energy transferred to the object is positive work, and energy transferred from the object is negative work.

$$W = \vec{F} \cdot \vec{d}$$

To calculate the work a force does on an object as the object moves through some displacement, we use only the force component along the object's displacement. The force component perpendicular to the displacement does zero work.

unit: joule

Lecture II : Work

Gravitational force:

Spring force:

Lecture II : Kinetic energy

Kinetic energy K is energy associated with the state of motion of an object. The faster the object moves, the greater is its kinetic energy. When the object is stationary, its kinetic energy is zero.

unit: joule

Lecture II : Work-kinetic Theorem

$$\Delta K = W$$

change in the kinetic energy of an object

=

net work done on an object

$$K_f = K_i + W$$

Lecture II : Power

The time rate at which work is done by a force is said to be the power due to the force.

$$P_{avg} = \frac{W}{\Delta t} \qquad P = \frac{dW}{dt}$$

unit: watt, horsepower

Lecture II : Conservative & Nonconservative force

In a situation in which W1 = - W2 is always true, the other type of energy is a potential energy and the force is said to be a conservative force.

Others are called nonconservative force.

Property of conservative force: Path independence

The net work done by a conservative force on a particle moving around any closed path is zero.

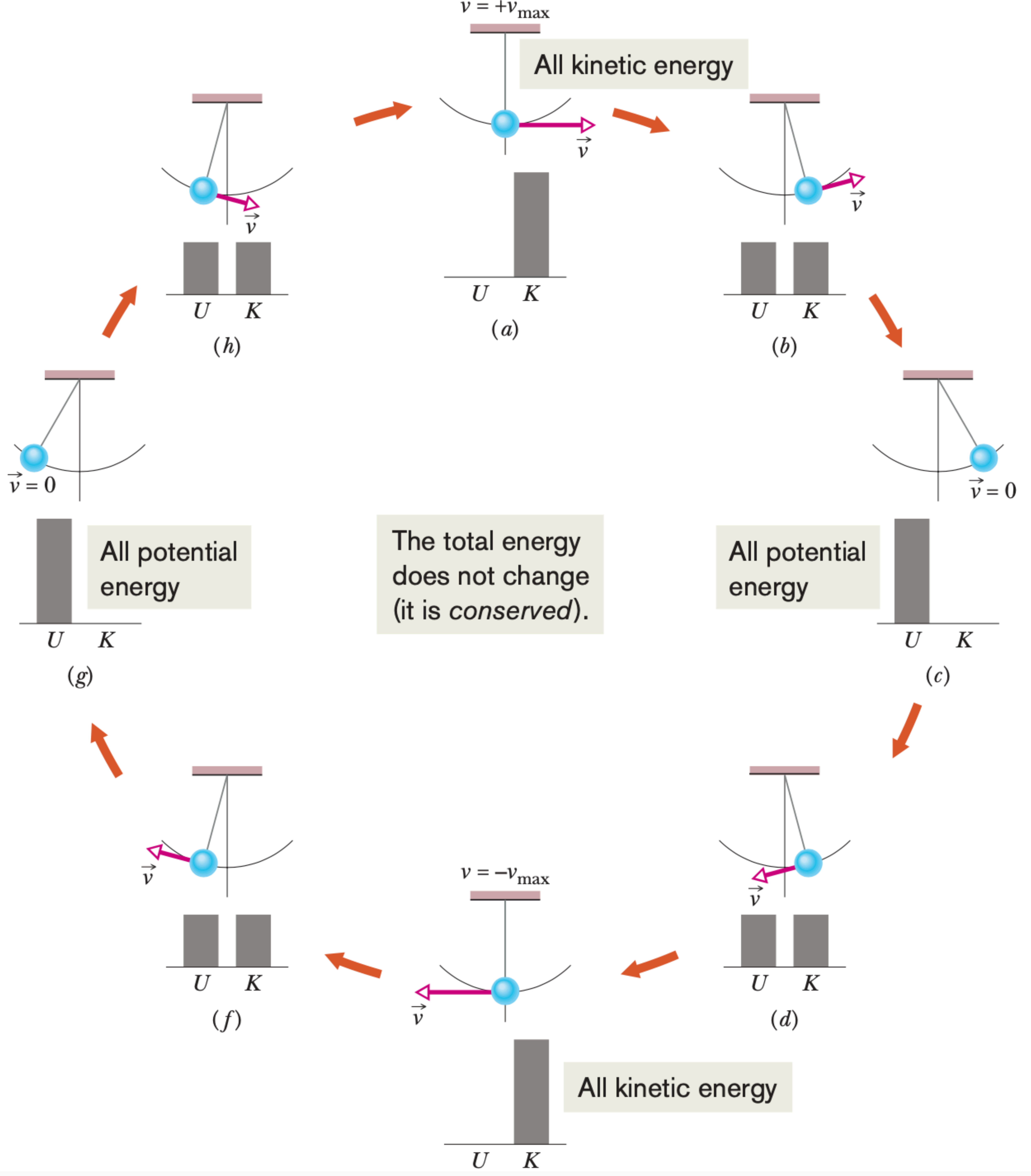
Lecture II : Conservation of Mechanical Energy

$$E_{mec} = K + U$$

In an isolated system where only conservative forces cause energy changes, the kinetic energy and potential energy can change, but their sum, the mechanical energy E_{mec} of the system, cannot change.

principle of conservation of mechanical energy.

$$\Delta E_{mec} = \Delta K + \Delta U = 0$$

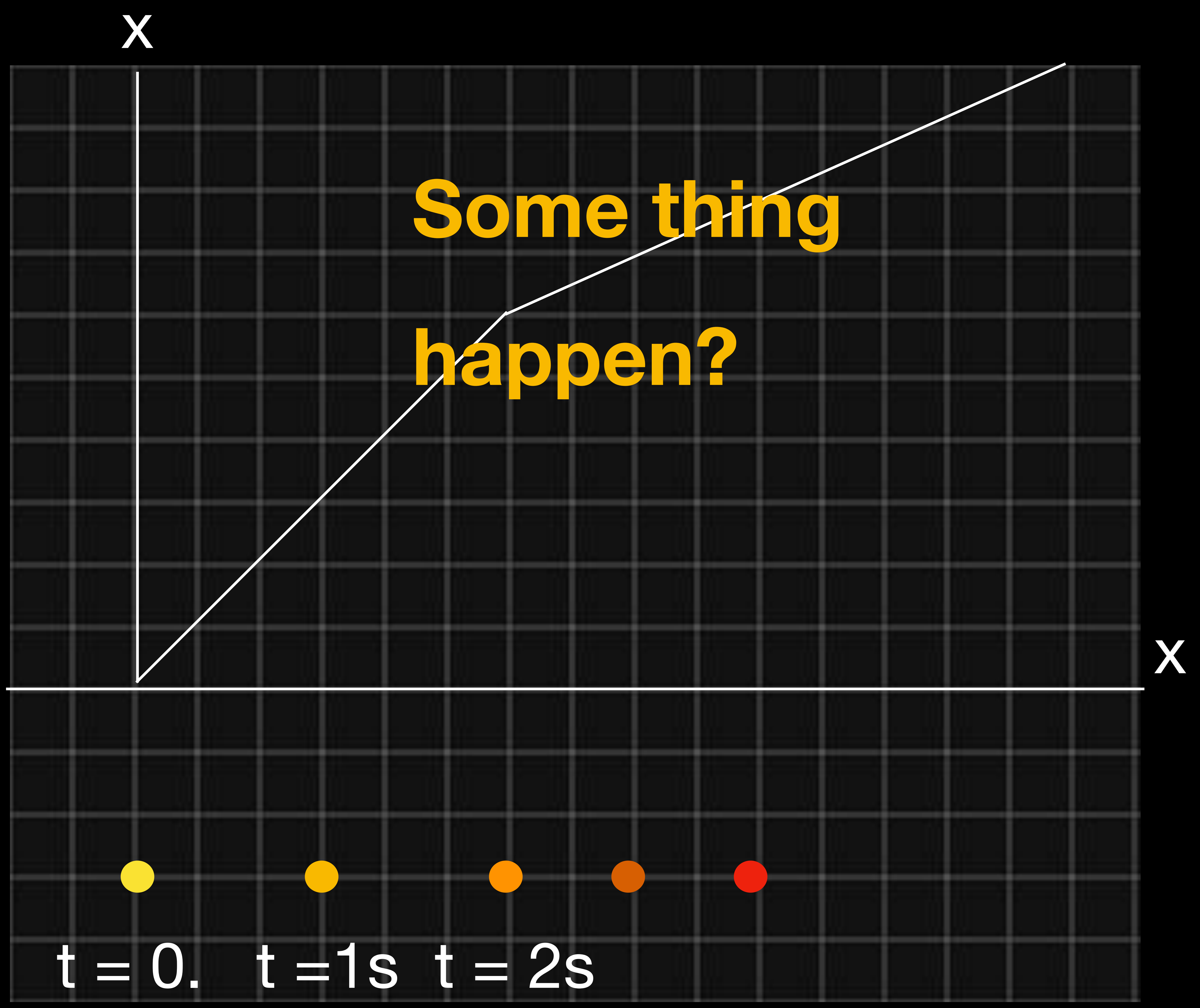


$v = +v_{max}$
All kinetic energy
$\vec{v}$
U K
(a)
U K
(b)
$\vec{v} = 0$
All potential energy
U K
(c)
U K
(d)
$v = -v_{max}$
All kinetic energy
U K
U K
(f)
$\vec{v} = 0$
All potential energy
U K
(g)
U K
(h)
The total energy does not change (it is *conserved*).

Lecture II : the change of velocity

Instantaneous velocity

$$\vec{V} = \lim_{t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$



Lecture II : the change of velocity

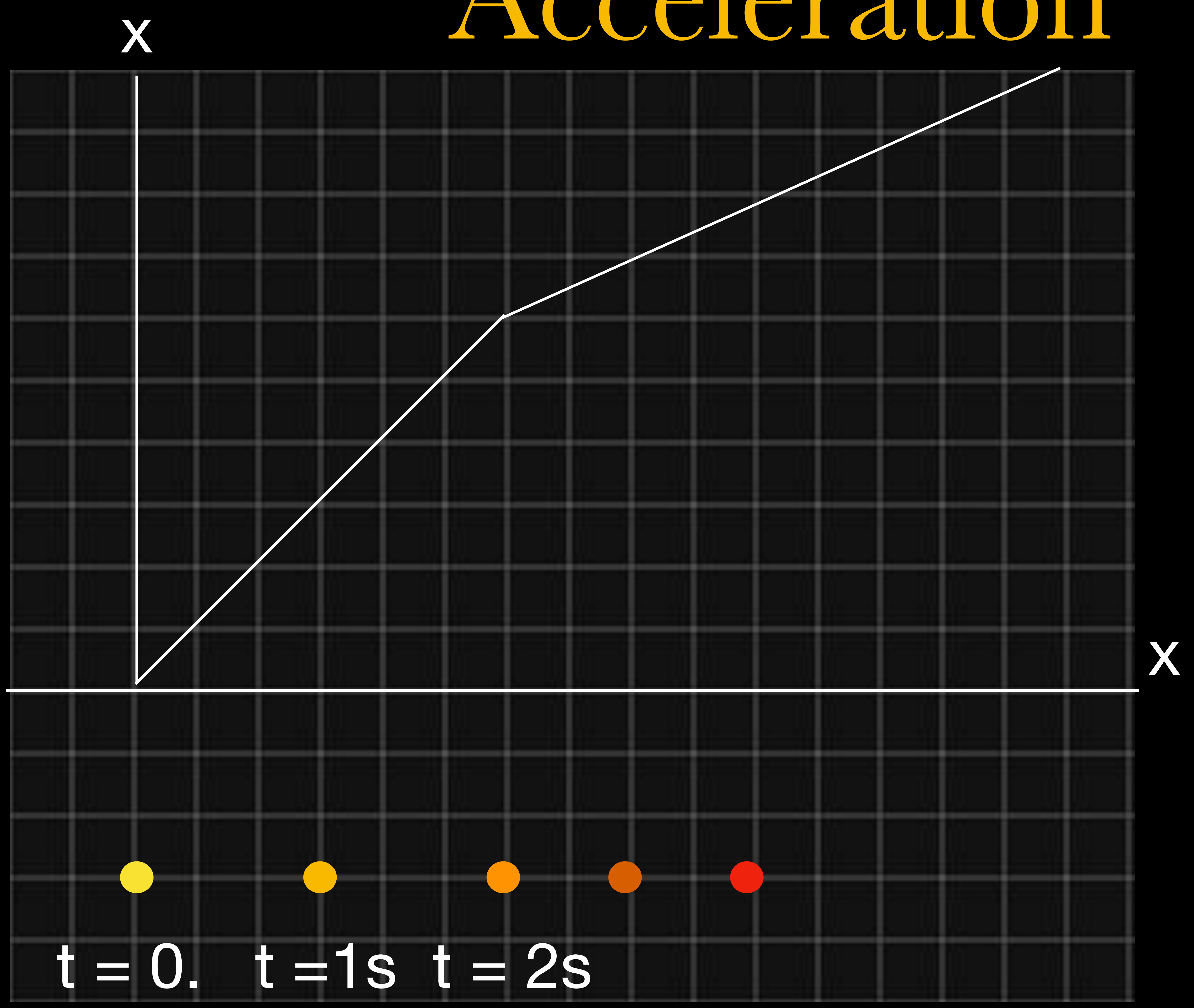
Instantaneous velocity

$$\vec{V} = \lim_{t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Acceleration:

