

# General Physics I

Mechanics, optics, thermal dynamics, and other basic fundamental things.

*TsungChe Liu*

# Grading

- Mid Exam 50%
- Final Exam 100%
- Others:
  - Class Interaction: +1% each positive feedback in class, please inform me the scores before the end of the day. (special rule: upper limit of electrophysics or physics department 30%, other department no upper limit)
  - Homework: 50% (special rule: I will insert some questions into the mid-exam and final exam, if you answer the question of home-work well but cannot complete the similar question in the mid-exam or final-exam, The score of that home work will be reset to zero)

# Lecture: Update of Grading and Rule

- 15 people don't like the original schedule(10:10-12:00 2:20-4:20) of the class and hope to complete the class ASAP, even with in 4 hrs.
  - Lunch break (30 mins or take a rest until 13:00)
- No one like the special rule. T\_T
- One people want to know the range of Mid-exam & Final-Exam.
- One people hope the loading of home work will not too heavy (<10 questions)
- Upload the slides: [pre.tir.tw/077/GP.html](http://pre.tir.tw/077/GP.html)

# Reminder of the Lecture I:

Keyword:

- Physical quantity:
  - Length, Time, & Mass
  - Unit (dimensional analysis)
  - Order of magnitude (f p n  $\mu$  m 0 k M G T P E )
  - Scale & Vector.

# Lecture II : Motion, Velocity, Force, & Energy

- Key word: Motion, time, displacement, vector, dot product, velocity, acceleration, energy, & kinetic energy.

# Lecture II : Motion along Straight line

- Positions of point

- $t = 0$

$$t = t_1 = 1\text{s}$$

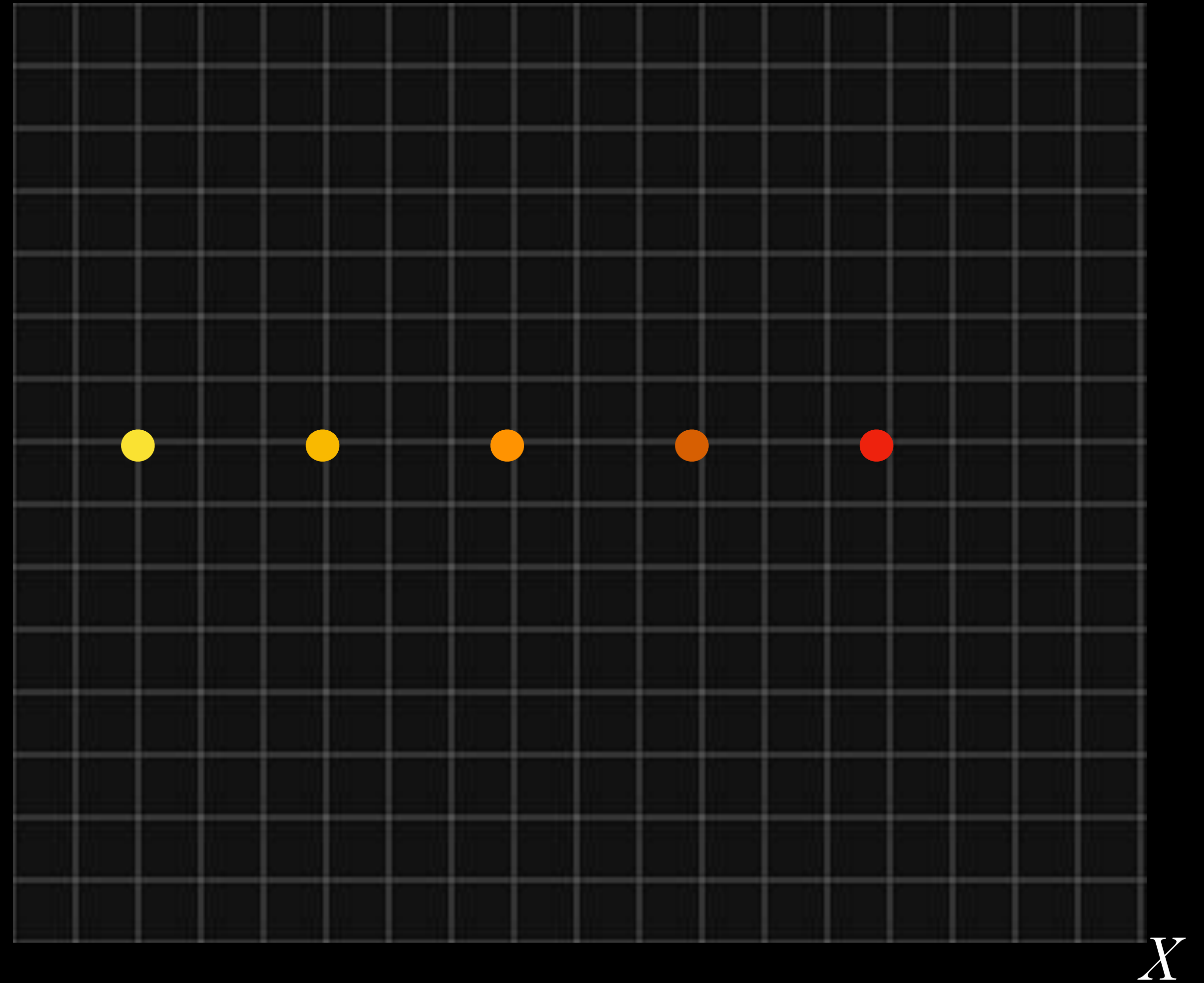
$$t = t_2 = 2\text{s}$$

$$t = t_3 = 3\text{s} \dots$$



# Lecture II : Motion along Straight line

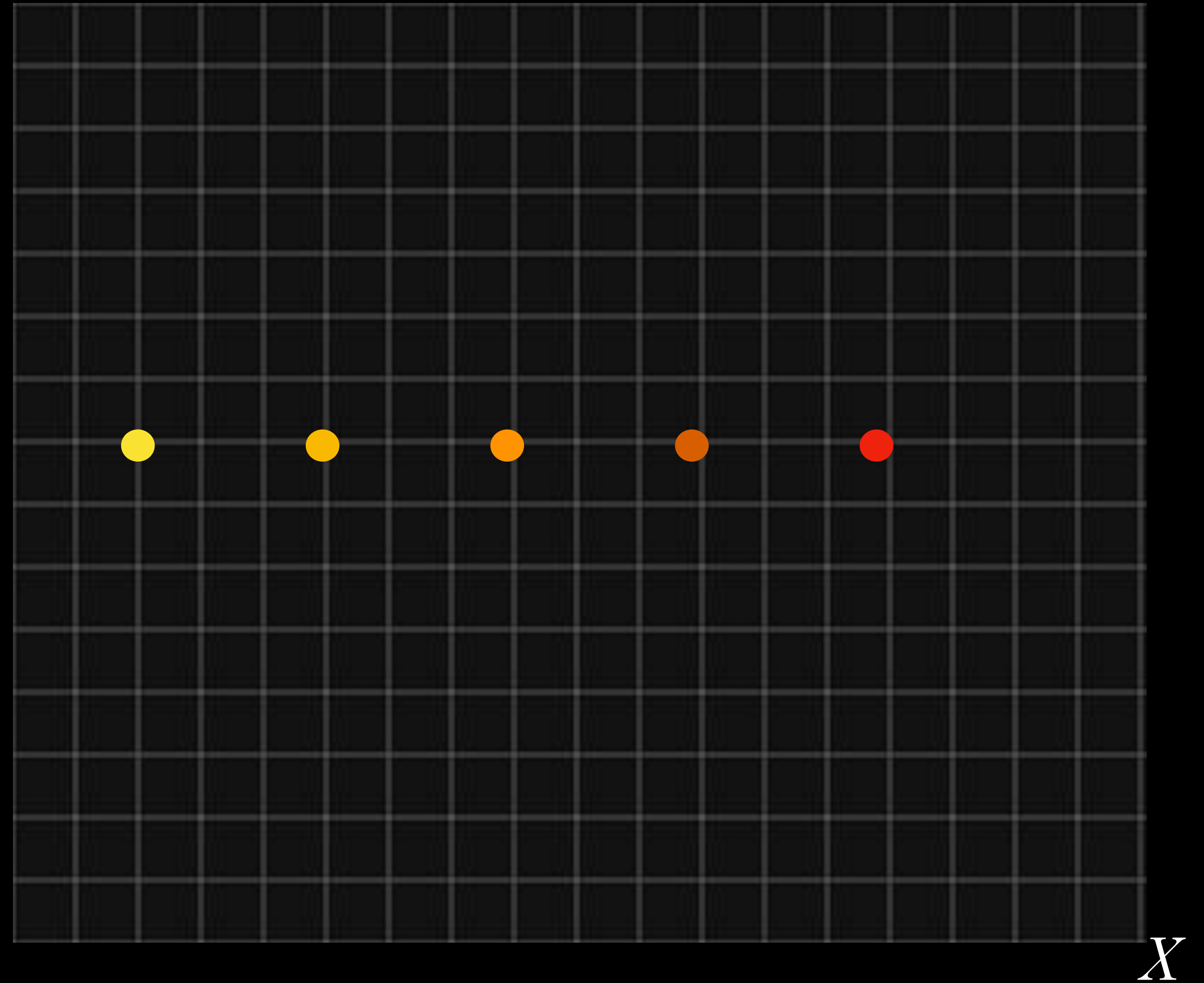
- Positions of point
  - $t = 0$
  - $t = t_1$
  - $t = t_2$
  - $t = t_3 \dots$



# Lecture II : Motion along Straight line

## Newton's first law

If no force acts on a body, the body's velocity cannot change; that is, the body cannot accelerate.





# Lecture II : Instantaneous velocity & Average velocity

Average velocity

$$\vec{V}_{avg} = \frac{\Delta x}{\Delta t}$$

Average speed

$$S_{avg} = \frac{\text{total distance}}{\Delta t}$$



x

# Lecture II : Instantaneous velocity & Average velocity

You drive a beat-up pickup truck along a straight road for 8.4 km at 70 km/h, at which point the truck runs out of gasoline and stops. Over the next 30 min, you walk another 2.0 km farther along the road to a gasoline station.

(a) What is your overall displacement from the beginning of your drive to your arrival at the station?



# Lecture II : Instantaneous velocity & Average velocity

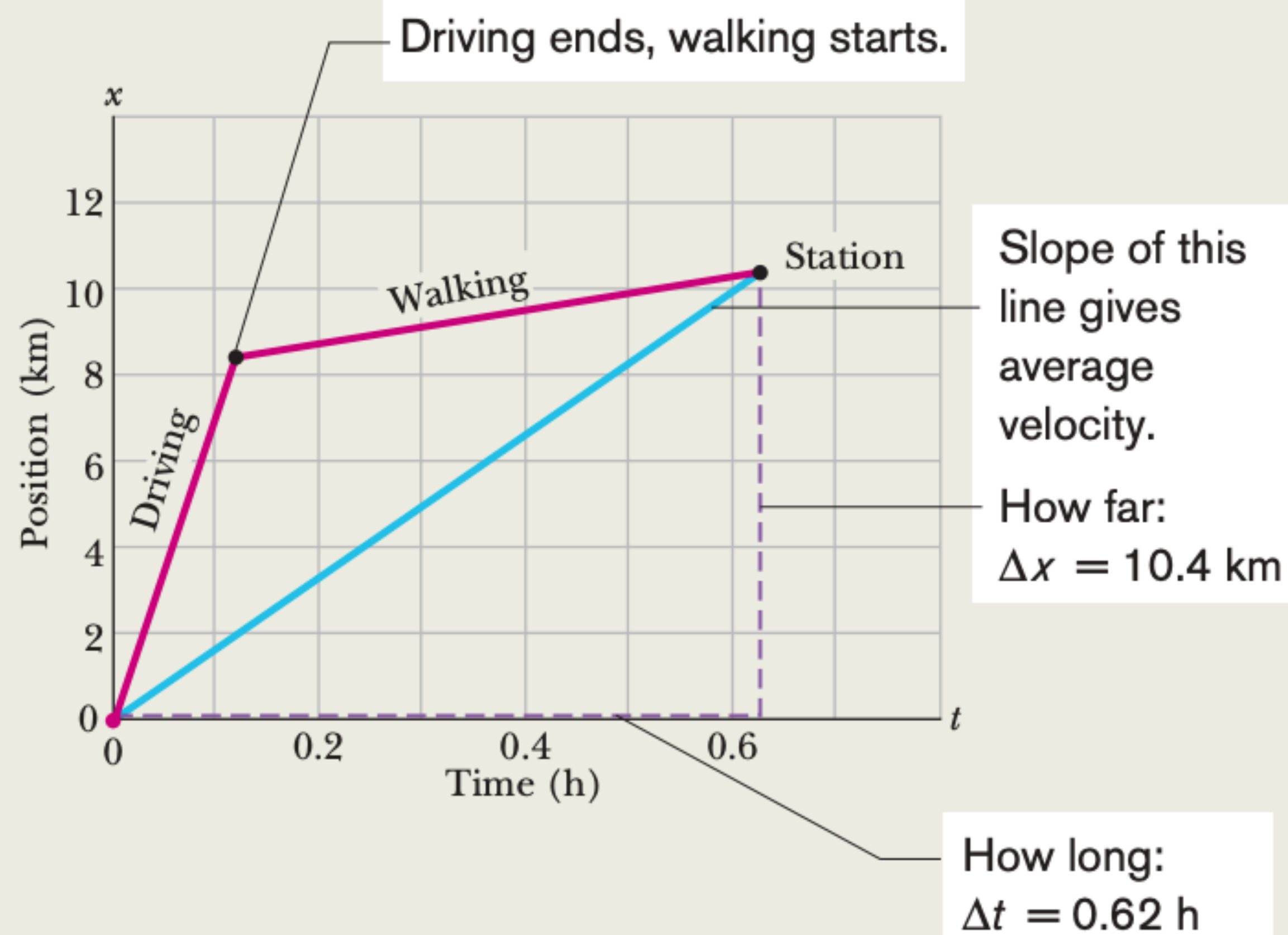
You drive a beat-up pickup truck along a straight road for 8.4 km at 70 km/h, at which point the truck runs out of gasoline and stops. Over the next 30 min, you walk another 2.0 km farther along the road to a gasoline station.

(a) What is your overall displacement from the beginning of your drive to your arrival at the station?

(b) What is the time interval  $\Delta t$  from the beginning of your drive to your arrival at the station?

(c) What is your average velocity  $v_{\text{avg}}$  from the beginning of your drive to your arrival at the station? Find it both numerically and graphically.

(d) Suppose that to pump the gasoline, pay for it, and walk back to the truck takes you another 45 min. What is your average speed from the beginning of your drive to your return to the truck with the gasoline?



**Figure 2-5** The lines marked “Driving” and “Walking” are the position–time plots for the driving and walking stages. (The plot for the walking stage assumes a constant rate of walking.) The slope of the straight line joining the origin and the point labeled “Station” is the average velocity for the trip, from the beginning to the station.

# Lecture II : Instantaneous velocity & Average velocity

You have now seen two ways to describe how fast something moves: average velocity and average speed, both of which are measured over a time interval  $t$ . However, the phrase “how fast” more commonly refers to how fast a particle is moving at a given instant—its **instantaneous velocity** (or simply **velocity**)  $v$

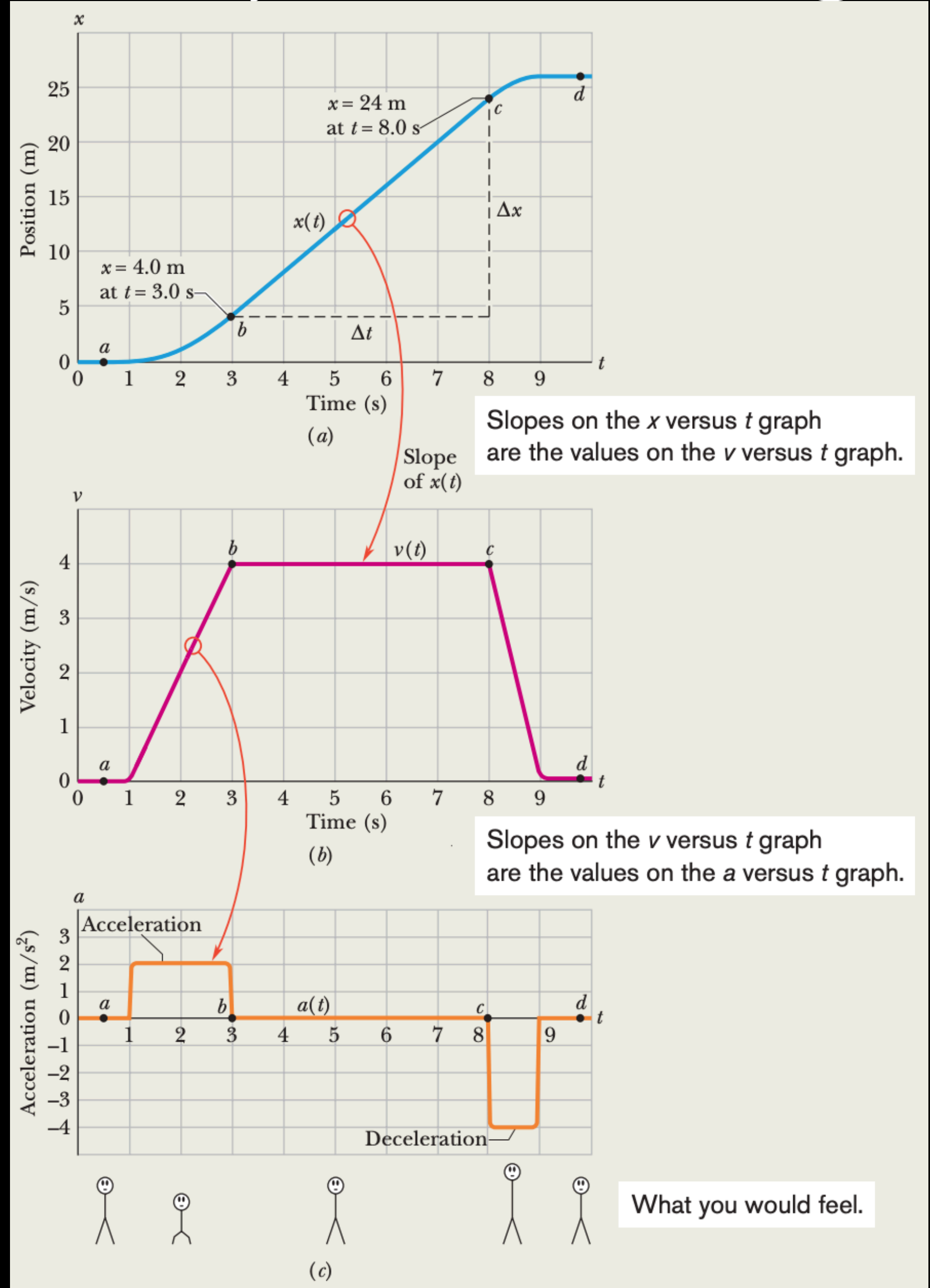
Instantaneous velocity

$$\vec{v} = \lim_{t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

# Lecture II : Instantaneous velocity & Average velocity

Instantaneous velocity

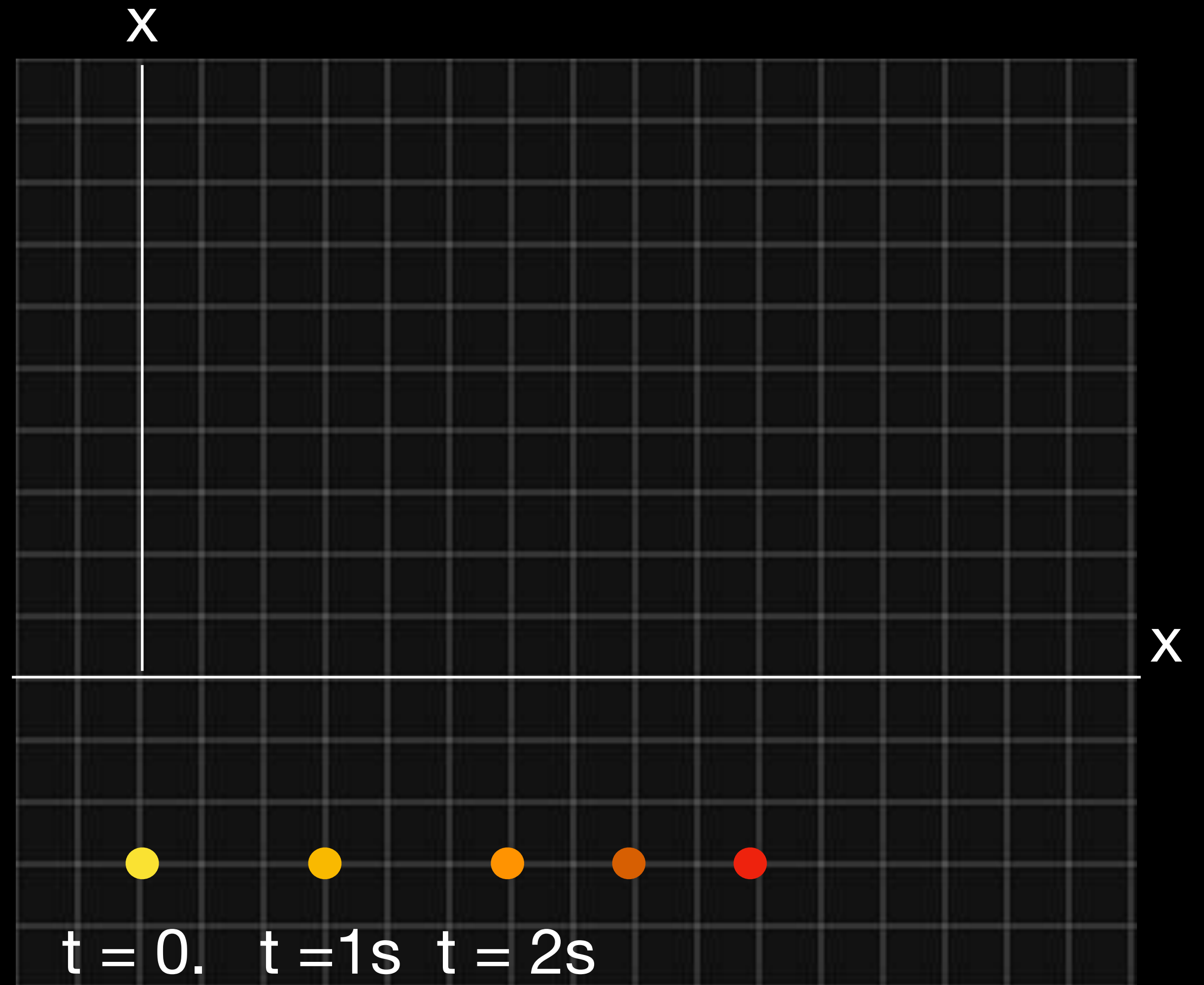
$$\vec{v} = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$



# Lecture II : the change of velocity

Instantaneous velocity

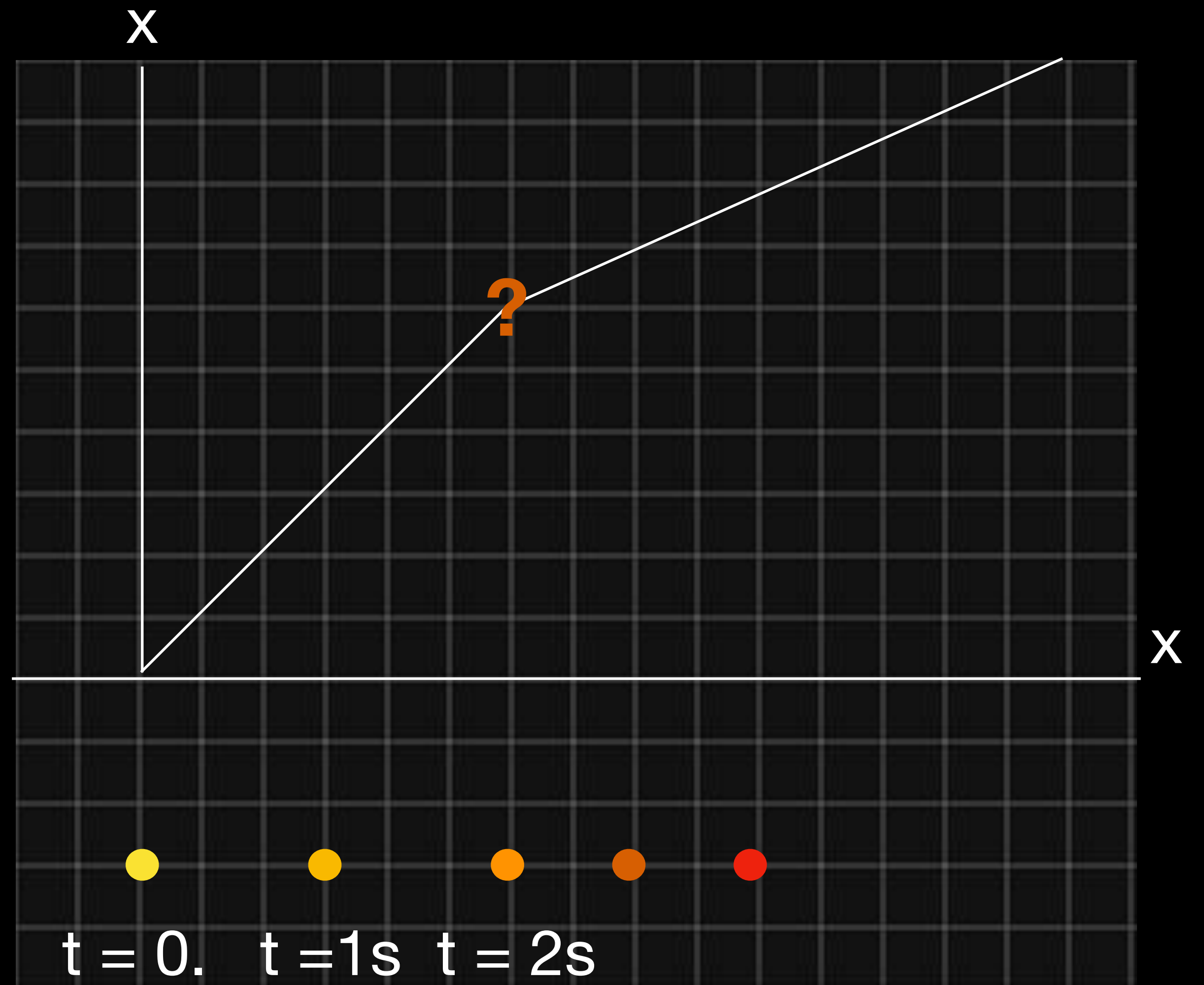
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# Lecture II : the change of velocity

Instantaneous velocity

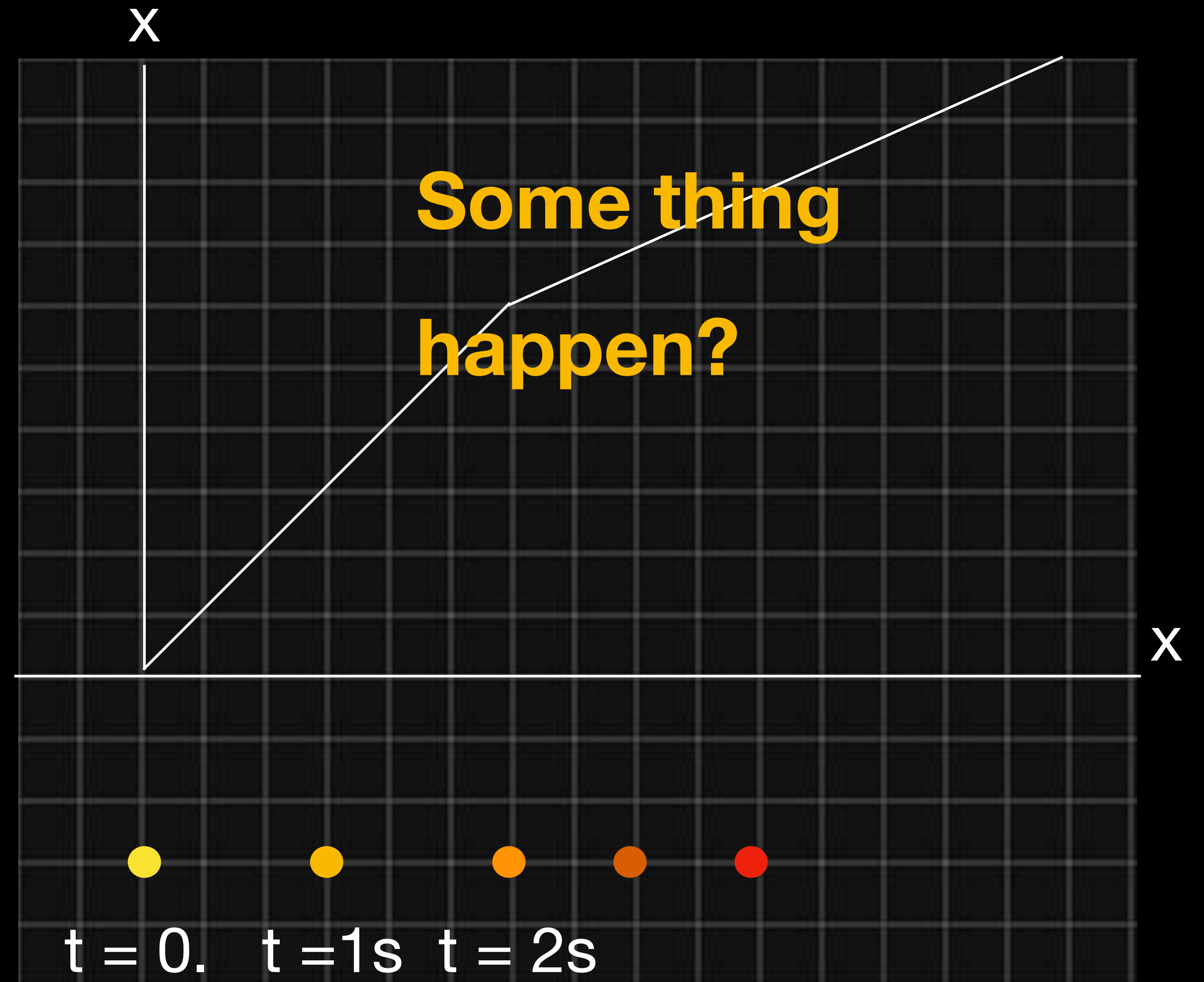
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# Lecture II : the change of velocity