$$? = \lim_{t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

Acceleration



Lecture II : Acceleration

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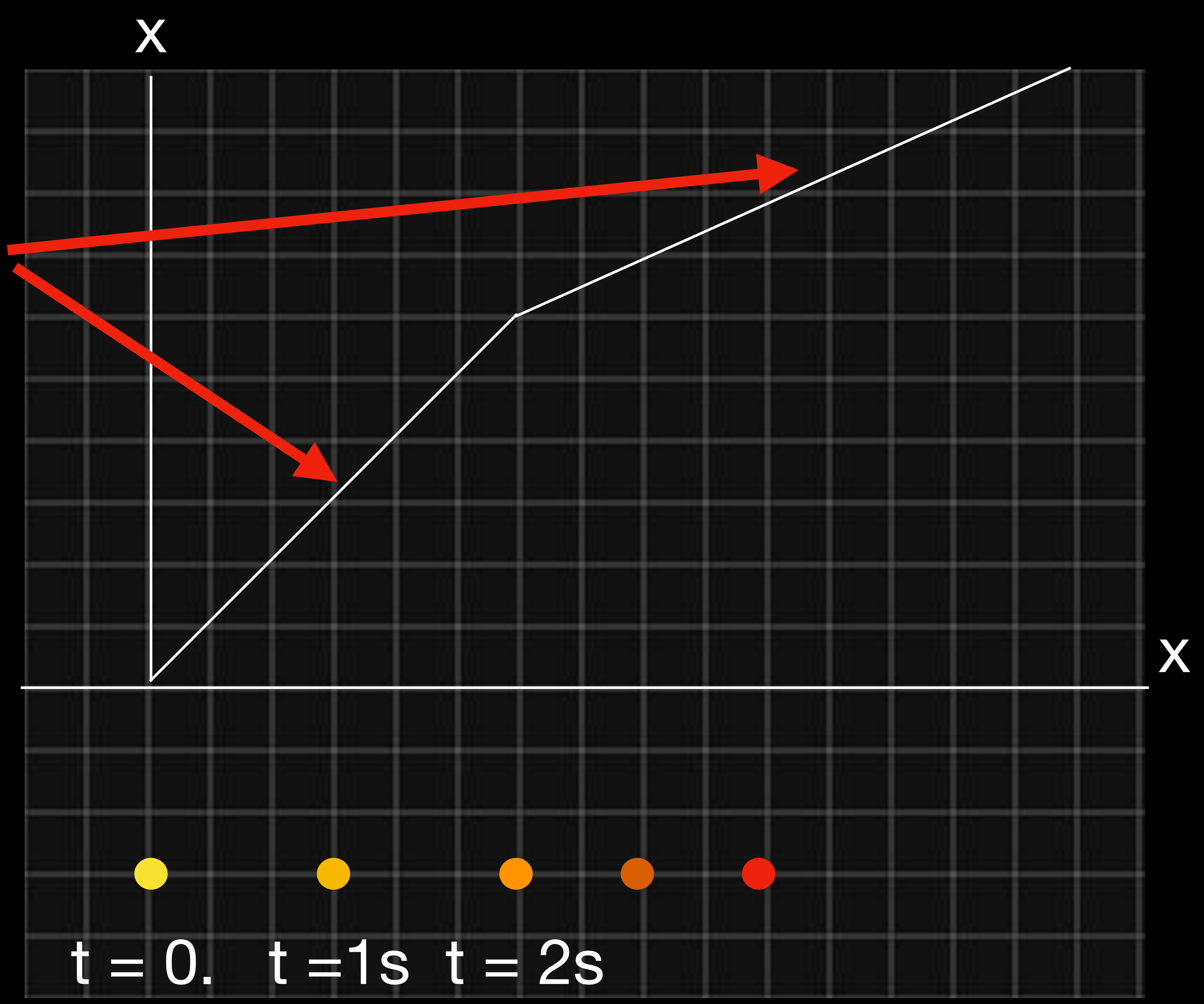


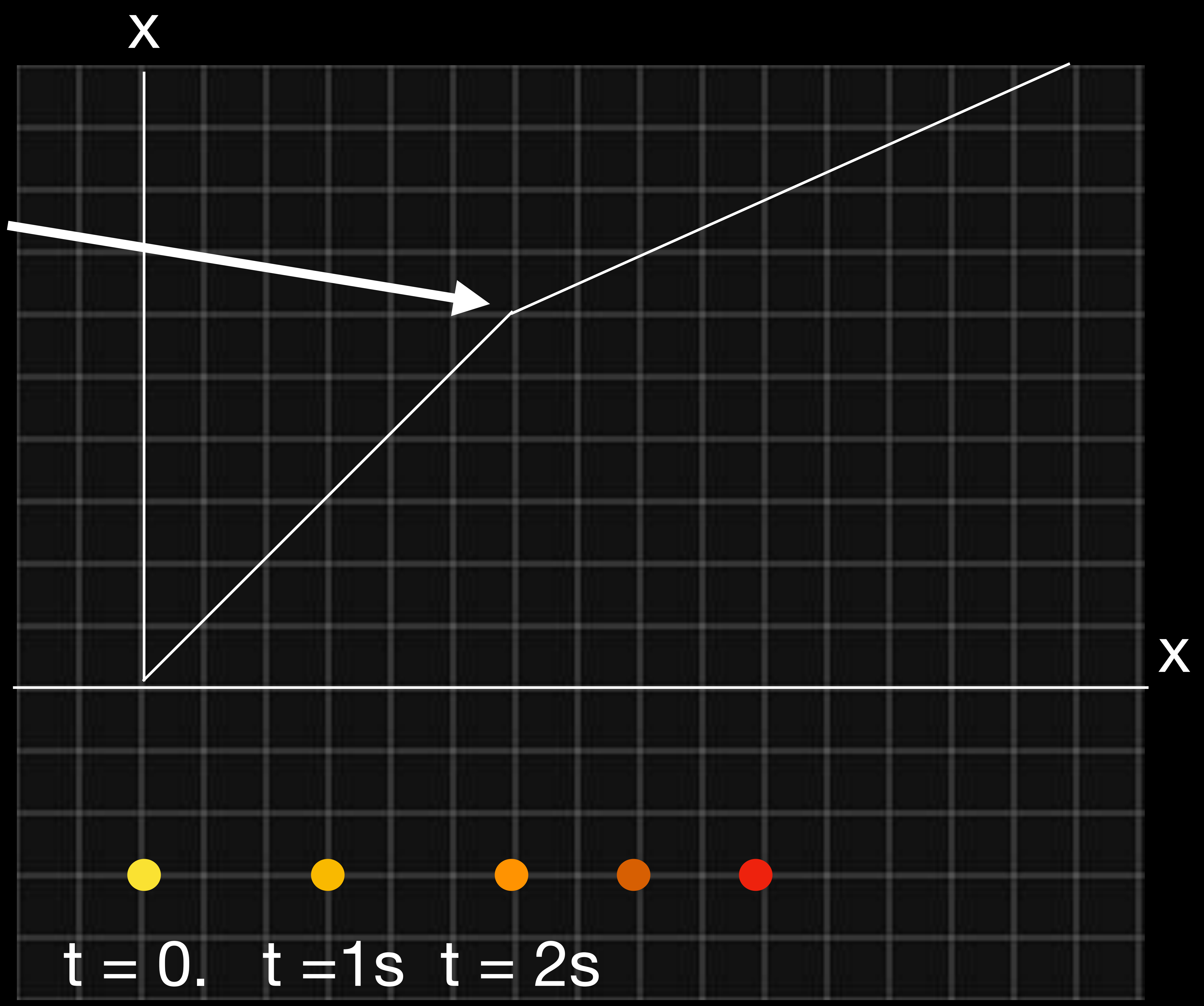
Lecture II : Acceleration

Newton's second law

The net force on a body is equal to the product of the body's mass and its acceleration.

Acceleration:

$$? = \lim_{t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$



Lecture II : Acceleration: ex

A particle's position on the x axis is given by

$$x = 4 - 27t + t^3,$$

with x in meters and t in seconds.

(a) Because position x depends on time t, the particle must be moving. Find the particle's velocity function $v(t)$ and acceleration function $a(t)$.