Instantaneous velocity

$$\vec{v} = \lim_{t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$





# Lecture II : the change of velocity

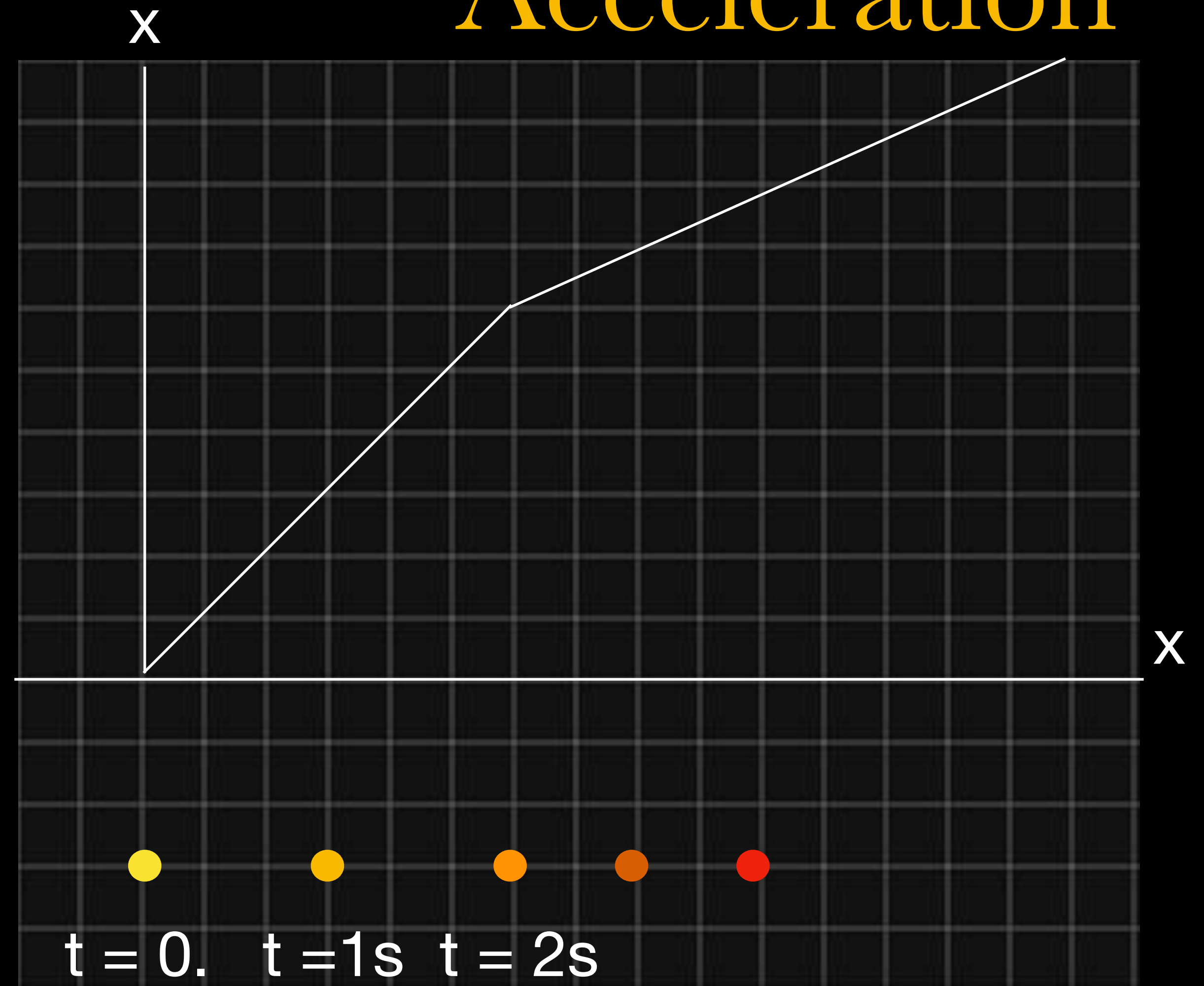
## Acceleration

Instantaneous velocity

$$\vec{v} = \lim_{t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Acceleration:

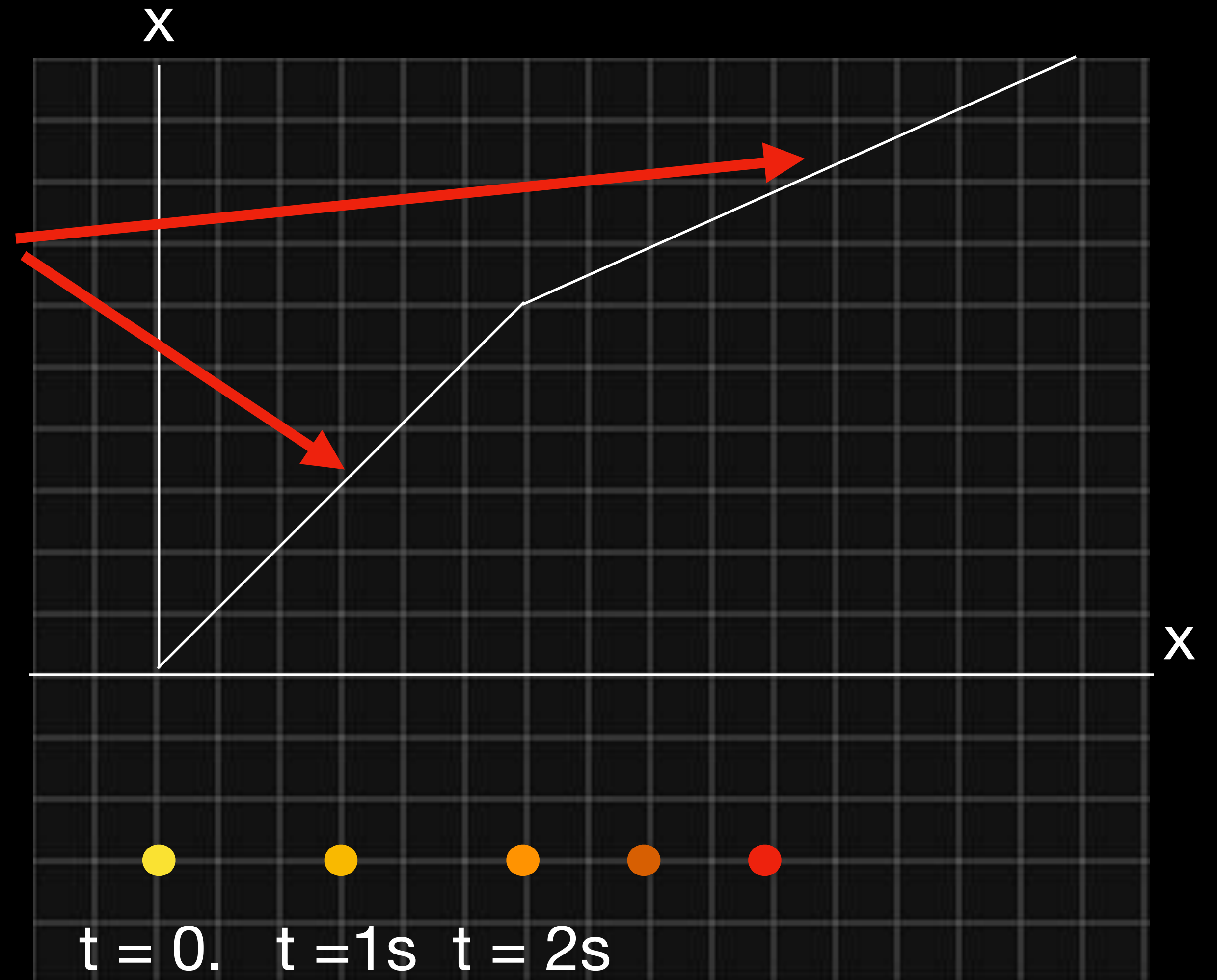
$$? = \lim_{t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$



# Lecture II : Acceleration

## Newton's first law

If no force acts on a body, the body's velocity cannot change; that is, the body cannot accelerate.



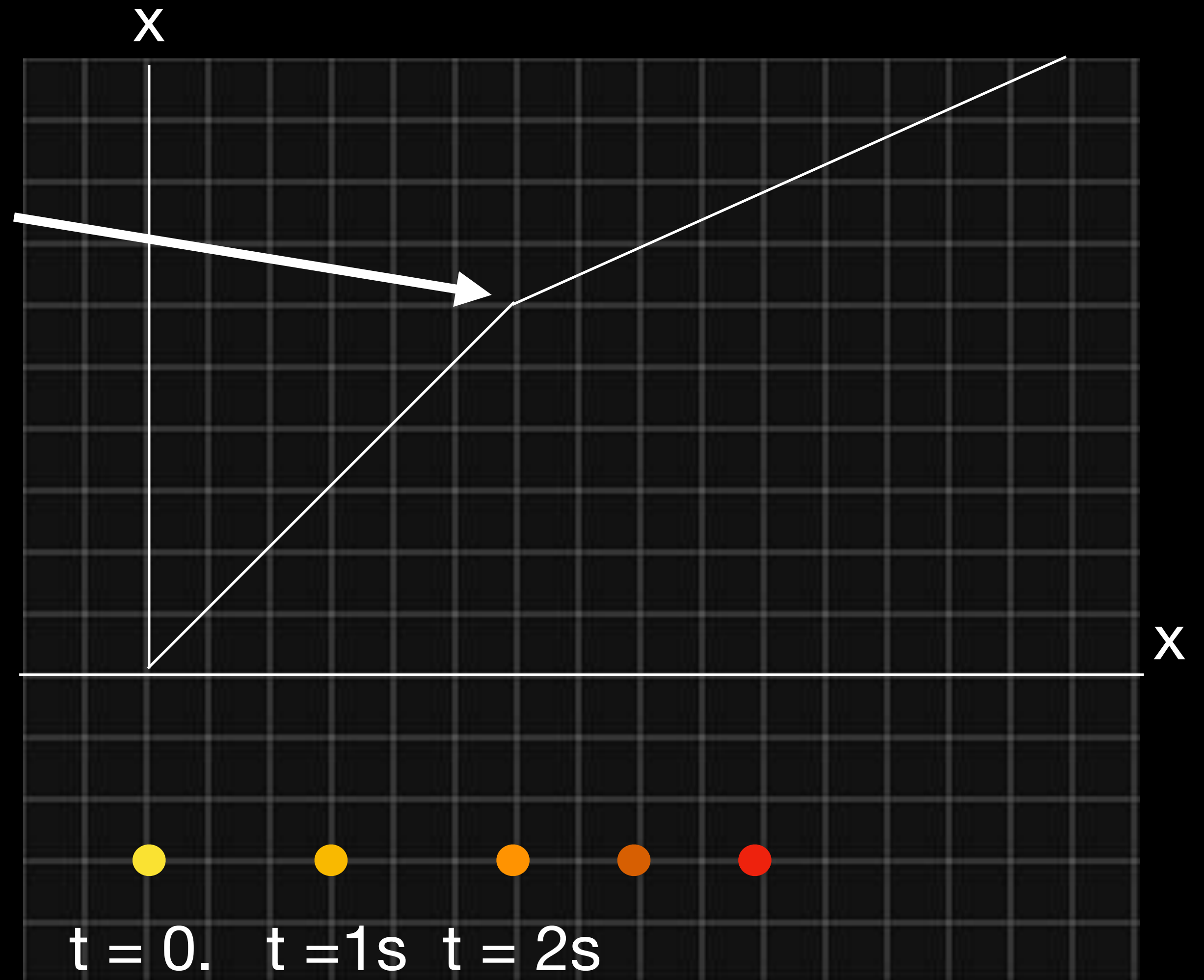
# Lecture II : Acceleration

## Newton's second law

The net force on a body is equal to the product of the body's mass and its acceleration.

Acceleration:

$$? = \lim_{t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$



# Lecture II : Acceleration: ex

A particle's position on the  $x$  axis is given by

$$x = 4 - 27t + t^3,$$

with  $x$  in meters and  $t$  in seconds.

(a) Because position  $x$  depends on time  $t$ , the particle must be moving. Find the particle's velocity function  $v(t)$  and acceleration function  $a(t)$ .

(b) Describe the particles motion for  $t > 0$

(c) Is there ever a time when  $v=0$

# Lecture II : Acceleration

A particle's position on the  $x$  axis is given by

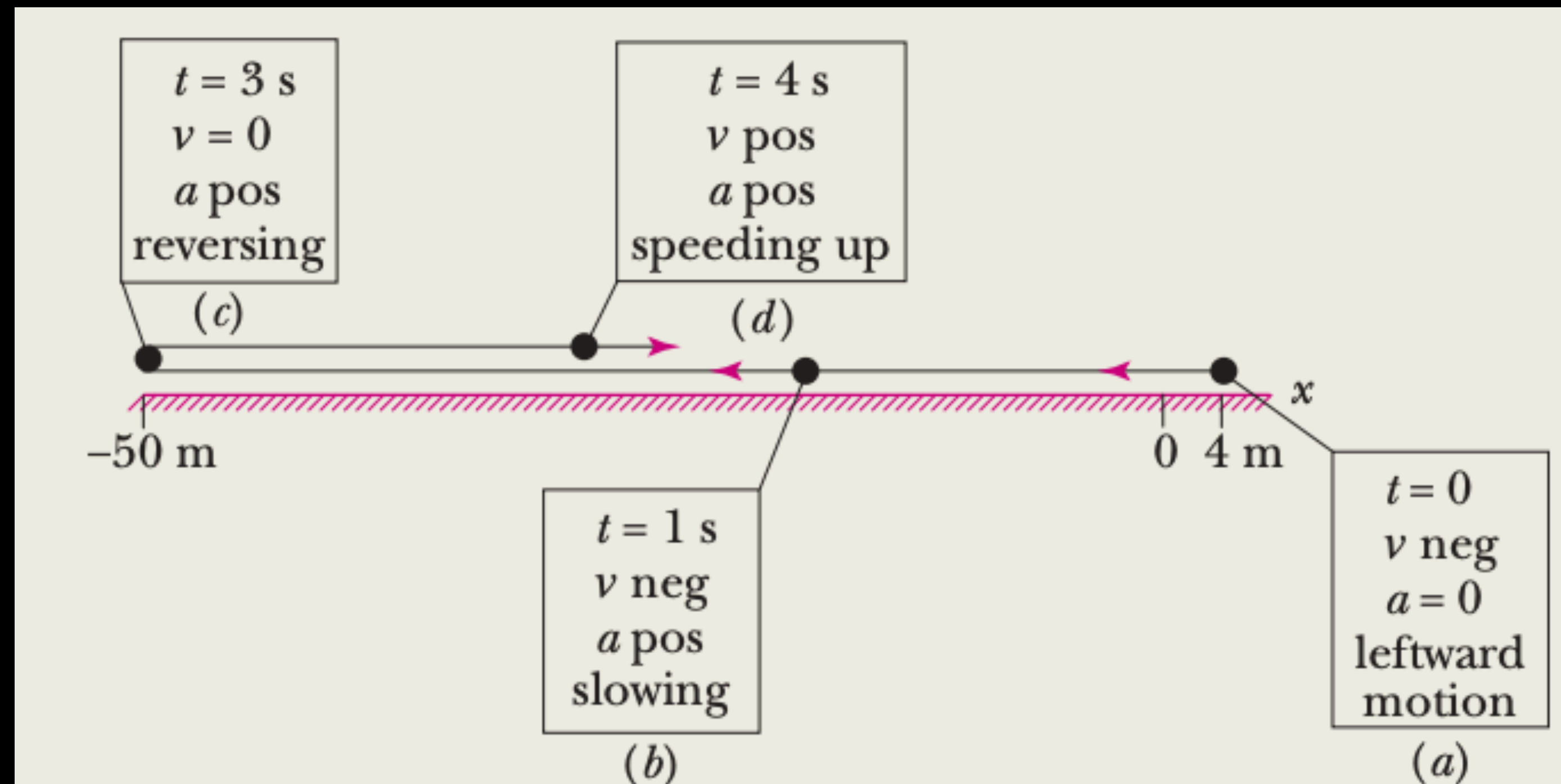
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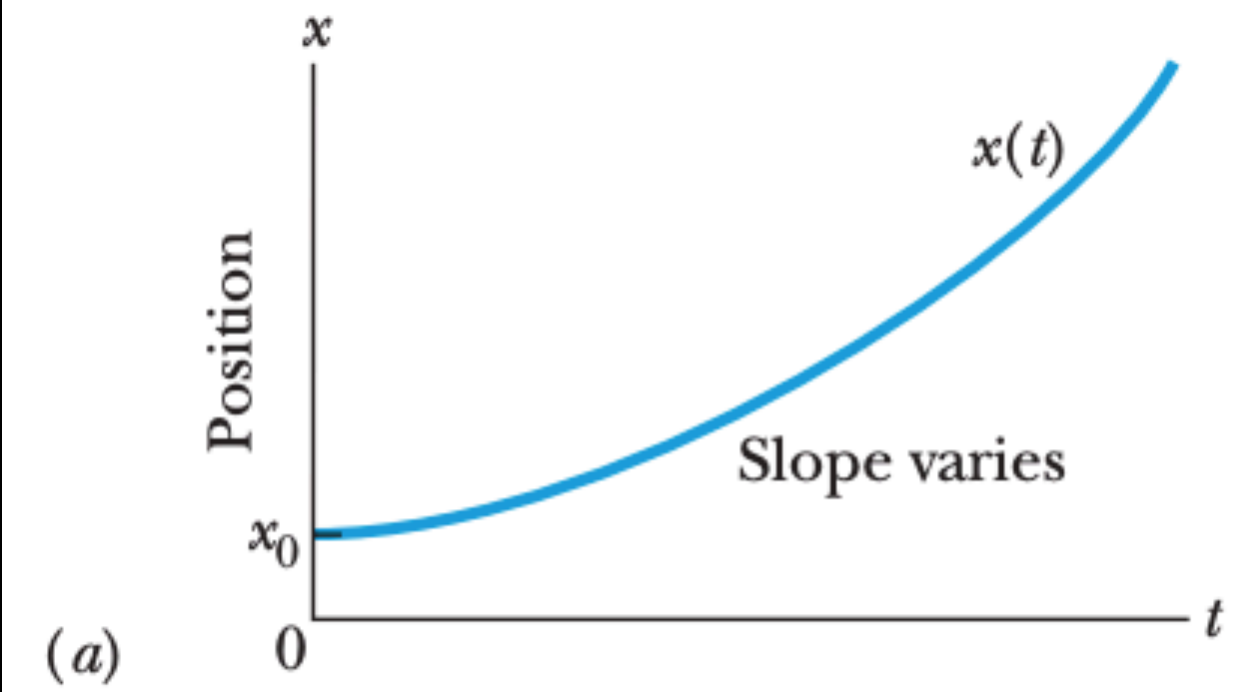


# Lecture II : Acceleration = C

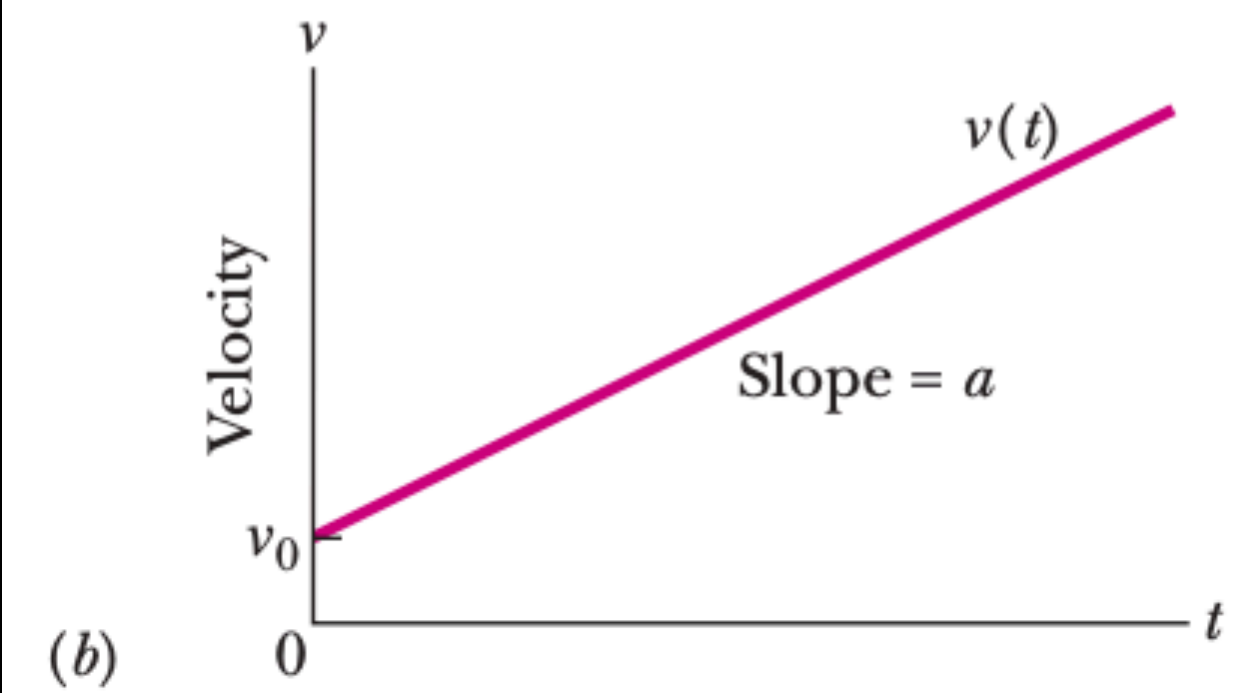
$$\vec{a} = \text{constant}$$

$$\vec{v} =$$

$$x =$$



Slopes of the position graph are plotted on the velocity graph.



Slope of the velocity graph is plotted on the acceleration graph.



# Lecture II : Acceleration = C

Question 1:

Spotting a police car, you brake a Ferrari from a speed 200 km/h to a speed 100 km/h during a displacement of 100m, at a constant acceleration.

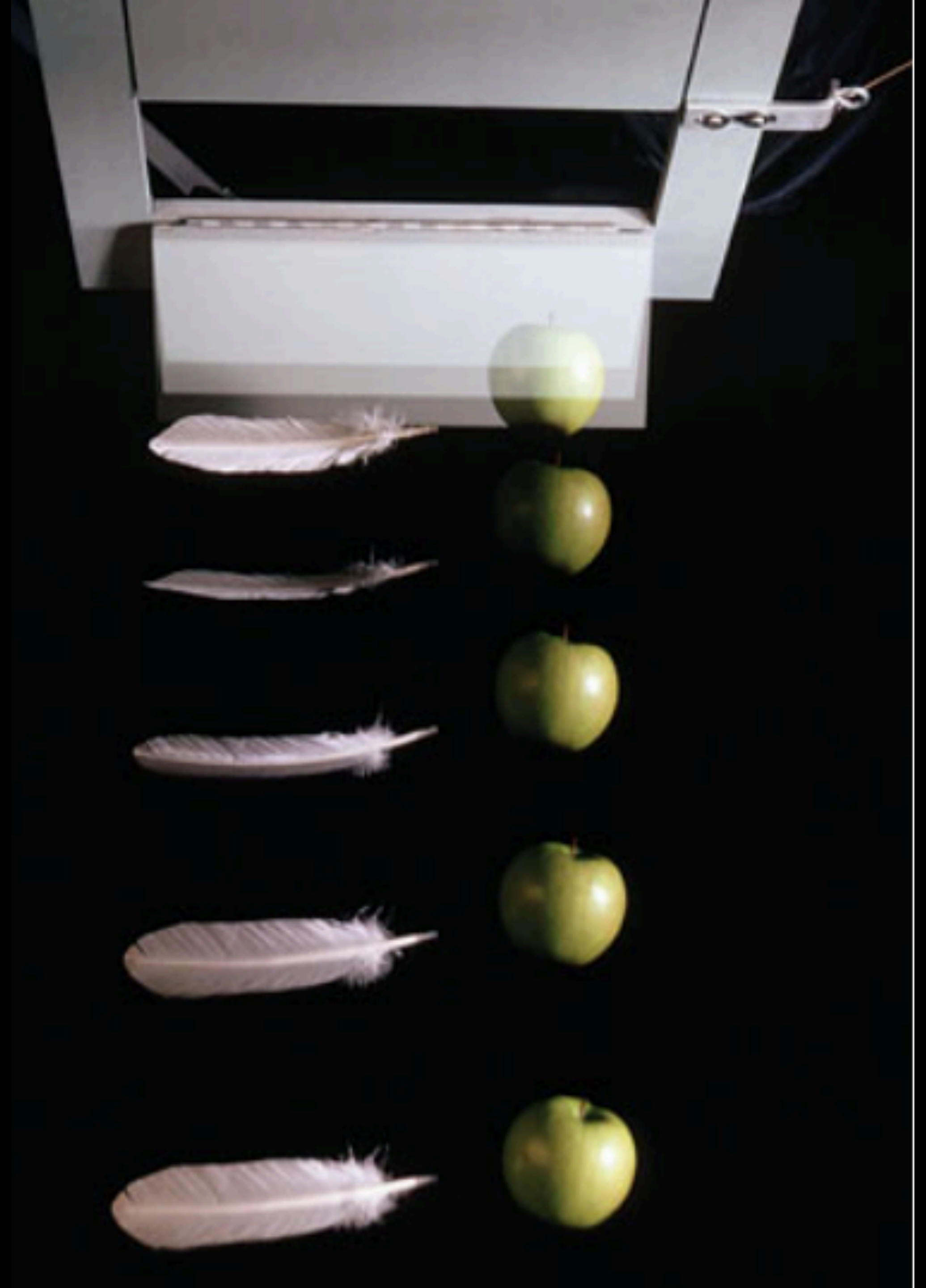
(a) what is that acceleration

(b) How many time is required for the given decrease in speed.



# Free-Fall Acceleration

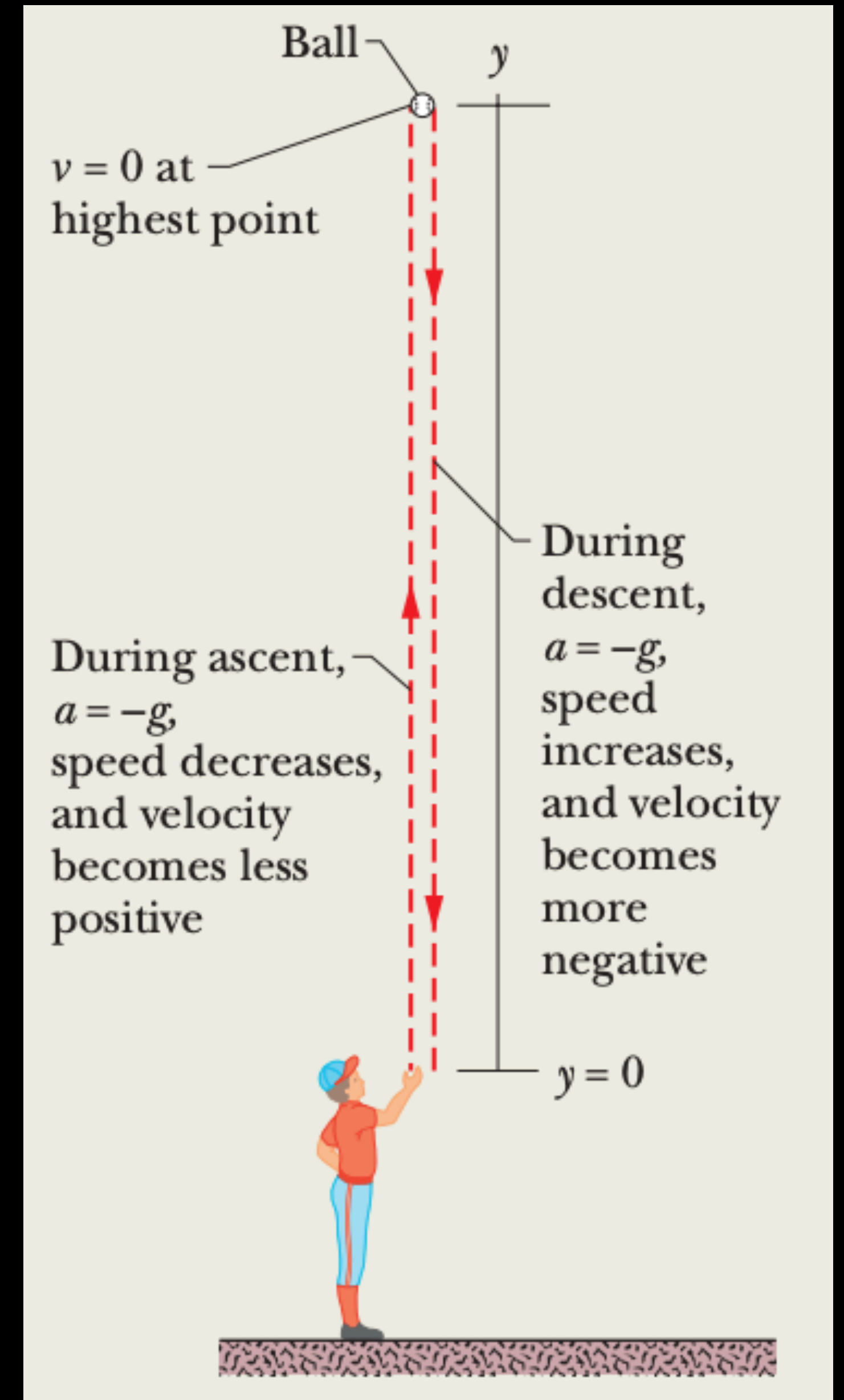
The free-fall acceleration near Earth's surface is  $a = -g = -9.8 \text{ m/s}^2$ , hence  $g = 9.8 \text{ m/s}^2$