(b) Describe the particles motion for t>0

(c) Is there ever a time when v=0

Lecture II : Acceleration

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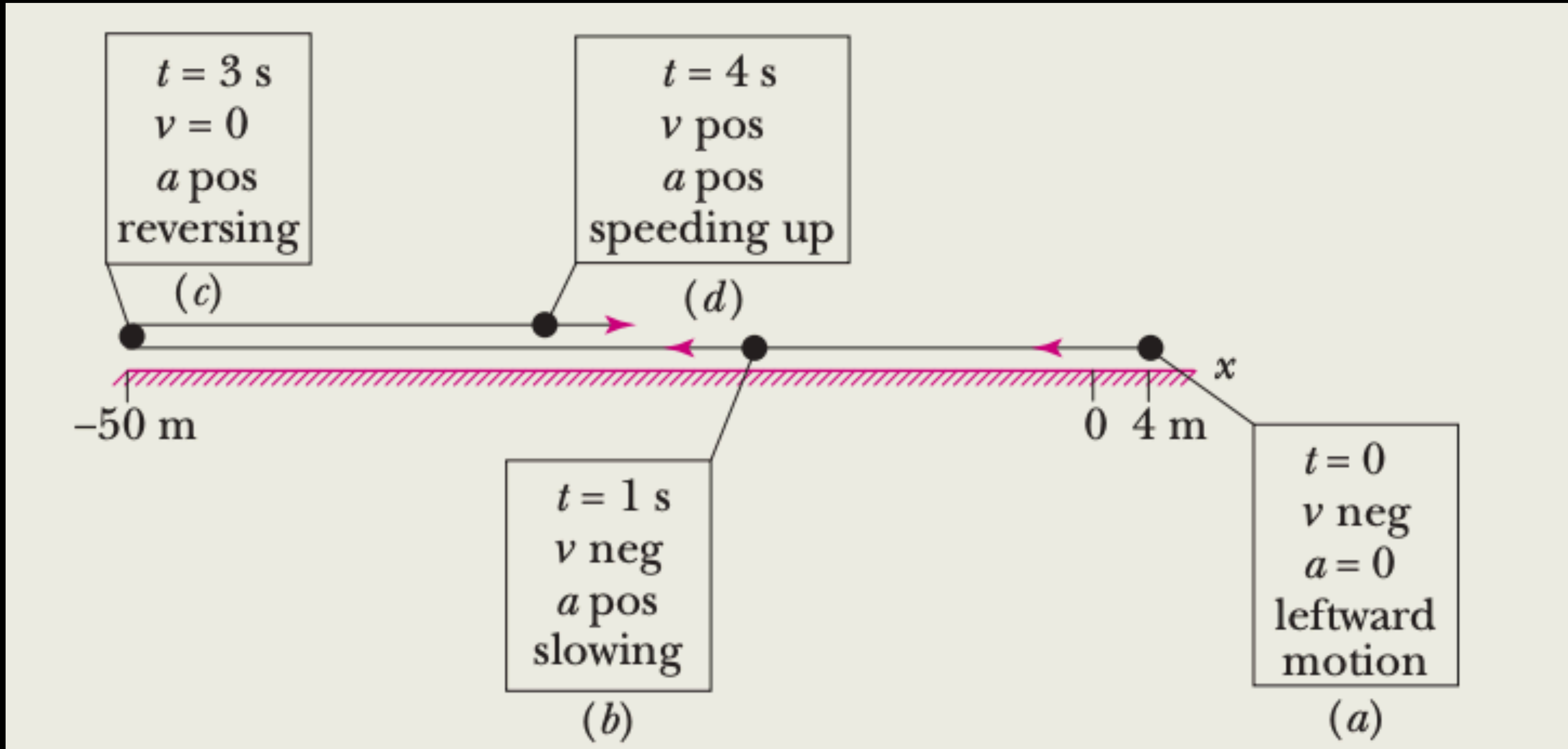
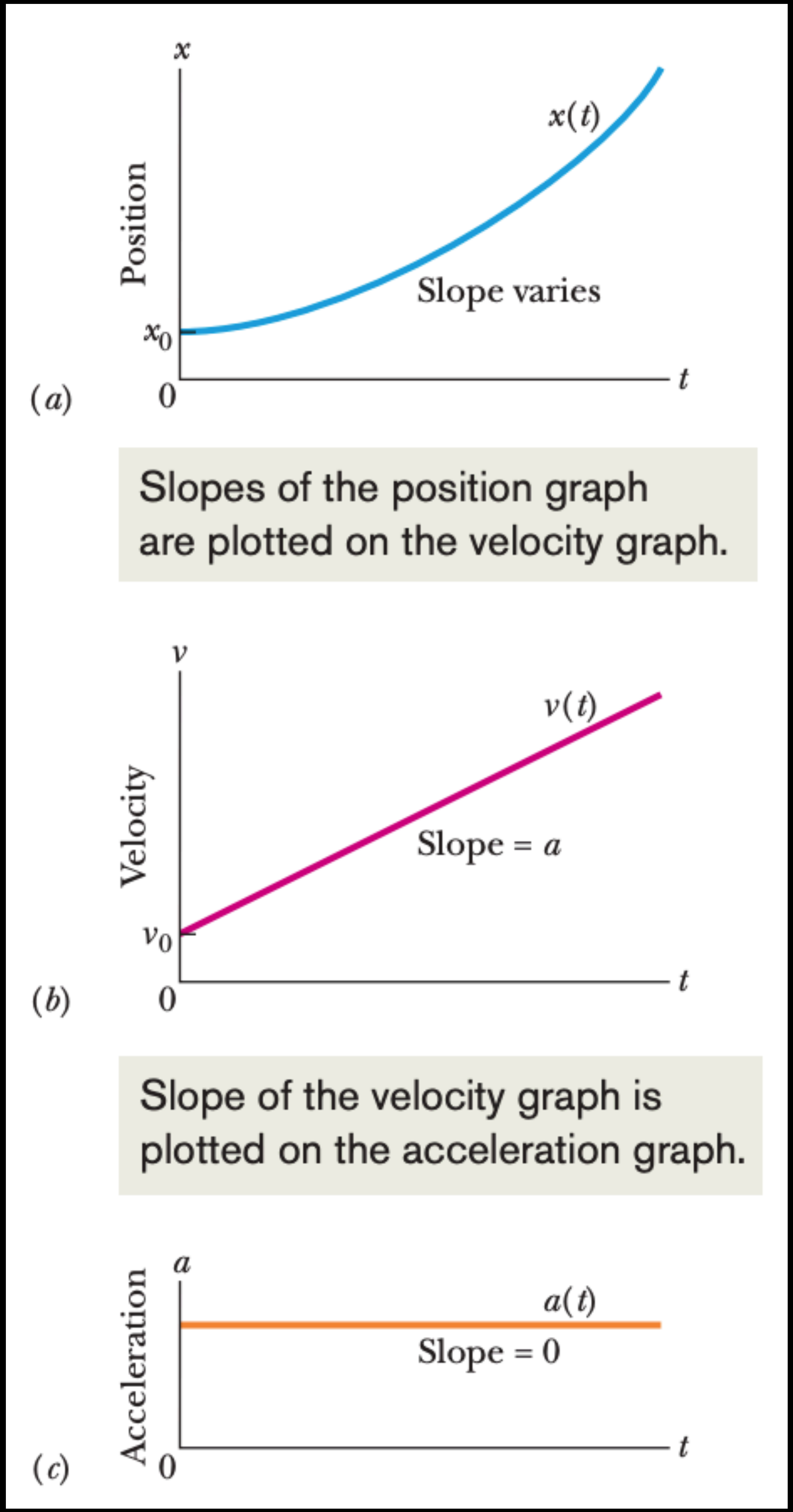


Lecture II : Acceleration = C

$$\vec{a} = \text{constant}$$

$$\vec{v} =$$

$$x =$$



Lecture II : Acceleration = C

Question 1:

Spotting a police car, you brake a Ferrari from a speed 200 km/h to a speed 100 km/h during a displacement of 100m, at a constant acceleration.

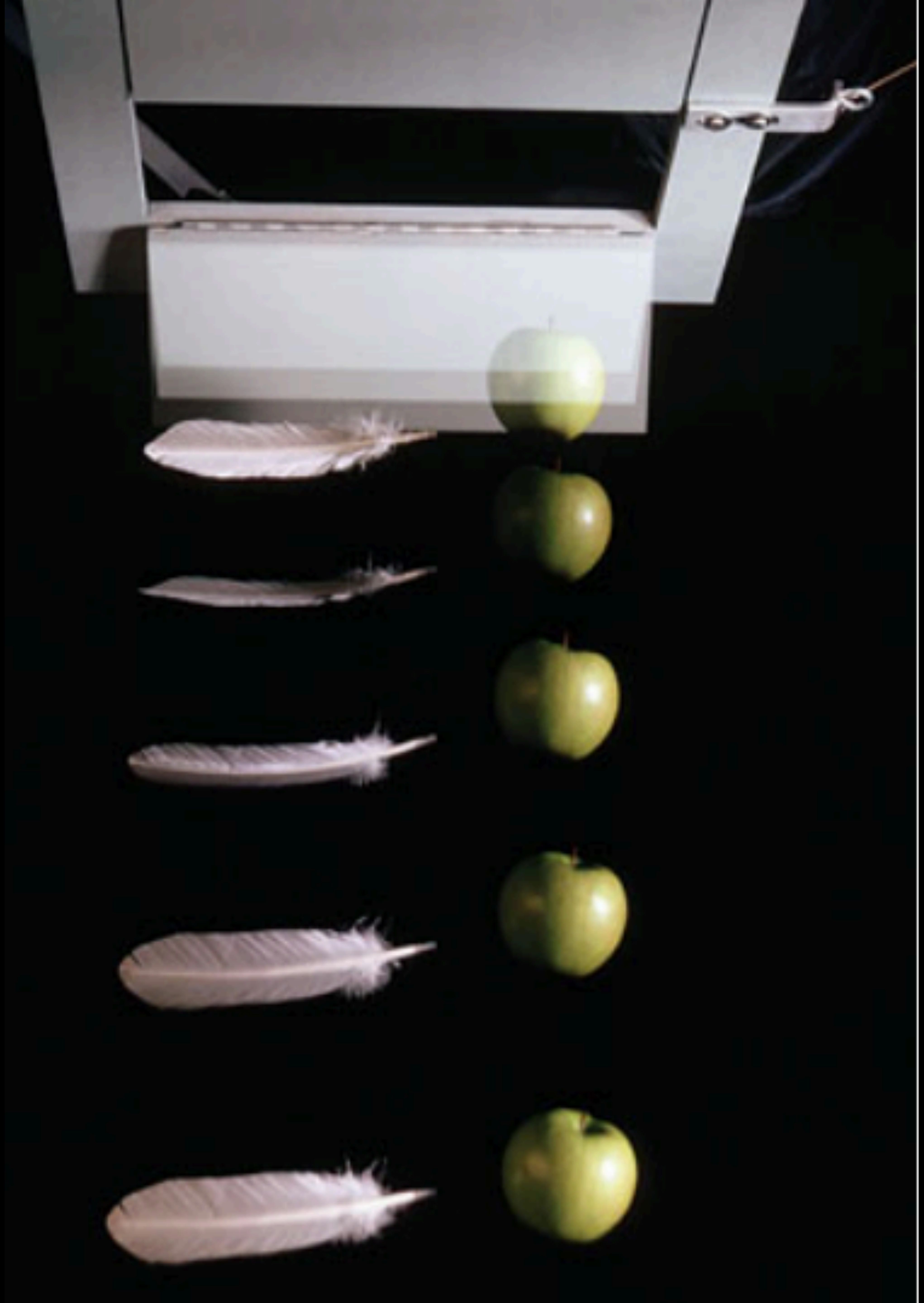
(a)what is that acceleration

(b) How many time is required for the given

decrease in speed.

Free-Fall Acceleration

The free-fall acceleration near Earth's surface is a = -g =-9.8 m/s, hence g = 9.8m/s



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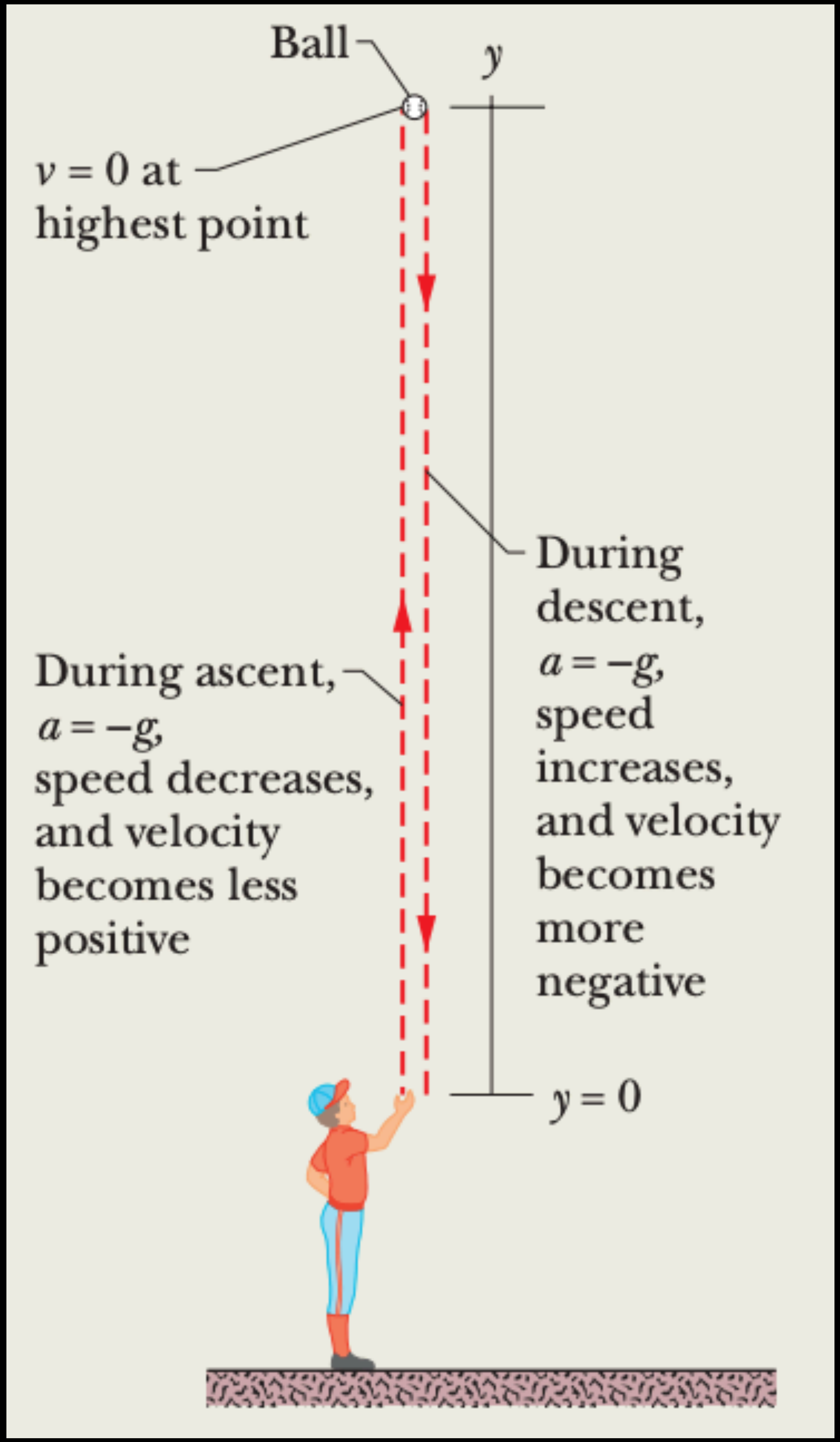
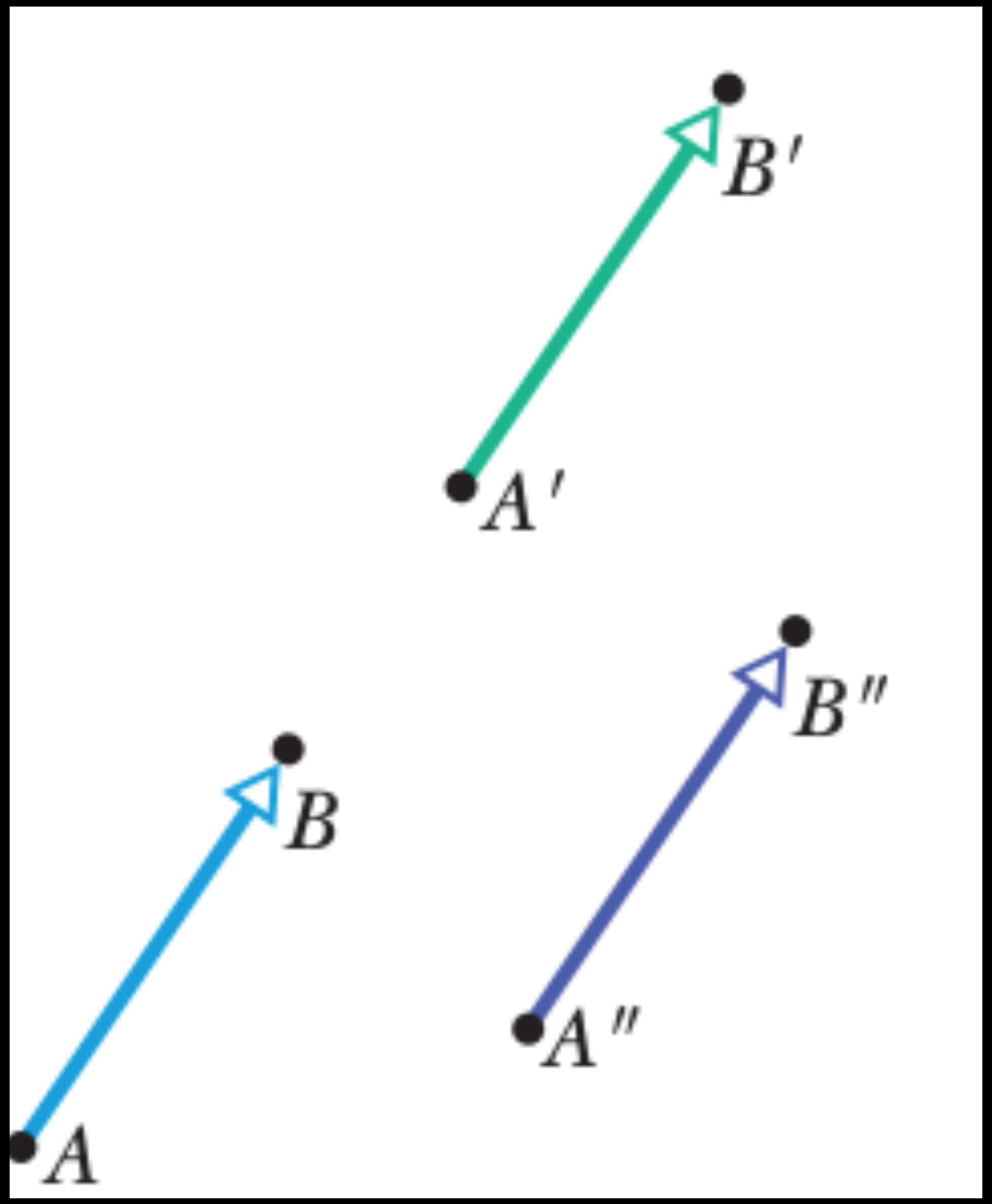
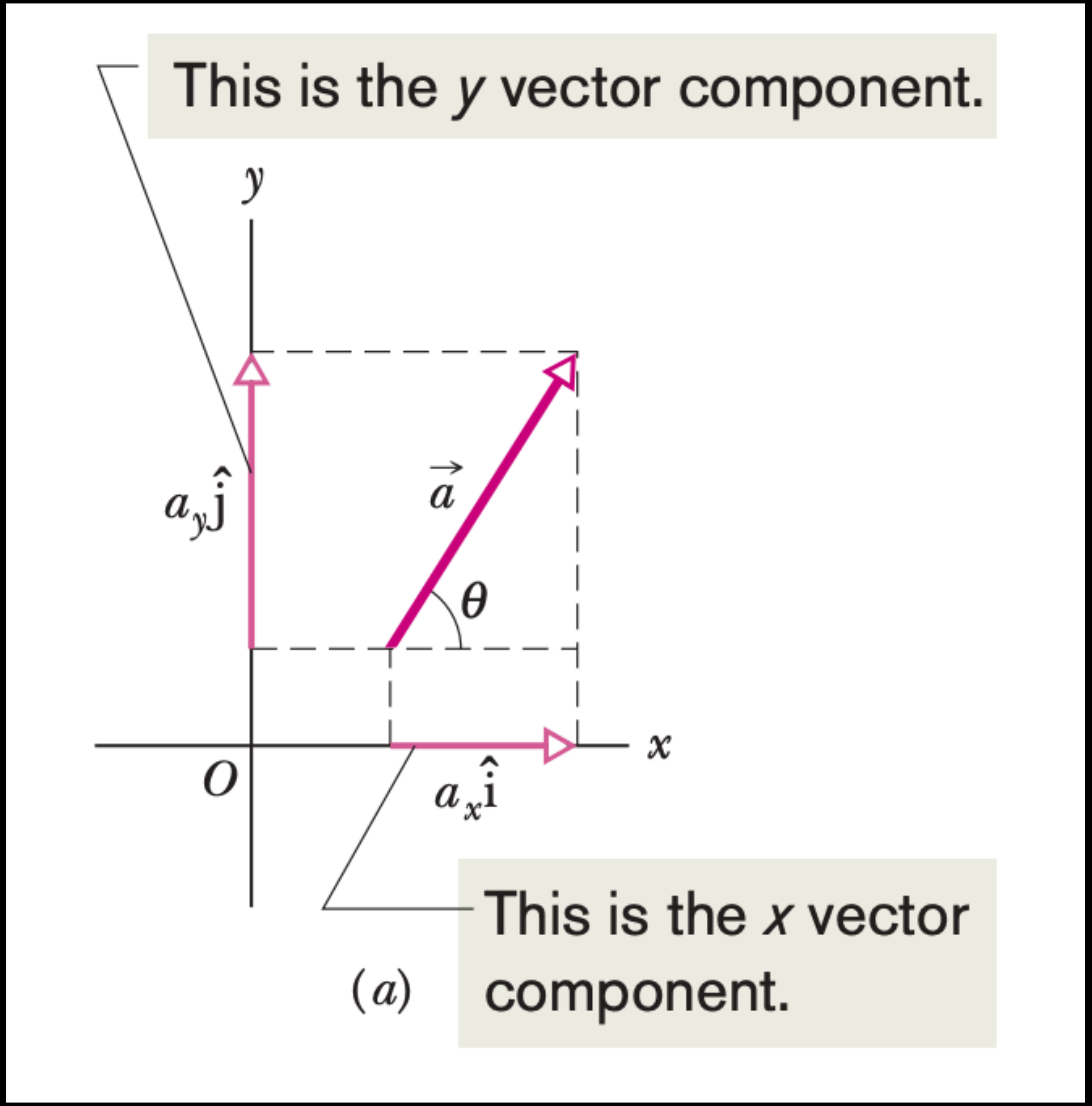


Vector

- Scalar:
  - A scalar is a physical quantity that has magnitude but no direction.
- Vector:
  - Vectors are physical quantities that possess both magnitude and direction.
  - Components of vectors
  - Adding vector