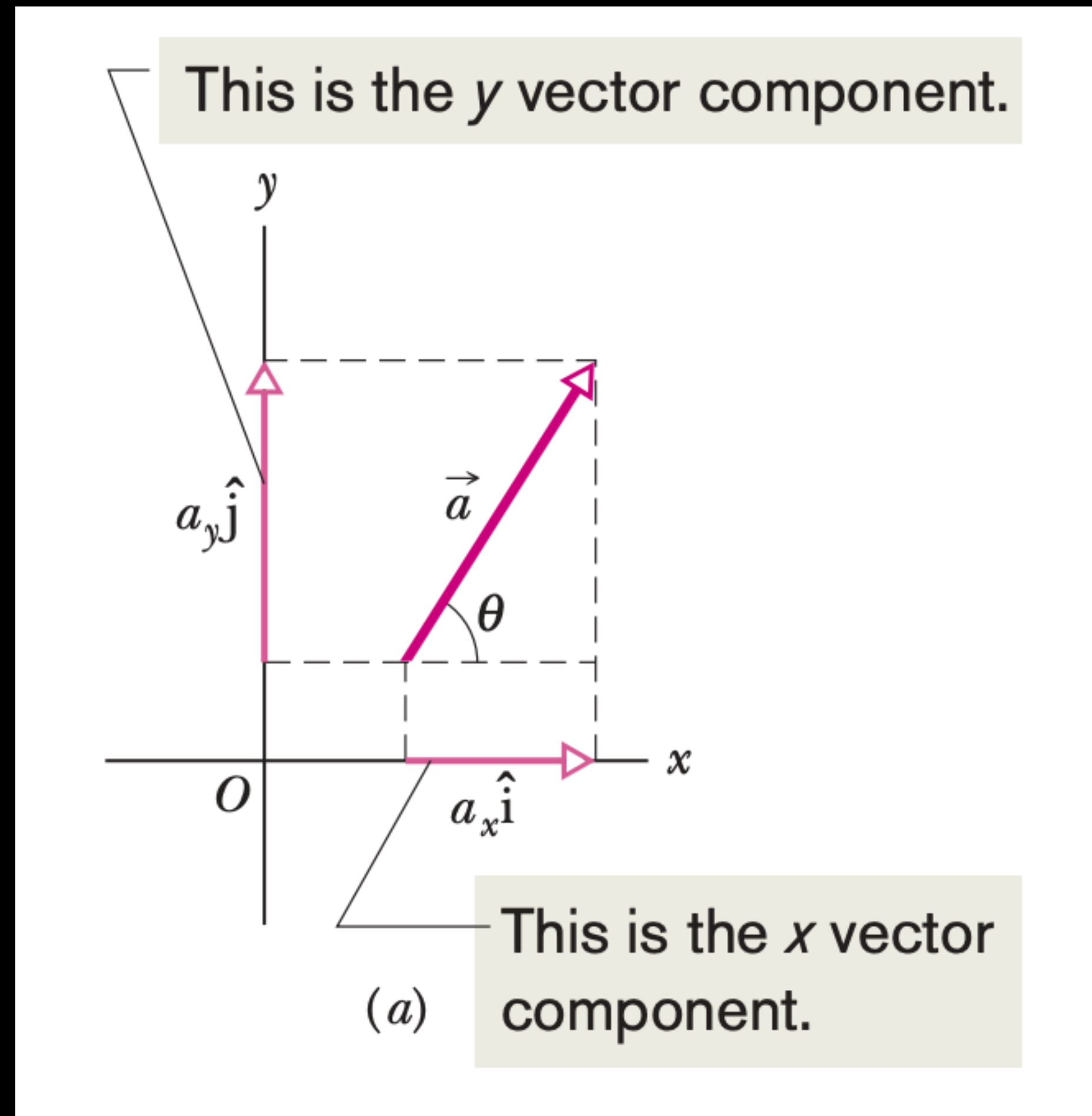
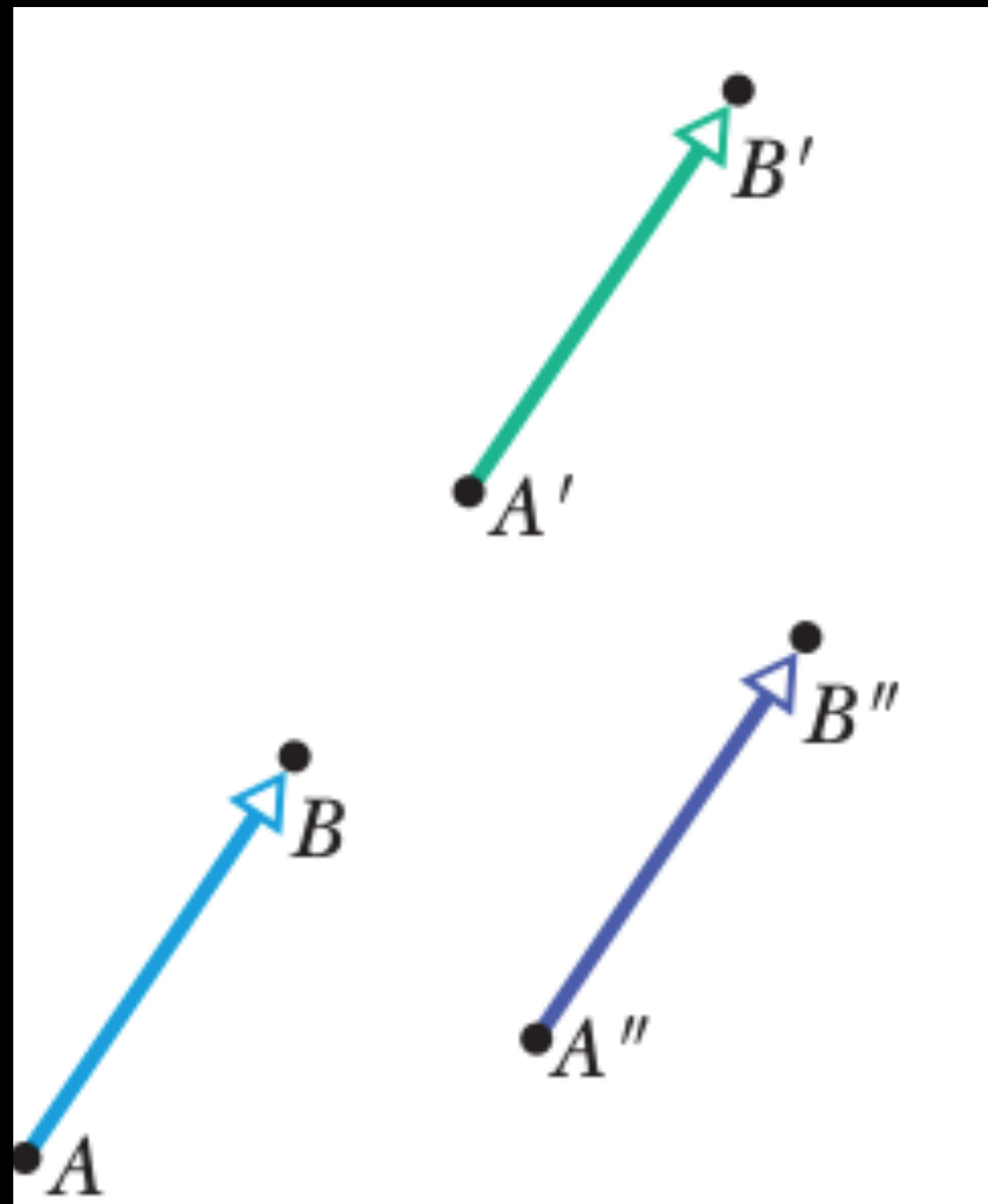
# Free-Fall Acceleration

The free-fall acceleration near Earth's surface is  $a = -g = -9.8 \text{ m/s}^2$ , hence  $g = 9.8 \text{ m/s}^2$



# Vector

- Scalar:
  - A scalar is a physical quantity that has **magnitude** but no direction.
- Vector:
  - Vectors are physical quantities that possess both **magnitude and direction**.
  - Components of vectors
  - Adding vector



# Vector

- Scalar:

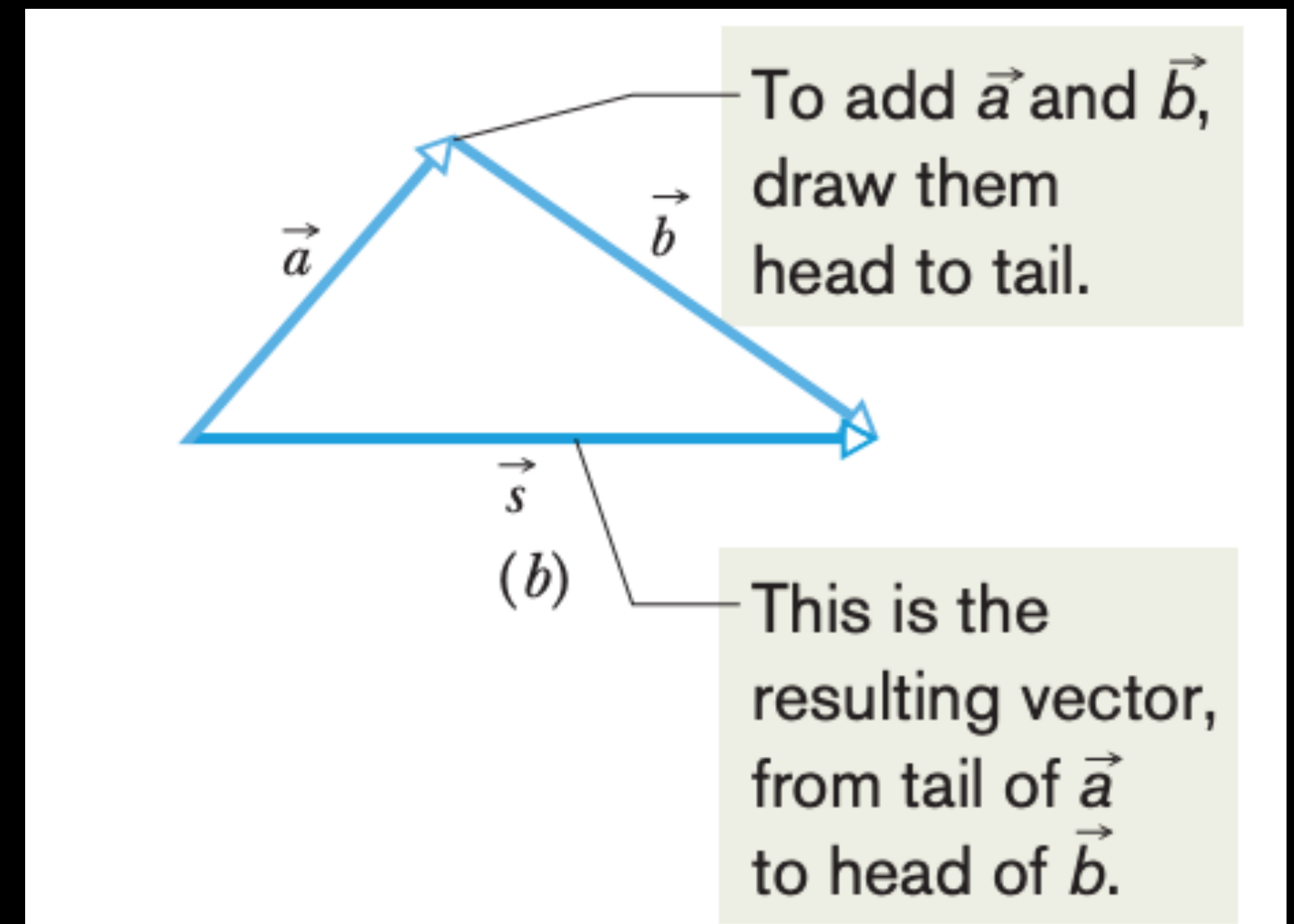
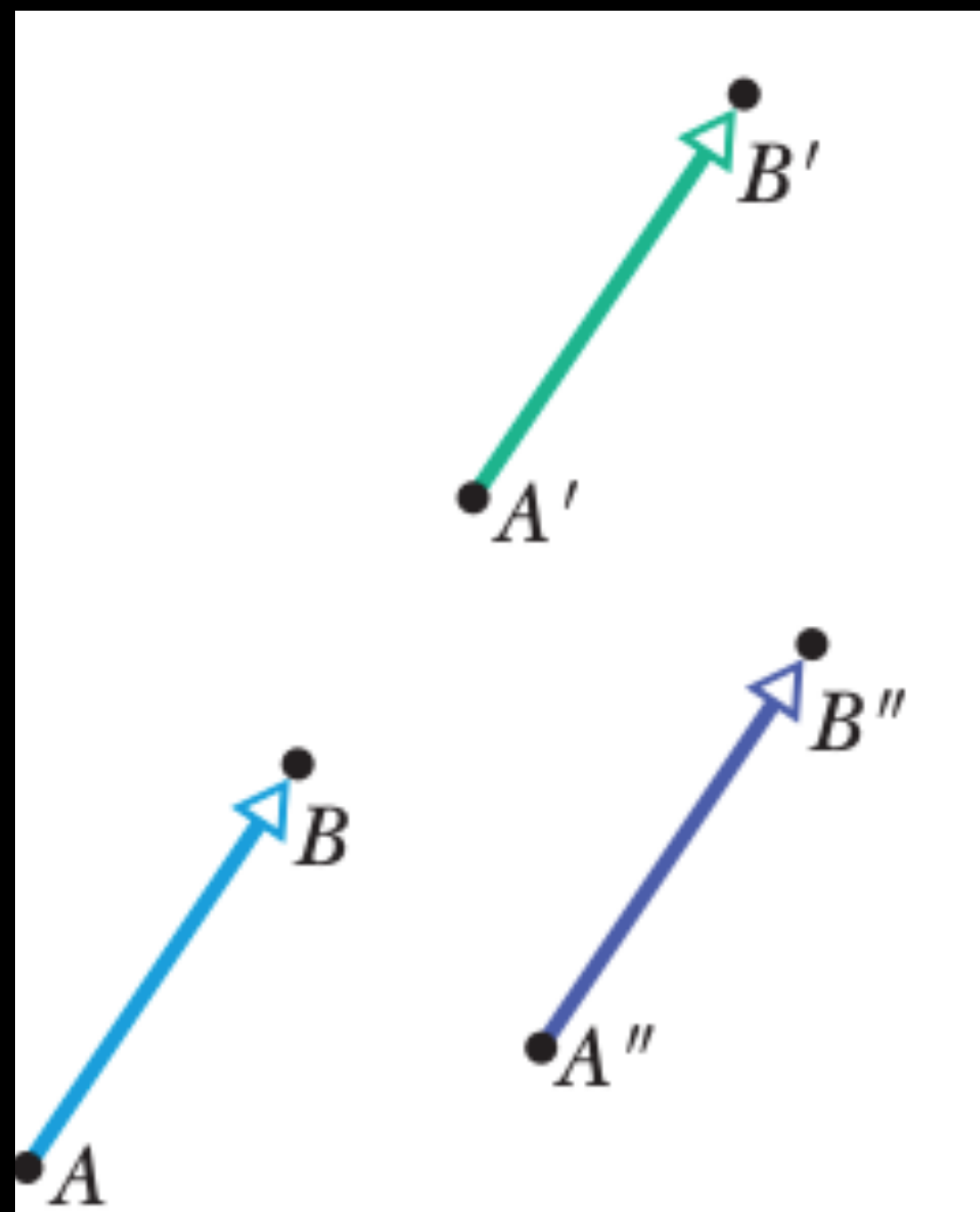
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- Vector:

- Vectors are physical quantities that possess both **magnitude and direction**.