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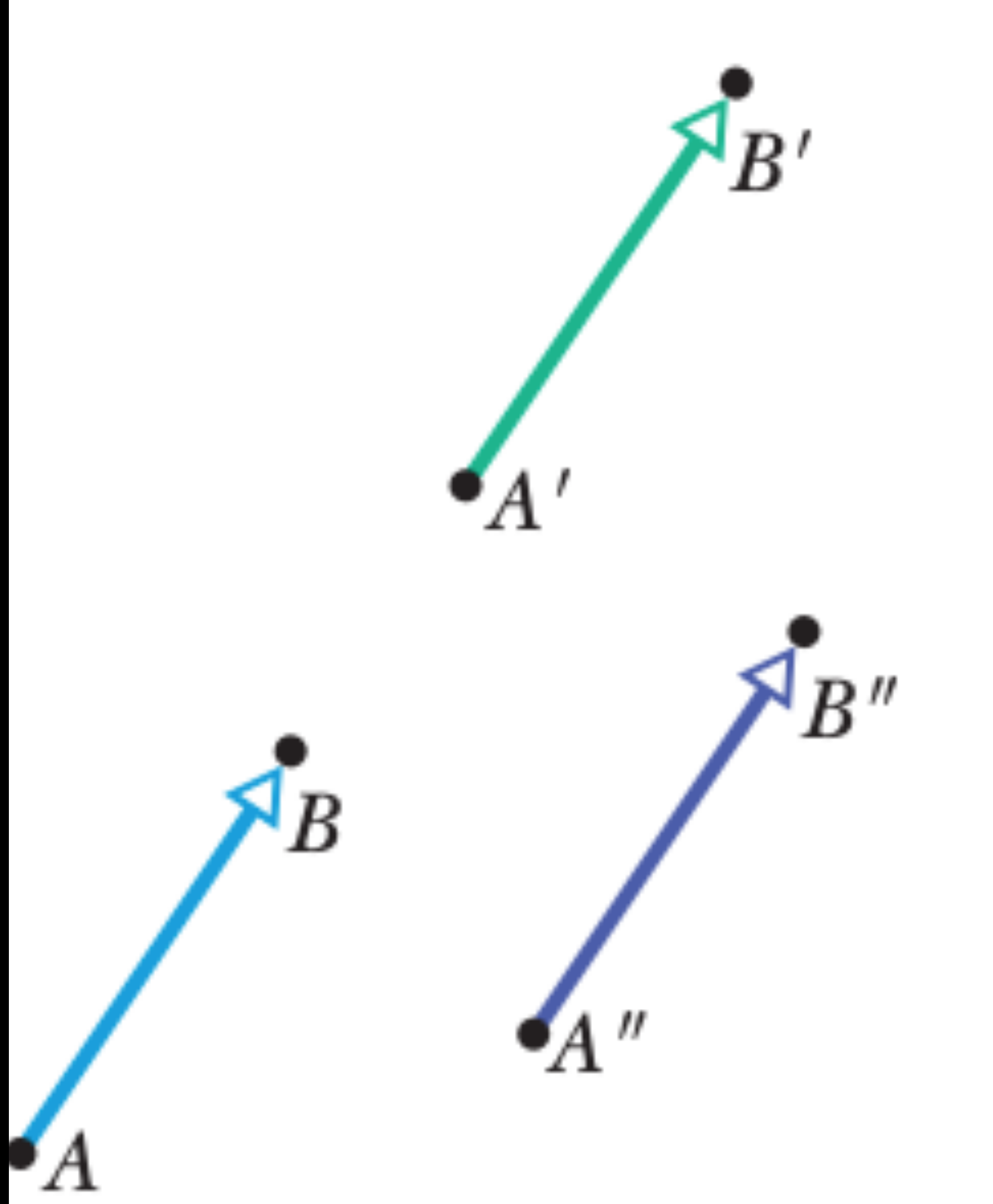
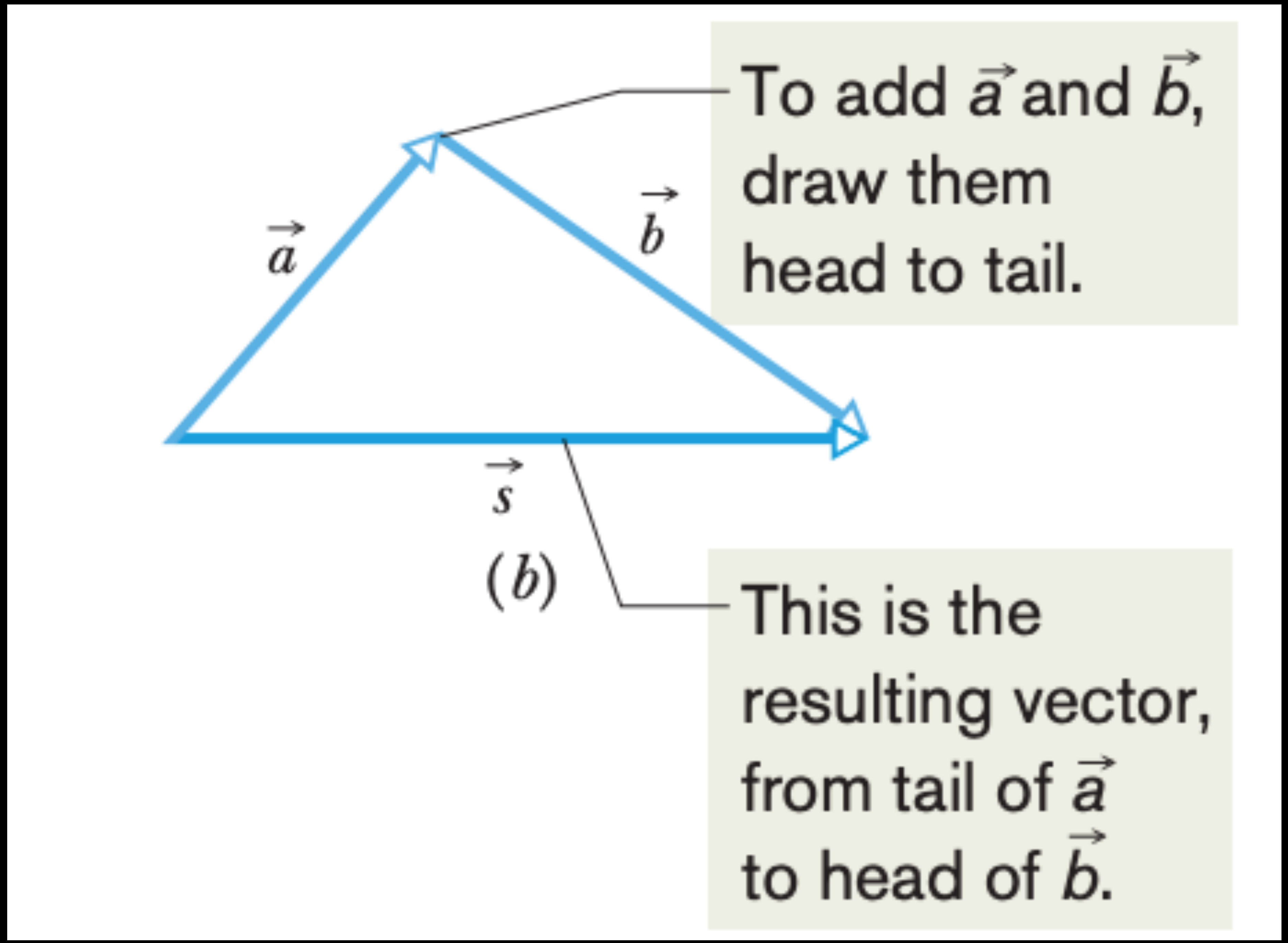
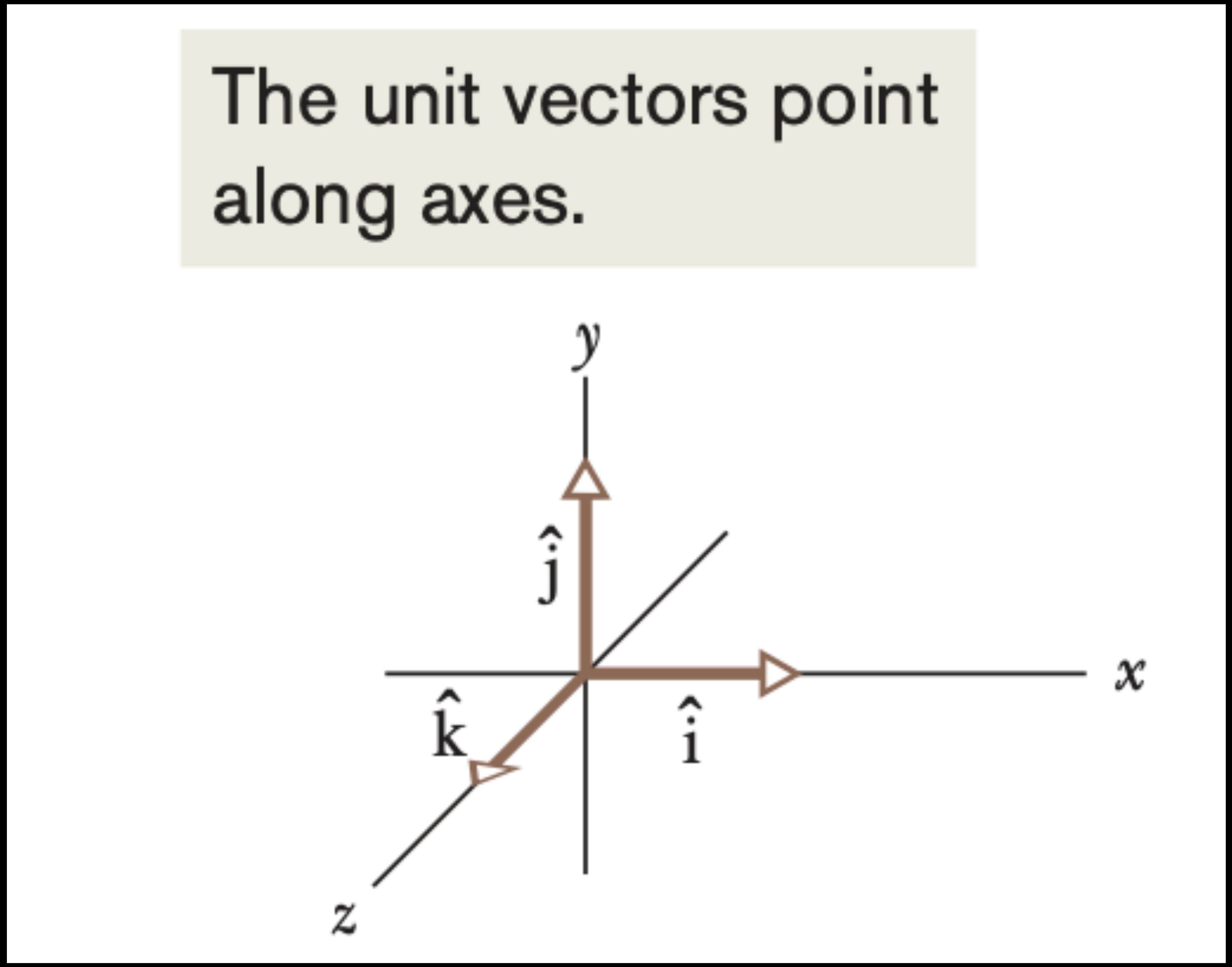




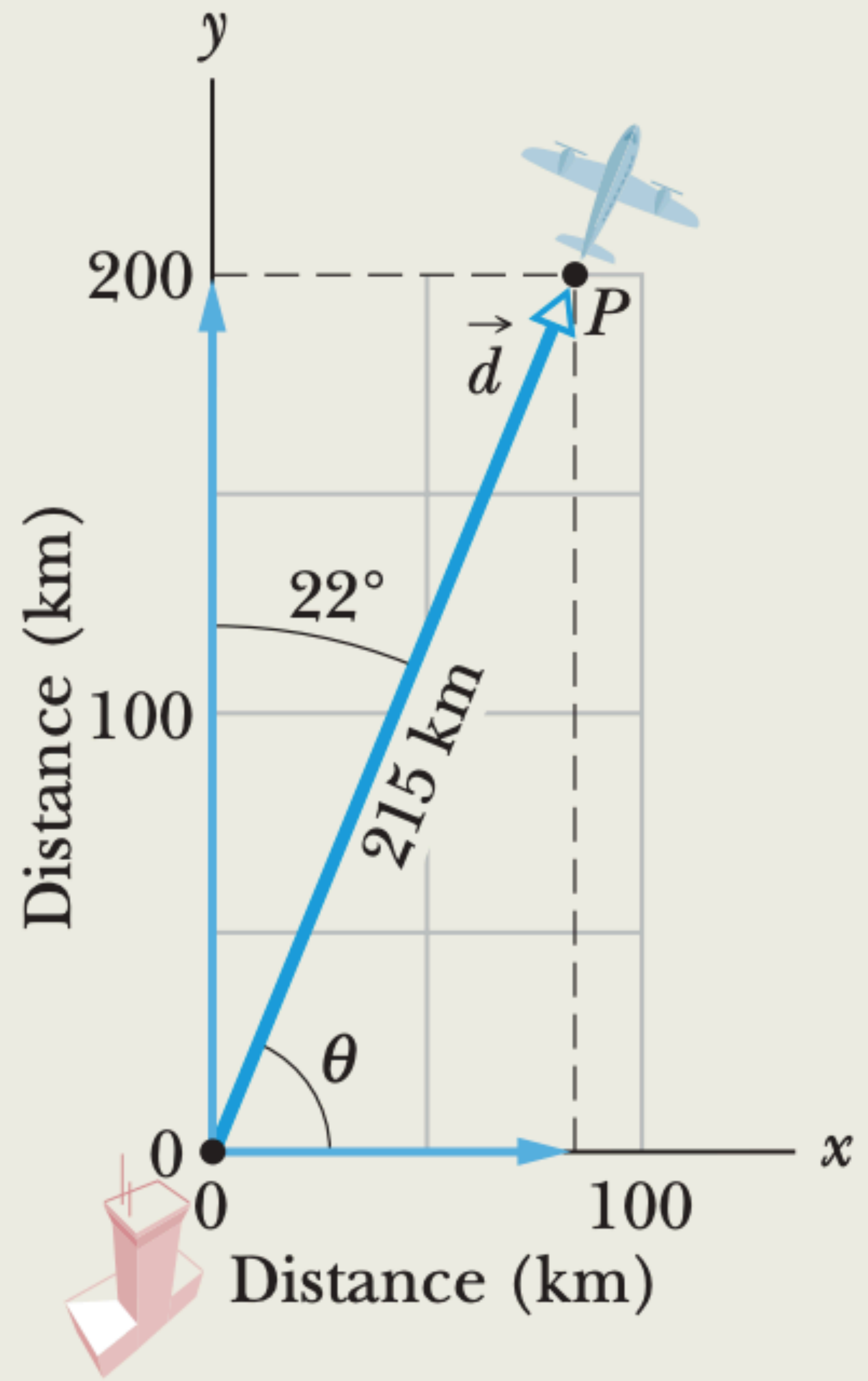
Unit Vector

- A unit vector is a vector that has a magnitude of exactly 1 and points in a particular direction. It lacks both dimension and unit. Its sole purpose is to point — that is, to specify a direction. The unit vectors in the positive directions of the x, y, and z. Unit vectors are very useful for expressing other vectors;



Position Vector

- Magnitude-angle notation

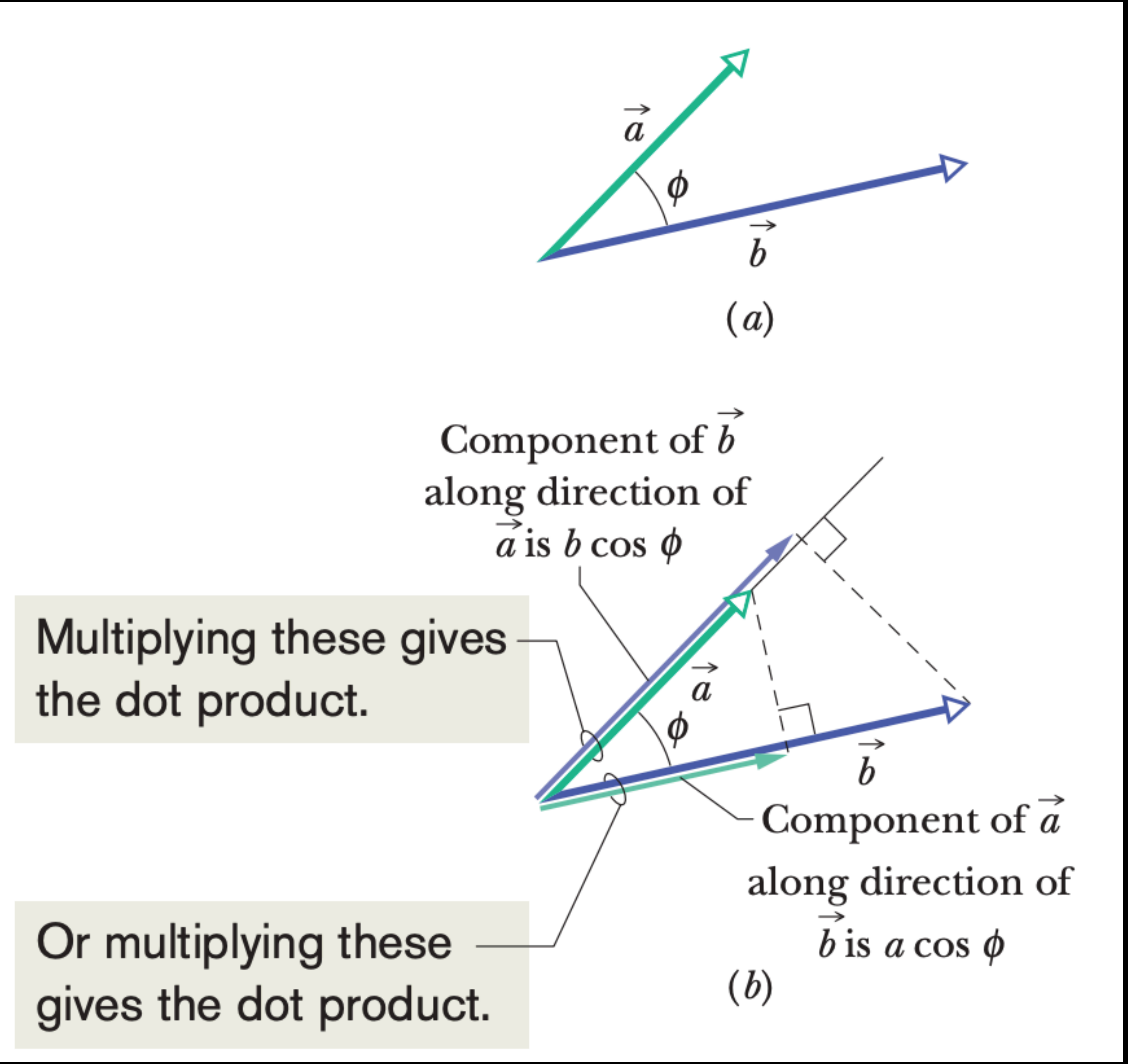


Multiplying Vectors

- Multiplying a vector by a scalar

Multiplying a vector by a vector (scalar product)

$$\vec{a} \cdot \vec{b} = ab\cos\phi$$



Multiplying Vectors

- Multiplying a vector by a scalar

Multiplying a vector by a vector

Scalar product:

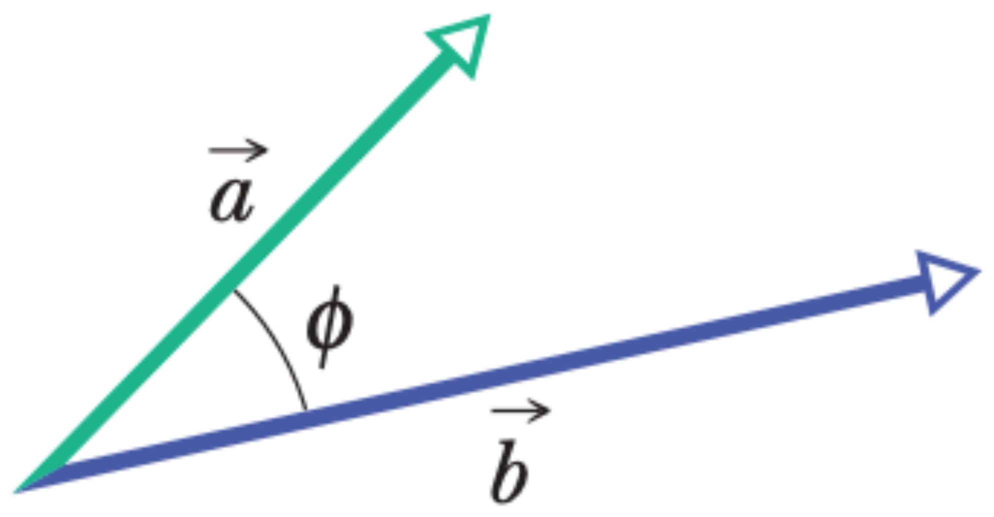
$\vec{a} \cdot \vec{b} = ab\cos\phi$

Vector product:

$\vec{a} \times \vec{b} = ab\sin\phi$

If $\vec{a}$ and $\vec{b}$ are parallel or anti-parallel,

What is the vector product of $\vec{a}$ and $\vec{b}$



$$\times \vec{b} = -\vec{b} \times \vec{a} = \begin{vmatrix} \hat{\mathrm{i}} & \hat{\mathrm{j}} & \hat{\mathrm{k}} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

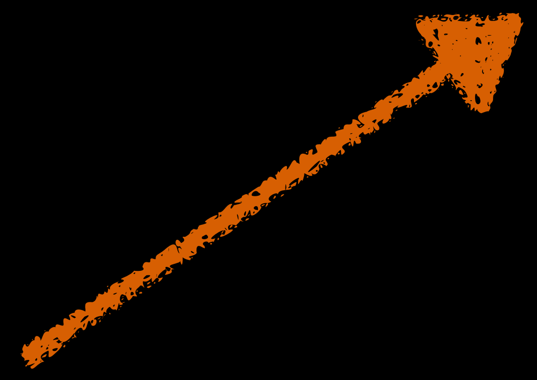
$$= \hat{\mathrm{i}} \begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix} - \hat{\mathrm{j}} \begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix} + \hat{\mathrm{k}} \begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix}$$

$$= (a_y b_z - b_y a_z)\hat{\mathrm{i}} + (a_z b_x - b_z a_x)\hat{\mathrm{j}} + (a_x b_y - b_x a_y$$

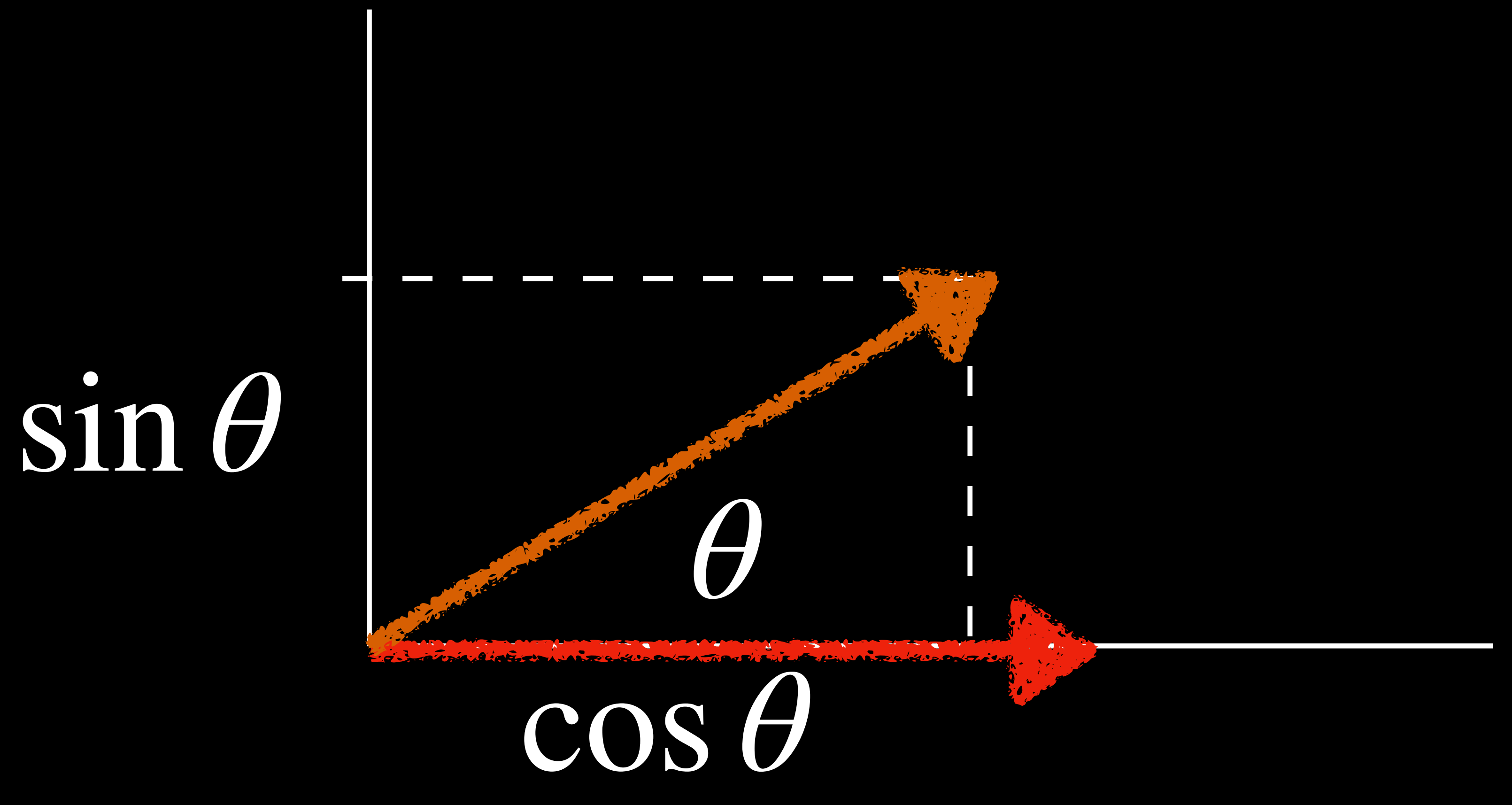
$$|\vec{a} \times \vec{b}| = ab \sin \theta$$

Rotation matrix & Vectors

$$\vec{a} = \begin{vmatrix} 1 \\ 0 \end{vmatrix}$$



$$\vec{a}' = \begin{vmatrix} \cos\theta \\ \sin\theta \end{vmatrix}$$



$$\begin{vmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{vmatrix} \begin{vmatrix} 1 \\ 0 \end{vmatrix} = \begin{vmatrix} \cos\theta \\ \sin\theta \end{vmatrix}$$

$$M_{11} * 1 + M_{12} * 0 = \cos\theta$$

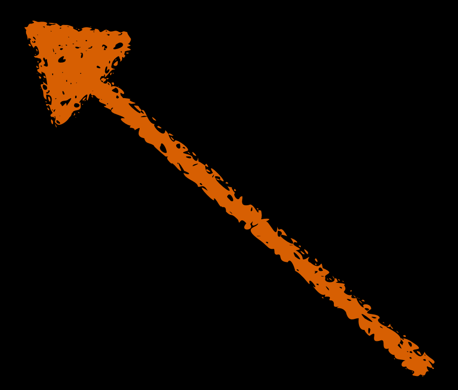
$$M_{21} * 1 + M_{22} * 0 = \sin\theta$$

$$\begin{vmatrix} \cos\theta & M_{12} \\ -\sin\theta & M_{22} \end{vmatrix}$$

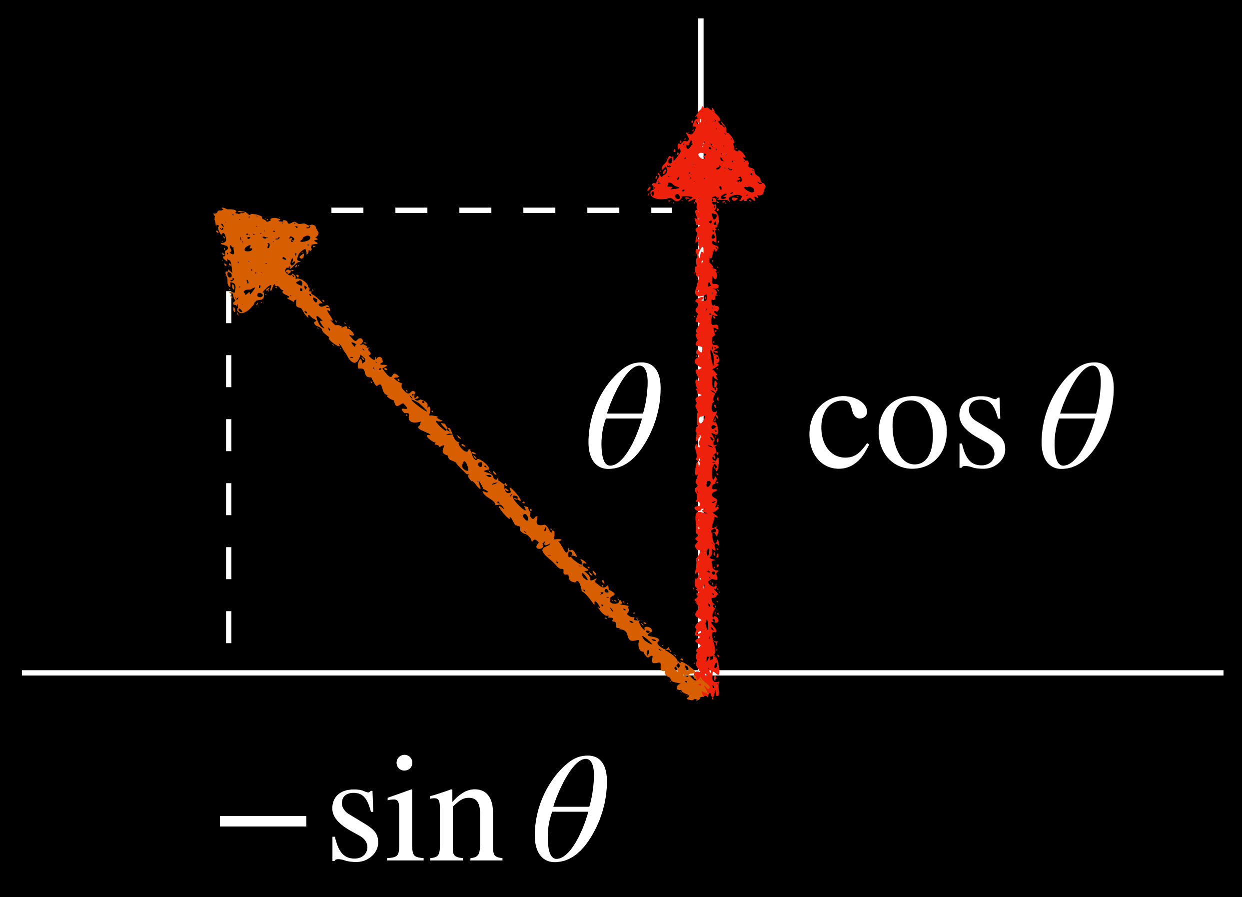
Rotation matrix & Vectors

$$\begin{vmatrix} \cos\theta & M_{12} \\ \sin\theta & M_{22} \end{vmatrix}$$

$$\vec{a} = \begin{vmatrix} 0 \\ 1 \end{vmatrix}$$



$$\vec{a}\,' = \begin{vmatrix} -\sin\theta \\ \cos\theta \end{vmatrix}$$



$$\begin{vmatrix} \cos\theta & M_{12} \\ \sin\theta & M_{22} \end{vmatrix} \begin{vmatrix} 0 \\ 1 \end{vmatrix} = \begin{vmatrix} -\sin\theta \\ \cos\theta \end{vmatrix}$$