- Components of vectors

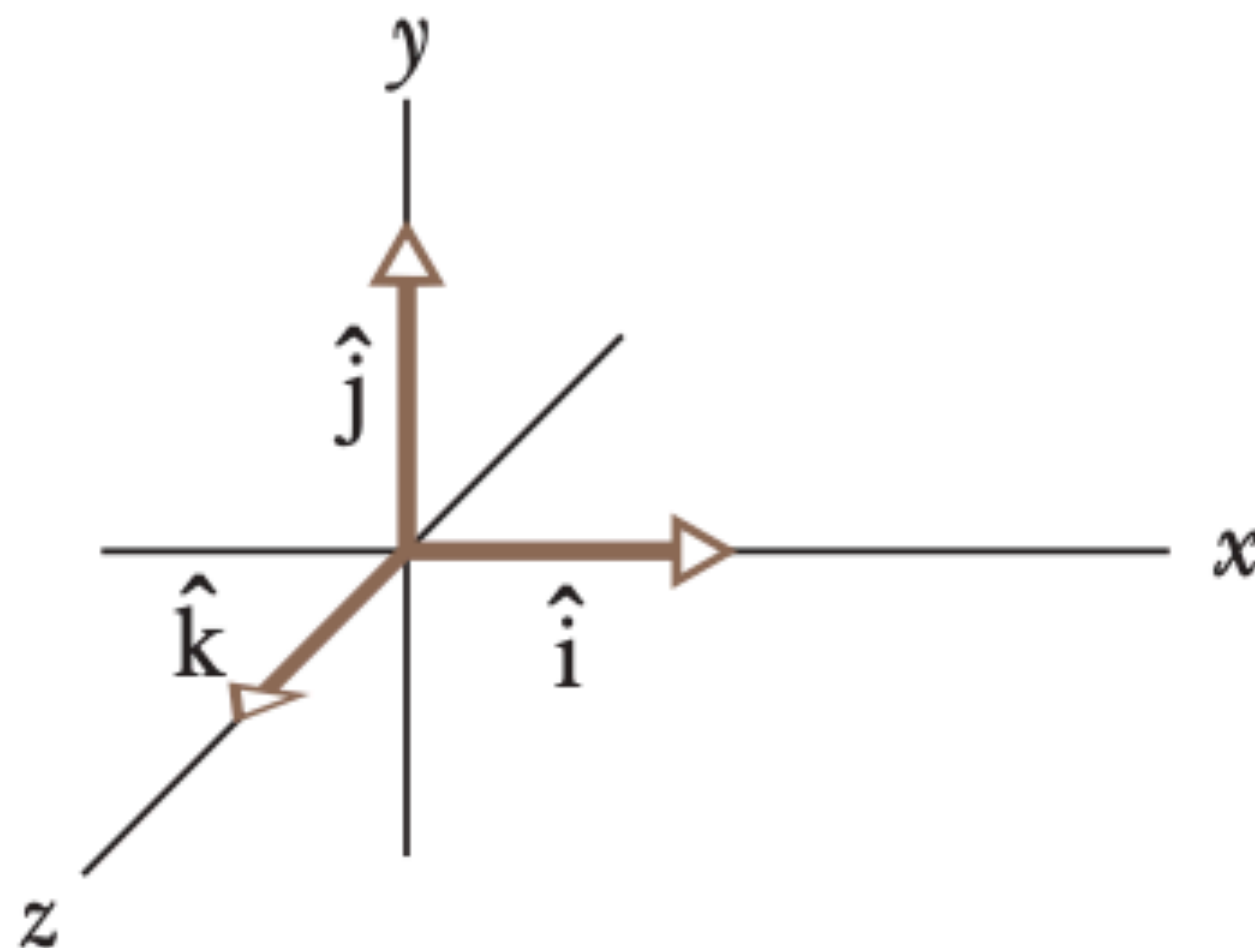
- Adding vector



# Unit Vector

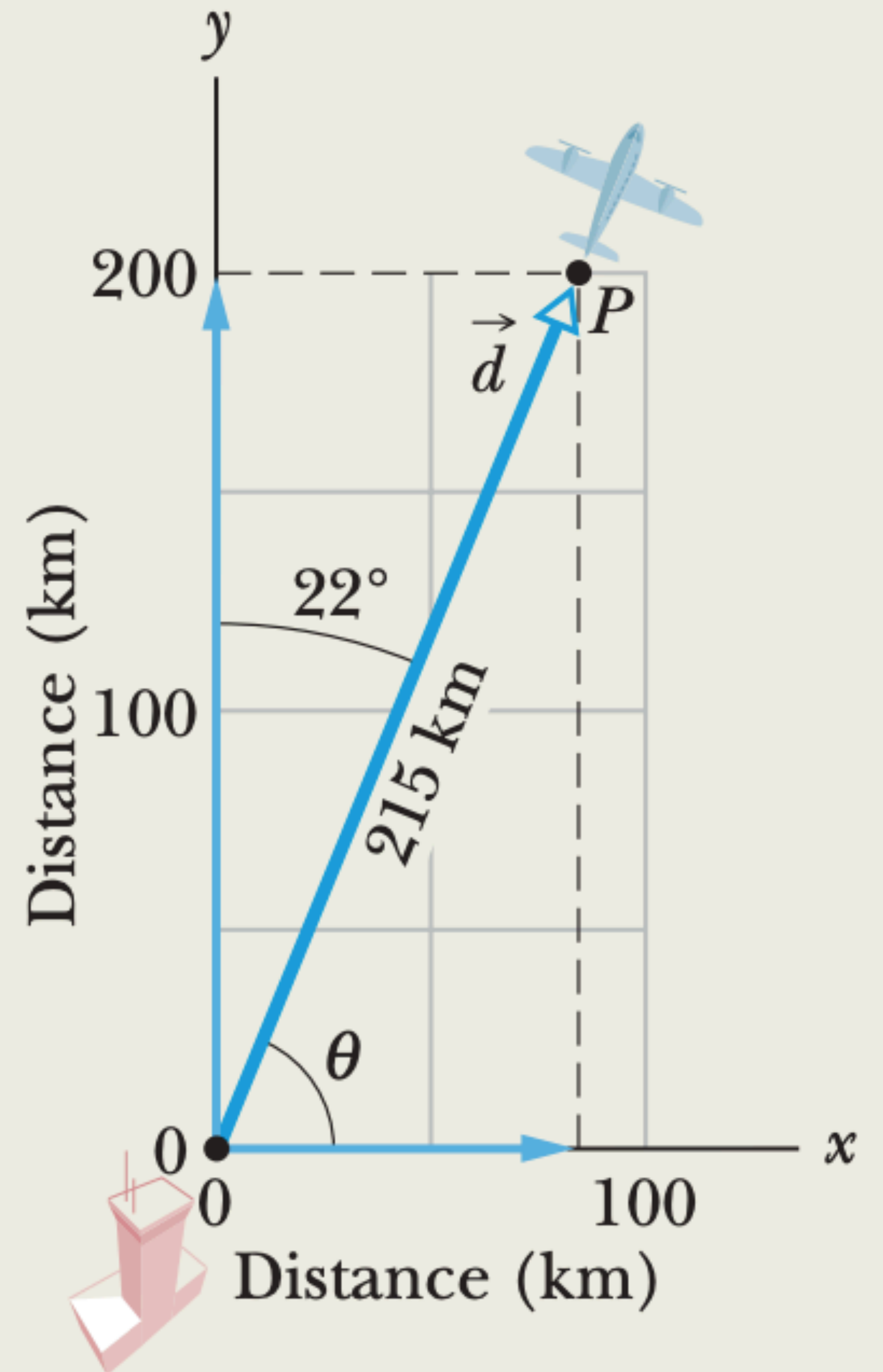
- A unit vector is a vector that has **a magnitude of exactly 1** and points **in a particular direction**. It lacks both dimension and unit. Its sole purpose is to point — that is, to specify a direction. The unit vectors in the **positive** directions of the  $x$ ,  $y$ , and  $z$ . Unit vectors are very useful for expressing other vectors;

The unit vectors point along axes.



# Position Vector

- Magnitude-angle notation

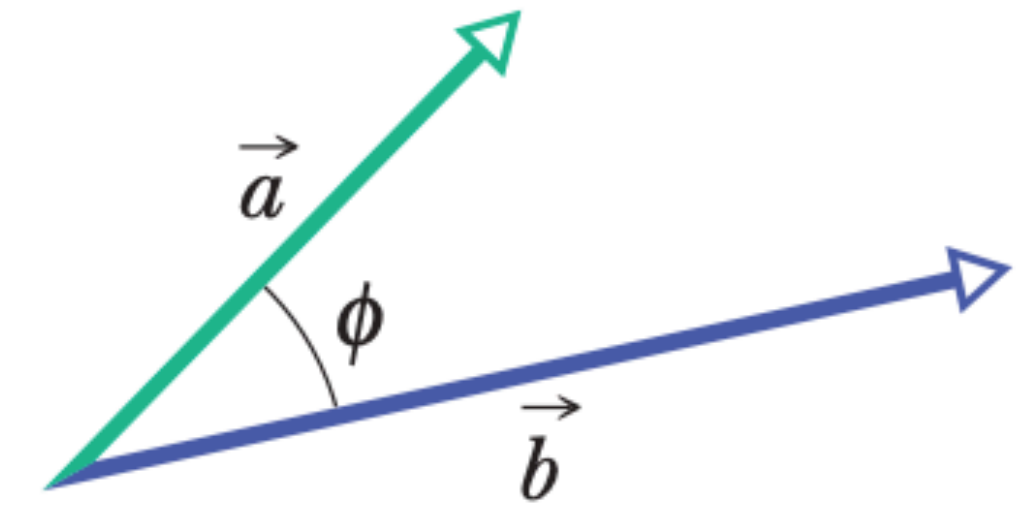


# Multiplying Vectors

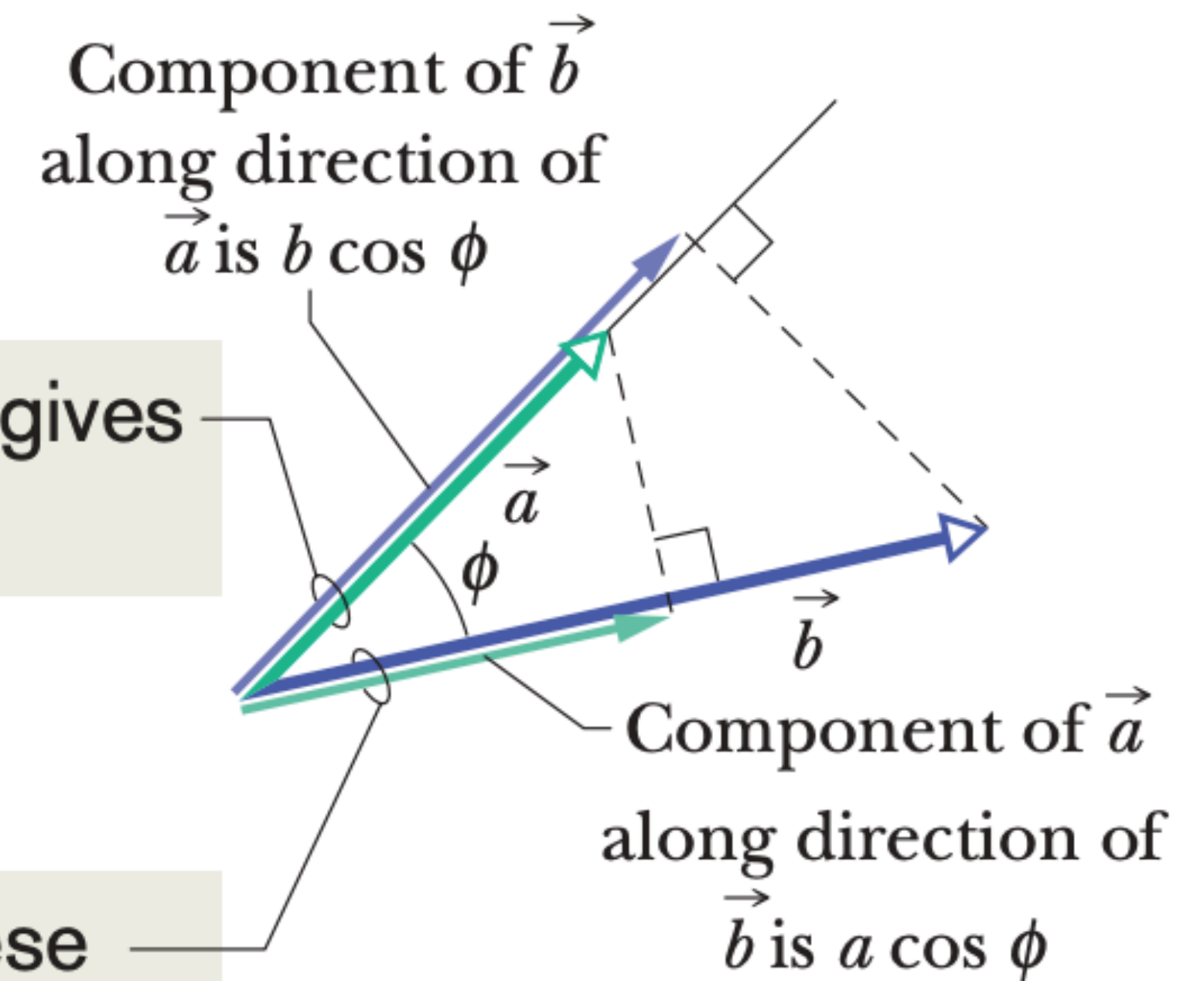
- Multiplying a vector by a scalar

Multiplying a vector by a vector  
(scalar product)

$$\vec{a} \cdot \vec{b} = ab \cos \phi$$



(a)



(b)

Multiplying these gives  
the dot product.

Or multiplying these  
gives the dot product.

# Multiplying Vectors

- Multiplying a vector by a scalar

Multiplying a vector by a vector

Scalar product:

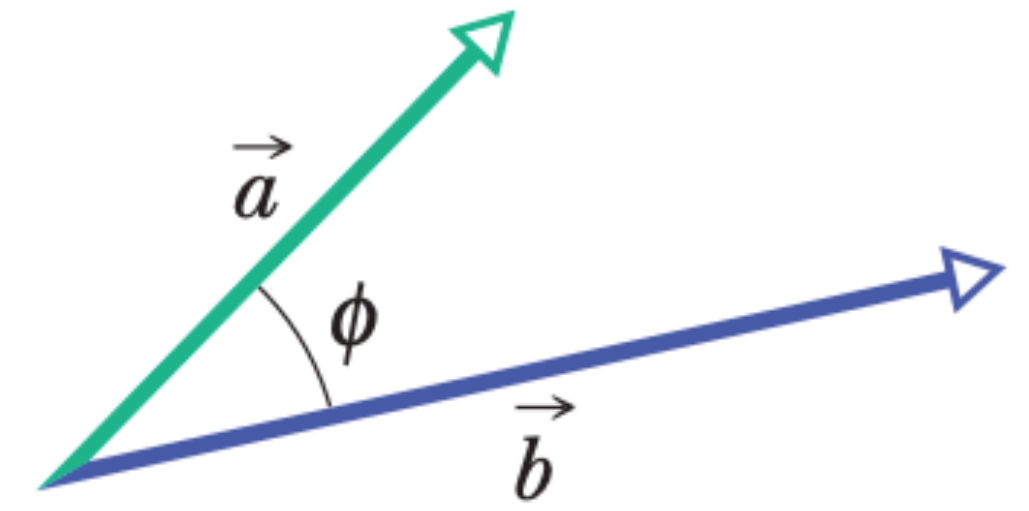
$$\vec{a} \cdot \vec{b} = ab \cos \phi$$

Vector product:

$$\vec{a} \times \vec{b} = ab \sin \phi$$

If  $\vec{a}$  and  $\vec{b}$  are parallel or anti-parallel,

What is the vector product of  $\vec{a}$  and  $\vec{b}$



$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

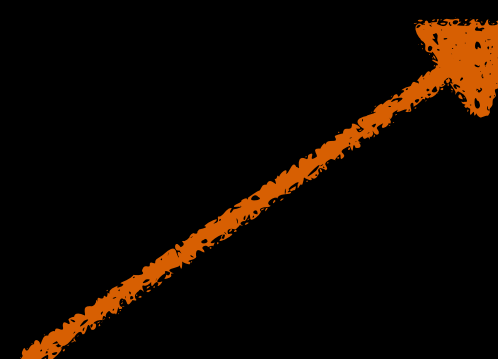
$$= \hat{i} \begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix} - \hat{j} \begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix} + \hat{k} \begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix}$$