$$\cos\theta * 0 + M_{12} * 1 = -\sin\theta$$

$$\sin\theta * 0 + M_{22} * 1 = \cos\theta$$

$$\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$$ For anti-clockwise

$$\begin{vmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{vmatrix}$$ For clockwise

Identity Matrix, Unit matrix

$$I = 1,\ \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix},\ \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix},\ \ldots$$

When A is m×n, it is a property of matrix multiplication that

$$I_m \times A = A,\ \ A \times I_n = A$$

Axis rotation (Ex: 3D)

$$R_x = \begin{vmatrix} 1 & 0 & 0 \\ 0 & cos\theta & sin\theta \\ 0 & -sin\theta & cos\theta \end{vmatrix}$$

$$R_y = \begin{vmatrix} cos\theta & 0 & sin\theta \\ 0 & 1 & 0 \\ -sin\theta & 0 & cos\theta \end{vmatrix}$$

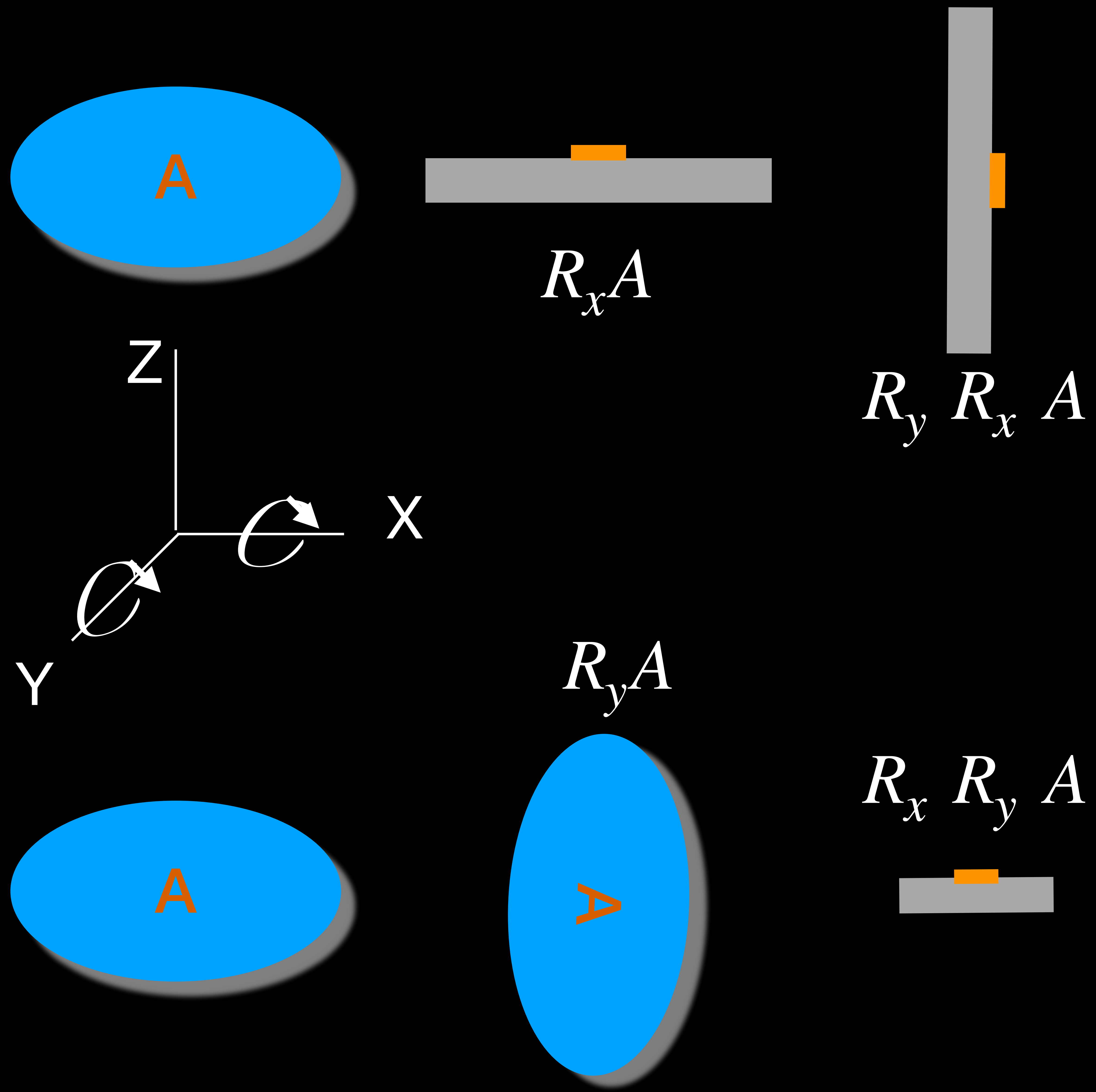
$$R_z = \begin{vmatrix} cos\theta & sin\theta & 0 \\ -sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Axis rotation (non-commutative)

$$R_x = \begin{vmatrix} 1 & 0 & 0 \\ 0 & cos\theta & sin\theta \\ 0 & -sin\theta & cos\theta \end{vmatrix}$$

$$R_y = \begin{vmatrix} cos\theta & 0 & sin\theta \\ 0 & 1 & 0 \\ -sin\theta & 0 & cos\theta \end{vmatrix}$$

$$R_z = \begin{vmatrix} cos\theta & sin\theta & 0 \\ -sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

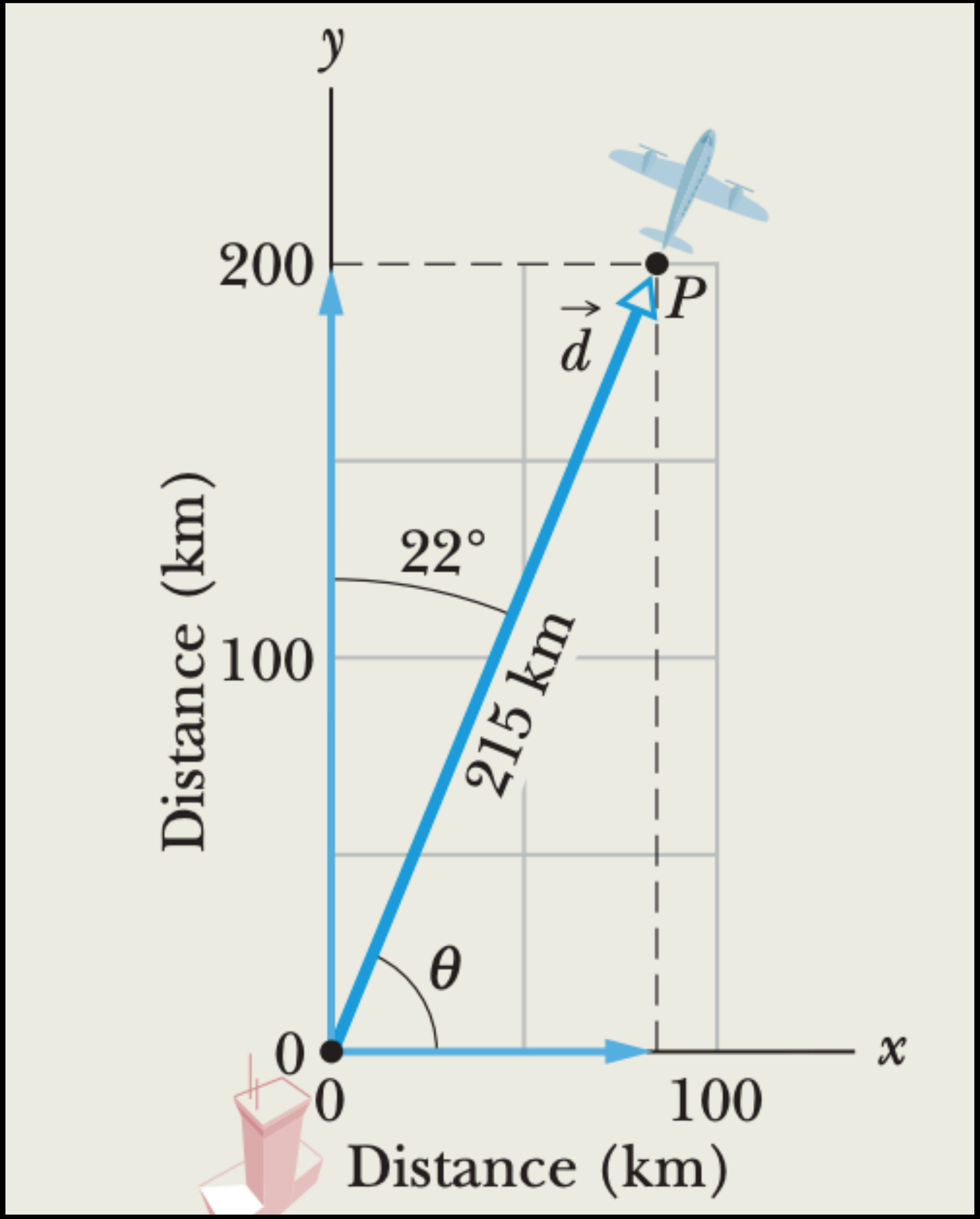


Position vector

position vector $\vec{r}$, which is a vector that extends from a reference point (usually the origin) to the particle. In the unit-vector