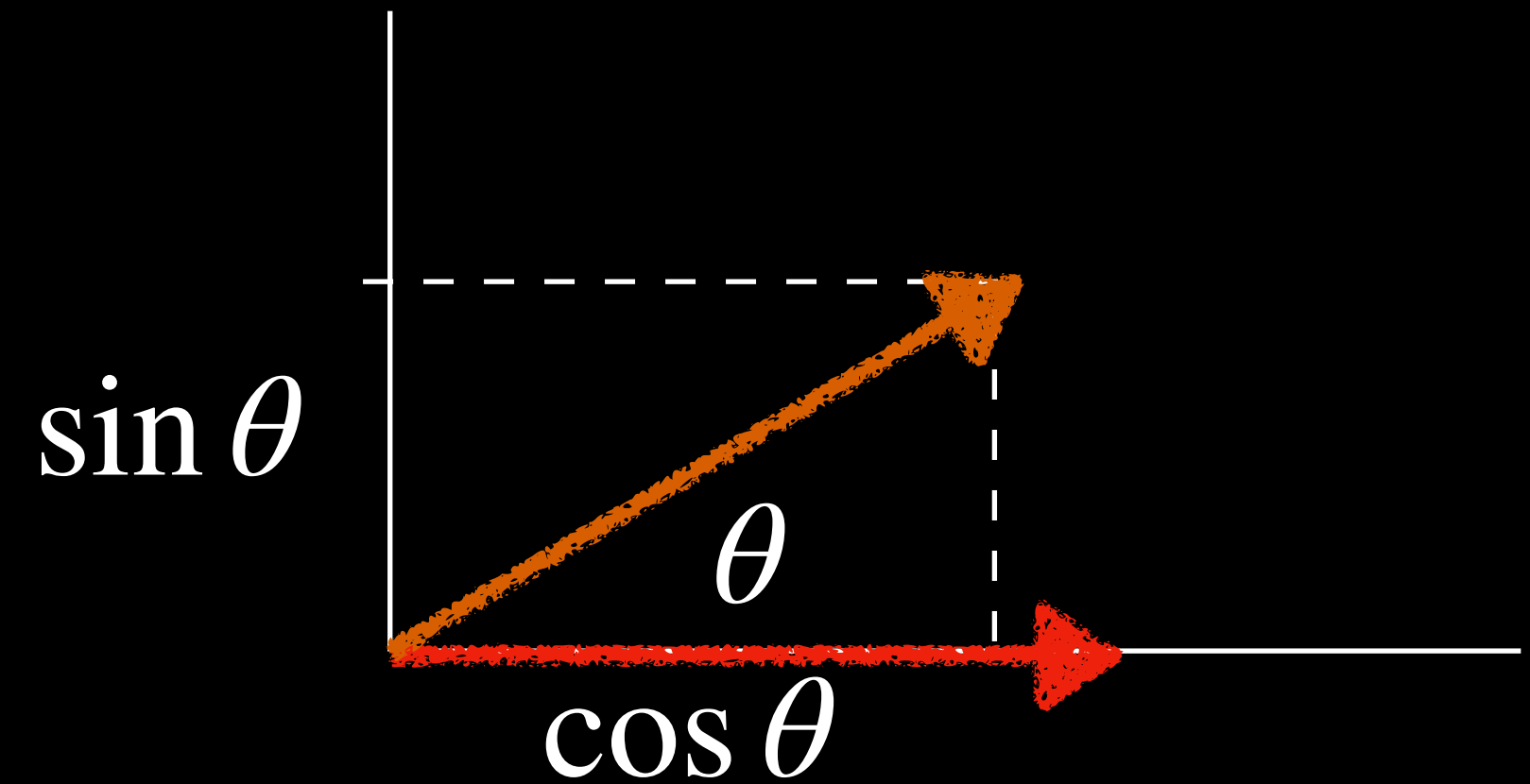
$$= (a_y b_z - b_y a_z) \hat{i} + (a_z b_x - b_z a_x) \hat{j} + (a_x b_y - b_x a_y) \hat{k}$$

$$|\vec{a} \times \vec{b}| = ab \sin \theta$$

# Rotation matrix & Vectors


$$\vec{a} = \begin{vmatrix} 1 \\ 0 \end{vmatrix}$$


$$\vec{a}' = \begin{vmatrix} \cos \theta \\ \sin \theta \end{vmatrix}$$



$$\begin{vmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{vmatrix} \begin{vmatrix} 1 \\ 0 \end{vmatrix} = \begin{vmatrix} \cos \theta \\ \sin \theta \end{vmatrix}$$

$$M_{11} * 1 + M_{12} * 0 = \cos \theta$$

$$M_{21} * 1 + M_{22} * 0 = \sin \theta$$

$$\begin{vmatrix} \cos \theta & M_{12} \\ -\sin \theta & M_{22} \end{vmatrix}$$



# Rotation matrix & Vectors

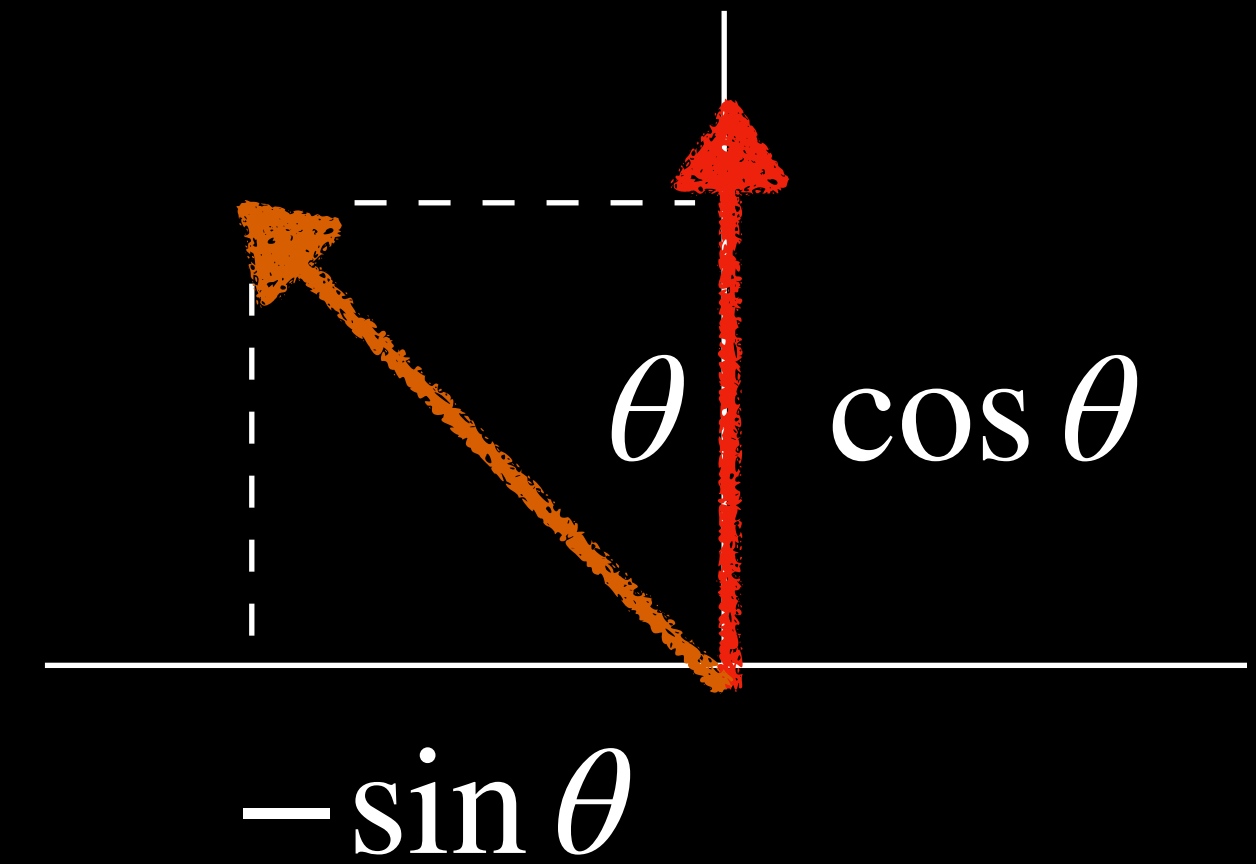
$$\begin{vmatrix} \cos \theta & M_{12} \\ \sin \theta & M_{22} \end{vmatrix}$$



$$\vec{a} = \begin{vmatrix} 0 \\ 1 \end{vmatrix}$$



$$\vec{a}' = \begin{vmatrix} -\sin \theta \\ \cos \theta \end{vmatrix}$$



$$\begin{vmatrix} \cos \theta & M_{12} \\ \sin \theta & M_{22} \end{vmatrix} \begin{vmatrix} 0 \\ 1 \end{vmatrix} = \begin{vmatrix} -\sin \theta \\ \cos \theta \end{vmatrix}$$

$$\begin{vmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{vmatrix}$$

For anti-clockwise

$$\cos \theta * 0 + M_{12} * 1 = -\sin \theta$$

$$\sin \theta * 0 + M_{22} * 1 = \cos \theta$$

$$\begin{vmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{vmatrix}$$

For clockwise

# Identity Matrix, Unit matrix

$$I = 1, \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}, \dots$$

When  $A$  is  $m \times n$ , it is a property of matrix multiplication that

$$I_m \times A = A, \quad A \times I_n = A$$

# Axis rotation (Ex: 3D)

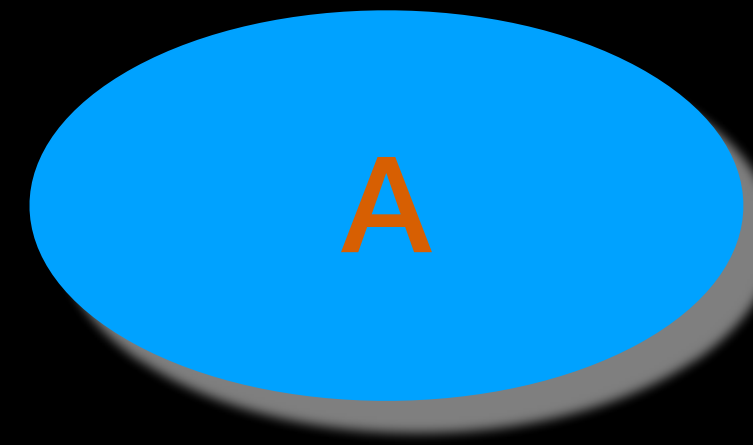
$$R_x = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{vmatrix}$$

$$R_y = \begin{vmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{vmatrix}$$

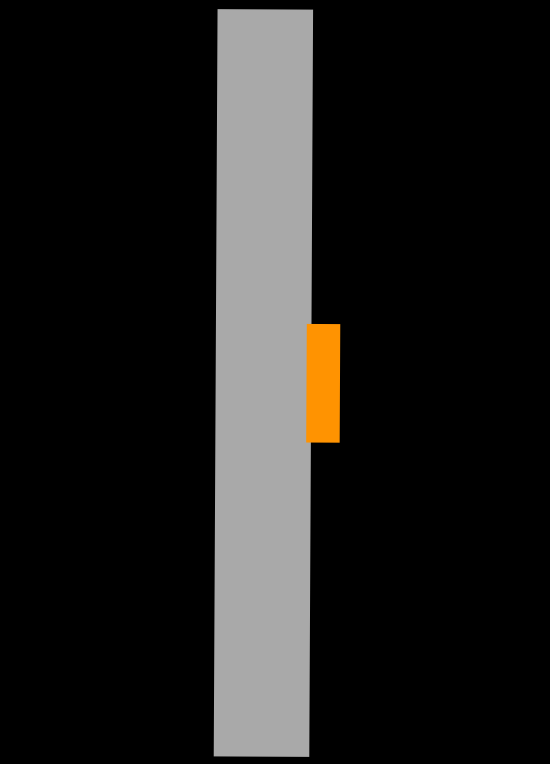
$$R_z = \begin{vmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

# Axis rotation (non-commutative)

$$R_x = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{vmatrix}$$

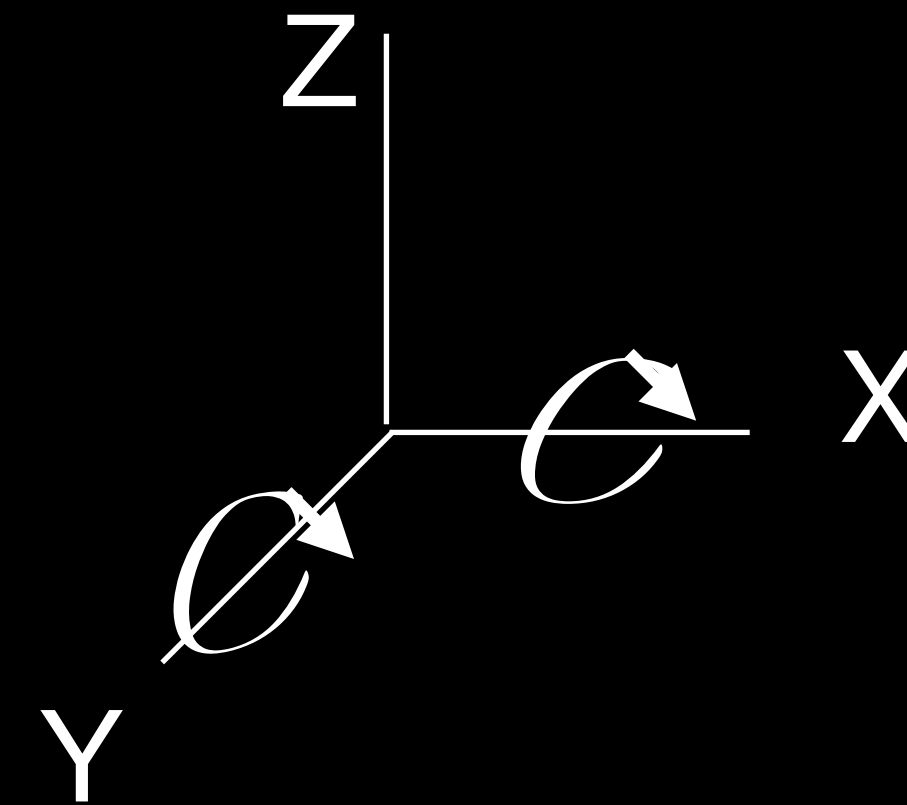


$R_x A$



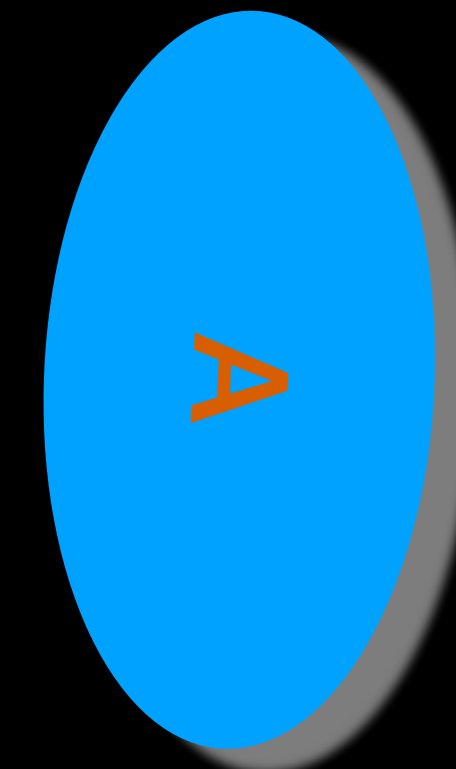
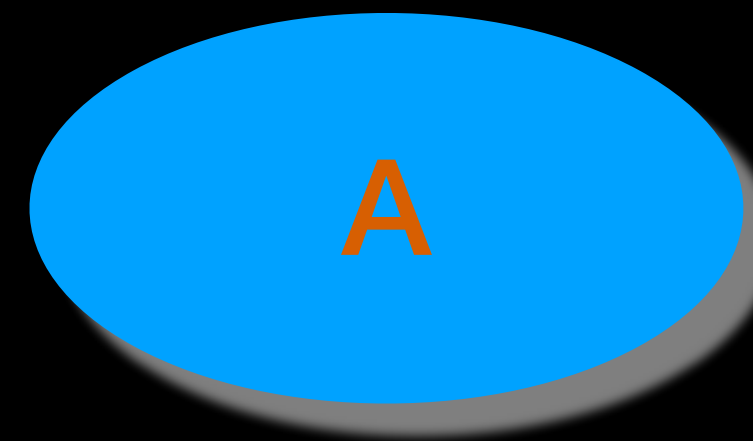
$R_y R_x A$

$$R_y = \begin{vmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{vmatrix}$$