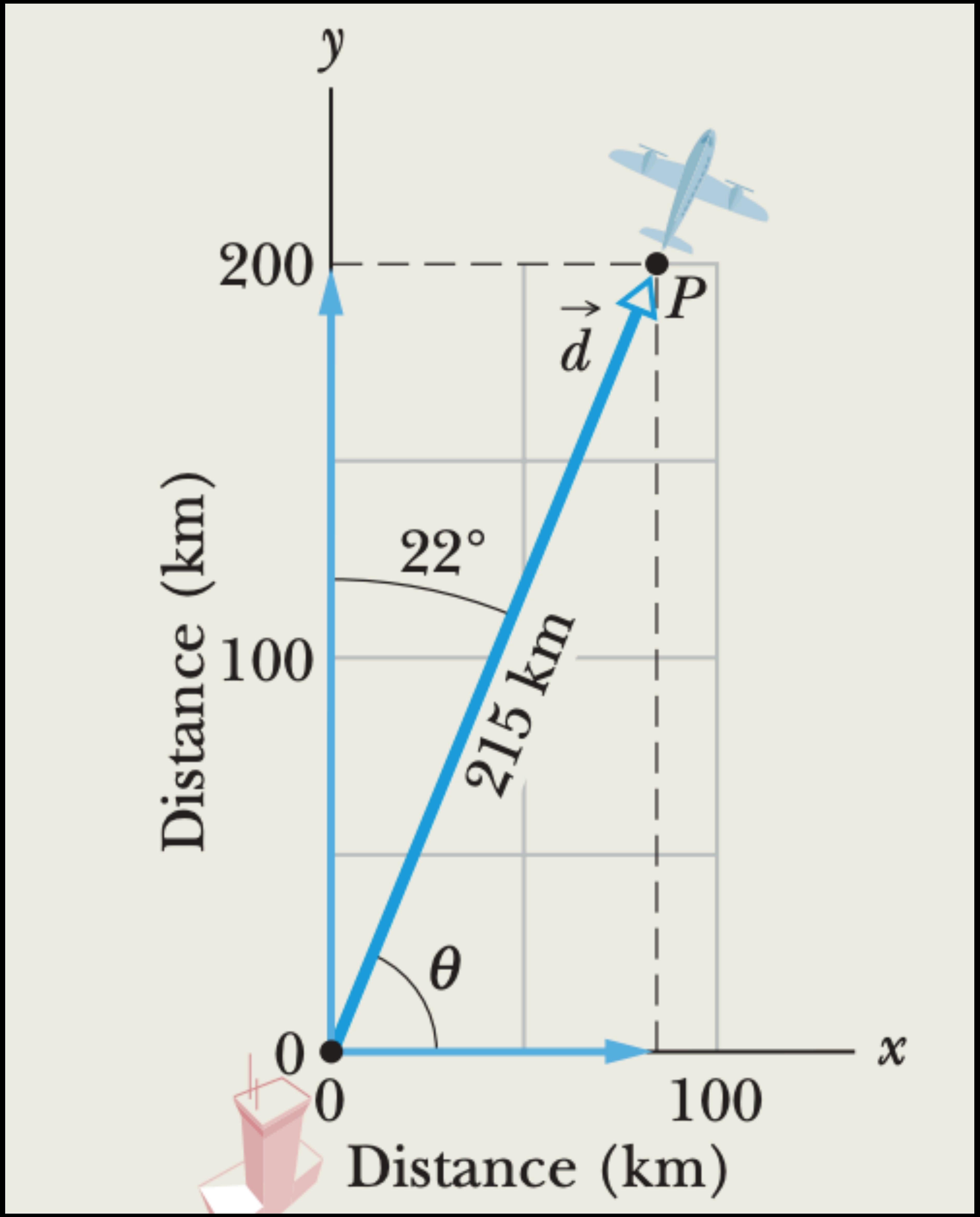
$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$$



Position vector

then the particle's displacement $\vec{r}$ during that time interval is



$$\vec{r}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$$

$$\vec{r}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$$

$$\Delta\vec{r} = \vec{r}(t_2) - \vec{r}(t_1)$$

$$= \vec{r}_2 - \vec{r}_1$$

$$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

Consider Peter's walk

$$\vec{r} = x(t)\hat{i} + y(t)\hat{j}$$

$$x(t) = -\,t^2 + 2t + 1$$

$$y(t) = t^2 - 2t + 1$$

What is the path of Peter from 0 s to 3 s?

Velocity again

Instantaneous velocity

$$\vec{V} = \lim_{t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

$$\vec{V} = \lim_{t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

Acceleration:

$$\vec{a} = \lim_{t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

$$\vec{a} = \lim_{t \to 0} \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{d\vec{v}}{dt}$$

Consider Peter's walk

$$\vec{r} = x(t)\hat{i} + y(t)\hat{j}$$

$$x(t) = -t^2 + 2t + 1$$

$$y(t) = t^2 - 2t + 1$$

What is the velocity of Peter?

Consider Peter's walk

$$\vec{r} = x(t)\hat{i} + y(t)\hat{j}$$

$$x(t) = -t^2 + 2t + 1$$

$$y(t) = t^2 - 2t + 1$$

What is the acceleration of Peter?

More practice

Here are four descriptions of the position (in meters) of a puck as it moves in an xy plane:

(1) $x = -3t^2 + 4t - 2$ and $y = 6t^2 - 4t$ (3) $\vec{r} = 2t^2\hat{\mathrm{i}} - (4t + 3)\hat{\mathrm{j}}$

(2) $x = -3t^3 - 4t$ and $y = -5t^2 + 6$ (4) $\vec{r} = (4t^3 - 2t)\hat{\mathrm{i}} + 3\hat{\mathrm{j}}$

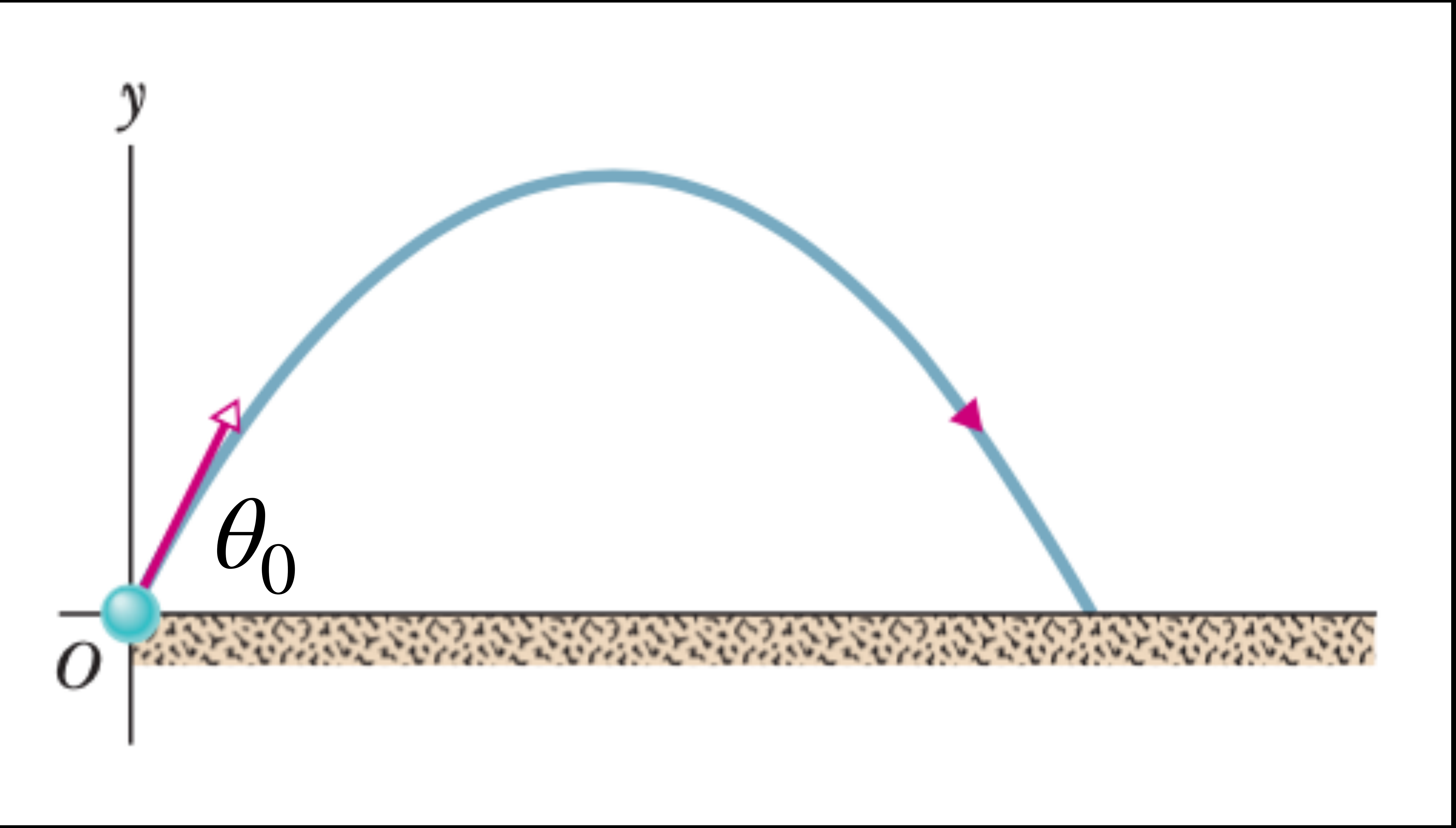
Are the x and y acceleration components constant? Is acceleration $\vec{a}$ constant?

Projectile motion

$$\overrightarrow{V} = V_{0x}\hat{i} + V_{oy}\hat{j}$$

$$V_{0x} = V_o \cos\theta_0$$

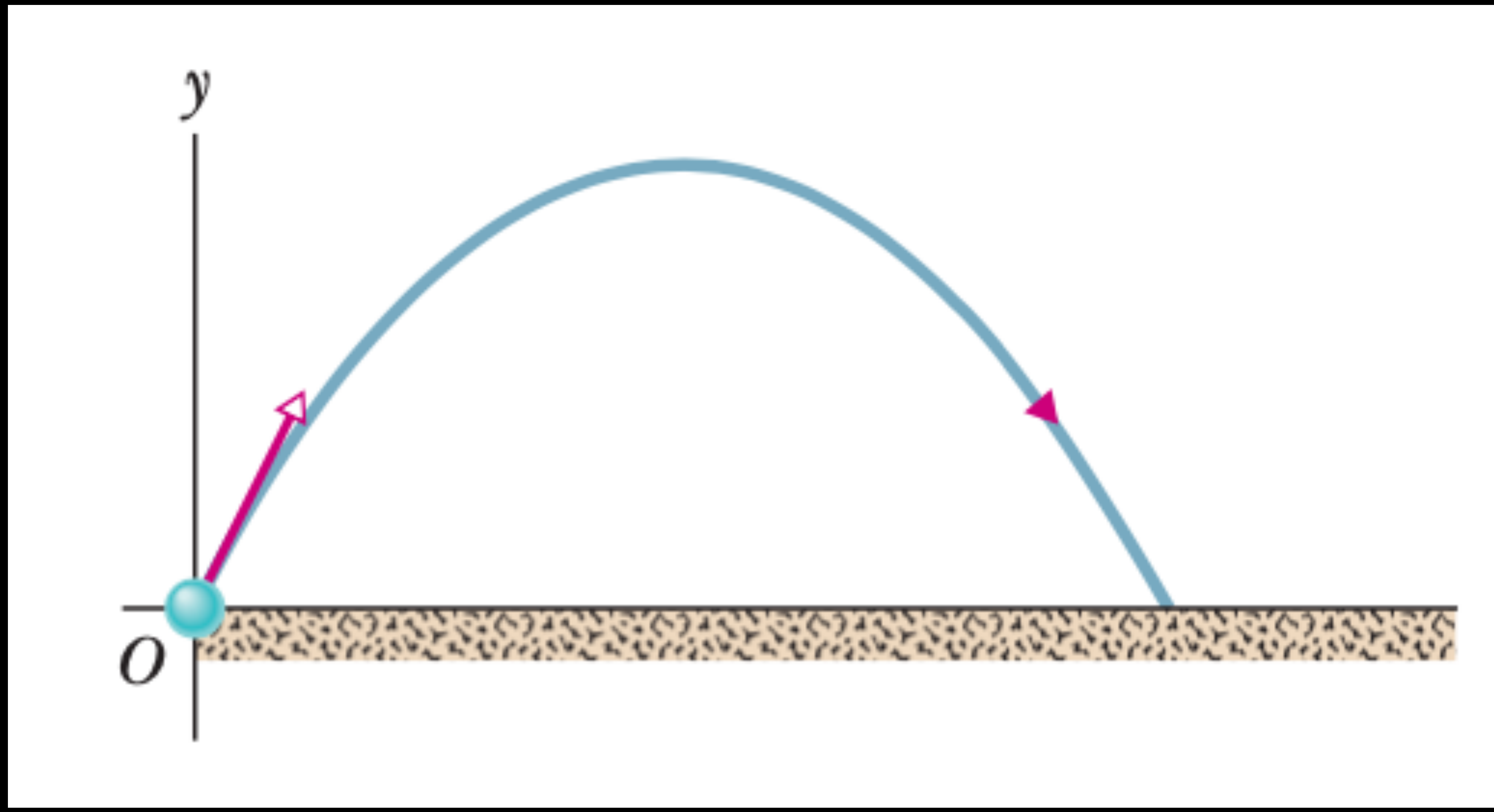
$$V_{0y} = V_o \sin\theta_0$$



Projectile motion

X axis:

$$X = V_{0x}t = V_o \cos\theta_0 t$$



Y axis:

$$Y = V_{0y}t - \frac{gt^2}{2} = V_0 \sin\theta_0 t - \frac{gt^2}{2}$$

$$V_y = V_{0y} + gt, \quad V_y^2 = V_{0y}^2 + 2g(y - y_0)$$

Projectile motion

X axis:

$$X = V_{0x}t = V_o \cos\theta_0 t \qquad t = \frac{x}{V_o \cos\theta_0}$$

Y axis:

$$Y = V_{0y}t - \frac{gt^2}{2} = V_0 \sin\theta_0 t - \frac{gt^2}{2}$$

$$= \tan\theta x - \frac{gx^2}{2V_0^2\cos^2\theta}$$

Trajectory!!