$R_y A$

$$R_z = \begin{vmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix}$$



$R_x R_y A$

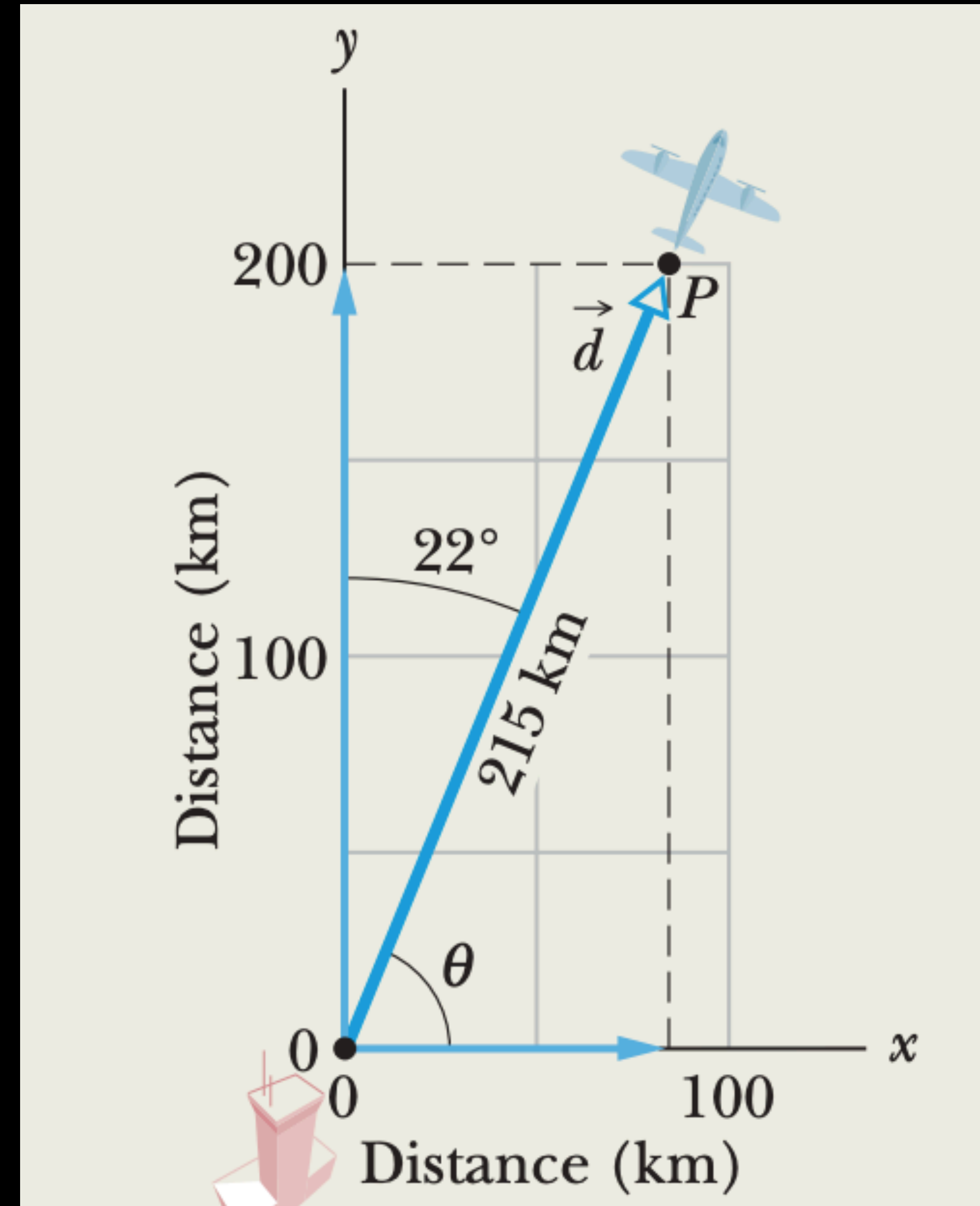


# Position vector

position vector  $\vec{r}$ , which is a vector that extends from a reference point (usually the origin) to the particle. In the unit-vector

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$$



# Position vector

then the particle's displacement  $\vec{r}$  during that time interval is

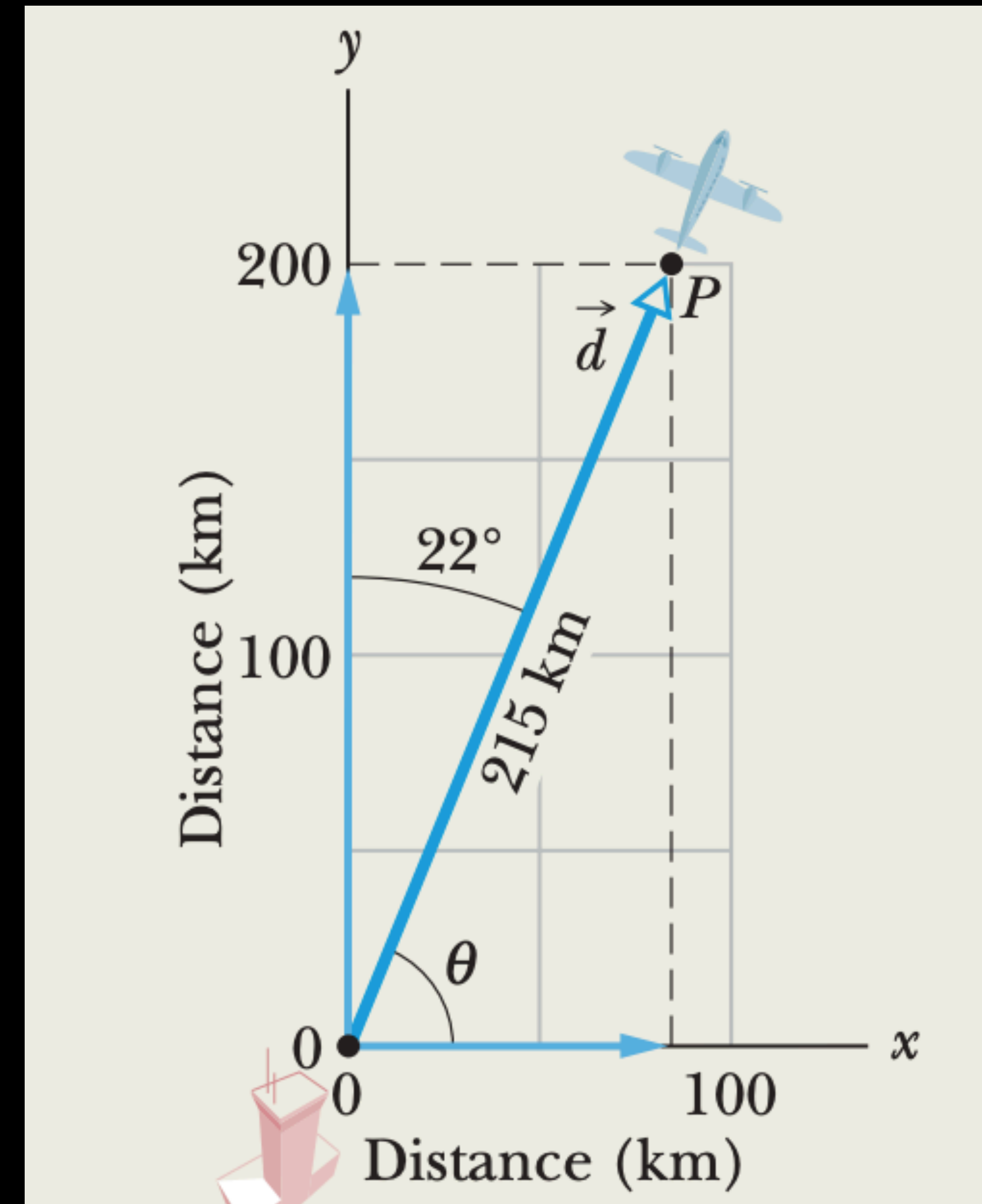
$$\vec{r}_1 = x_1 \hat{i} + y_1 \hat{j} + z_1 \hat{k}$$

$$\vec{r}_2 = x_2 \hat{i} + y_2 \hat{j} + z_2 \hat{k}$$

$$\Delta \vec{r} = \vec{r}(t_2) - \vec{r}(t_1)$$

$$= \vec{r}_2 - \vec{r}_1$$

$$= (x_2 - x_1) \hat{i} + (y_2 - y_1) \hat{j} + (z_2 - z_1) \hat{k}$$



Consider Peter's walk

$$\vec{r} = x(t)\hat{i} + y(t)\hat{j}$$

$$x(t) = -t^2 + 2t + 1$$

$$y(t) = t^2 - 2t + 1$$

What is the path of Peter from 0 s to 3 s?



# Velocity again

Instantaneous velocity

$$\vec{v} = \lim_{t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

$$\vec{v} = \lim_{t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

Acceleration:

$$\vec{a} = \lim_{t \rightarrow 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

$$\vec{a} = \lim_{t \rightarrow 0} \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{d\vec{v}}{dt}$$



Consider Peter's walk

$$\vec{r} = x(t)\hat{i} + y(t)\hat{j}$$

$$x(t) = -t^2 + 2t + 1$$

$$y(t) = t^2 - 2t + 1$$

What is the velocity of Peter?



Consider Peter's walk

$$\vec{r} = x(t)\hat{i} + y(t)\hat{j}$$

$$x(t) = -t^2 + 2t + 1$$

$$y(t) = t^2 - 2t + 1$$

What is the acceleration of Peter?



# More practice

Here are four descriptions of the position (in meters) of a puck as it moves in an  $xy$  plane:

(1)  $x = -3t^2 + 4t - 2$  and  $y = 6t^2 - 4t$  (3)  $\vec{r} = 2t^2\hat{i} - (4t + 3)\hat{j}$

(2)  $x = -3t^3 - 4t$  and  $y = -5t^2 + 6$  (4)  $\vec{r} = (4t^3 - 2t)\hat{i} + 3\hat{j}$

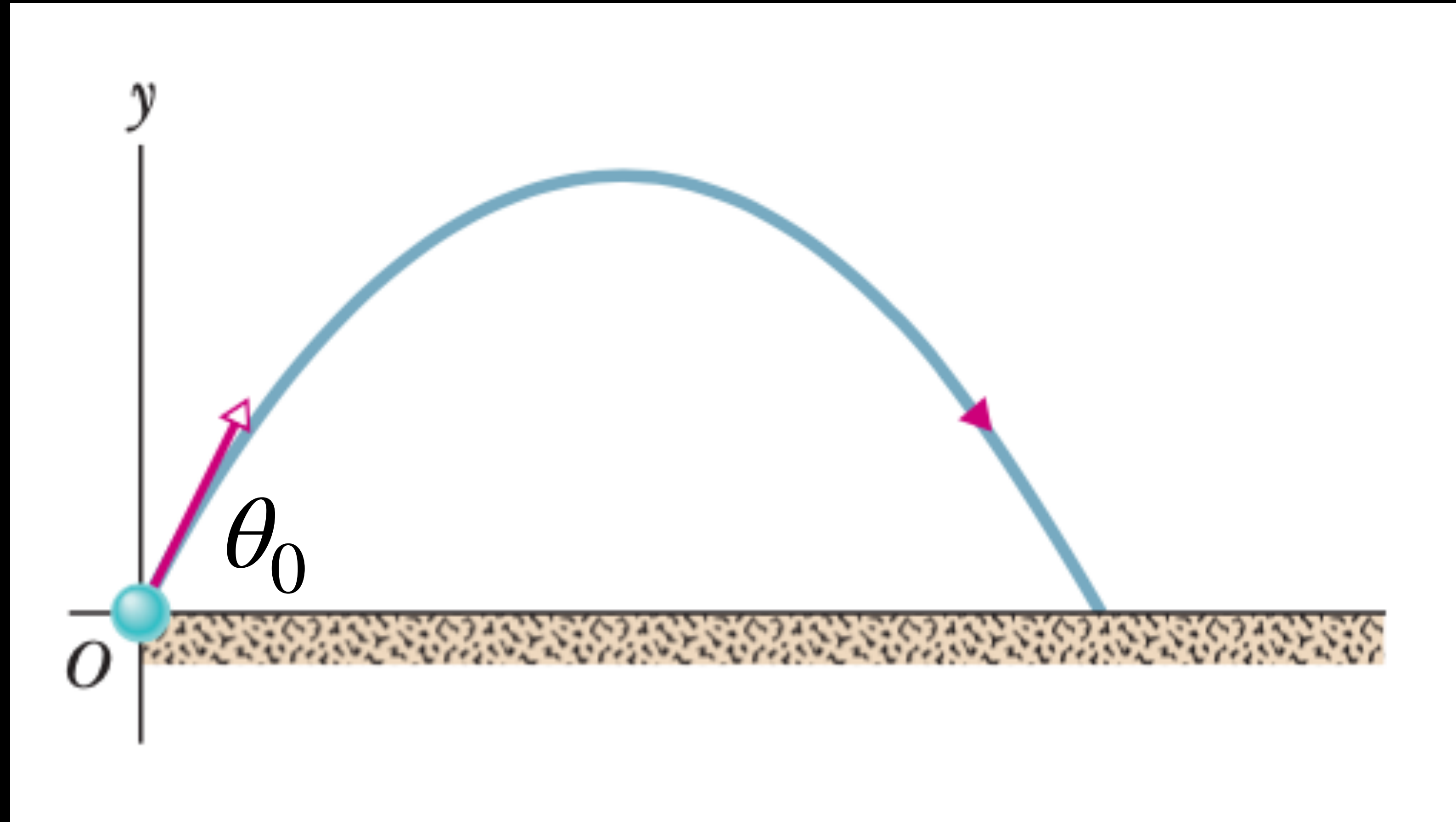
Are the  $x$  and  $y$  acceleration components constant? Is acceleration  $\vec{a}$  constant?

# Projectile motion

$$\vec{V} = V_{0x}\hat{i} + V_{0y}\hat{j}$$

$$V_{0x} = V_o \cos \theta_0$$

$$V_{0y} = V_o \sin \theta_0$$



# Projectile motion

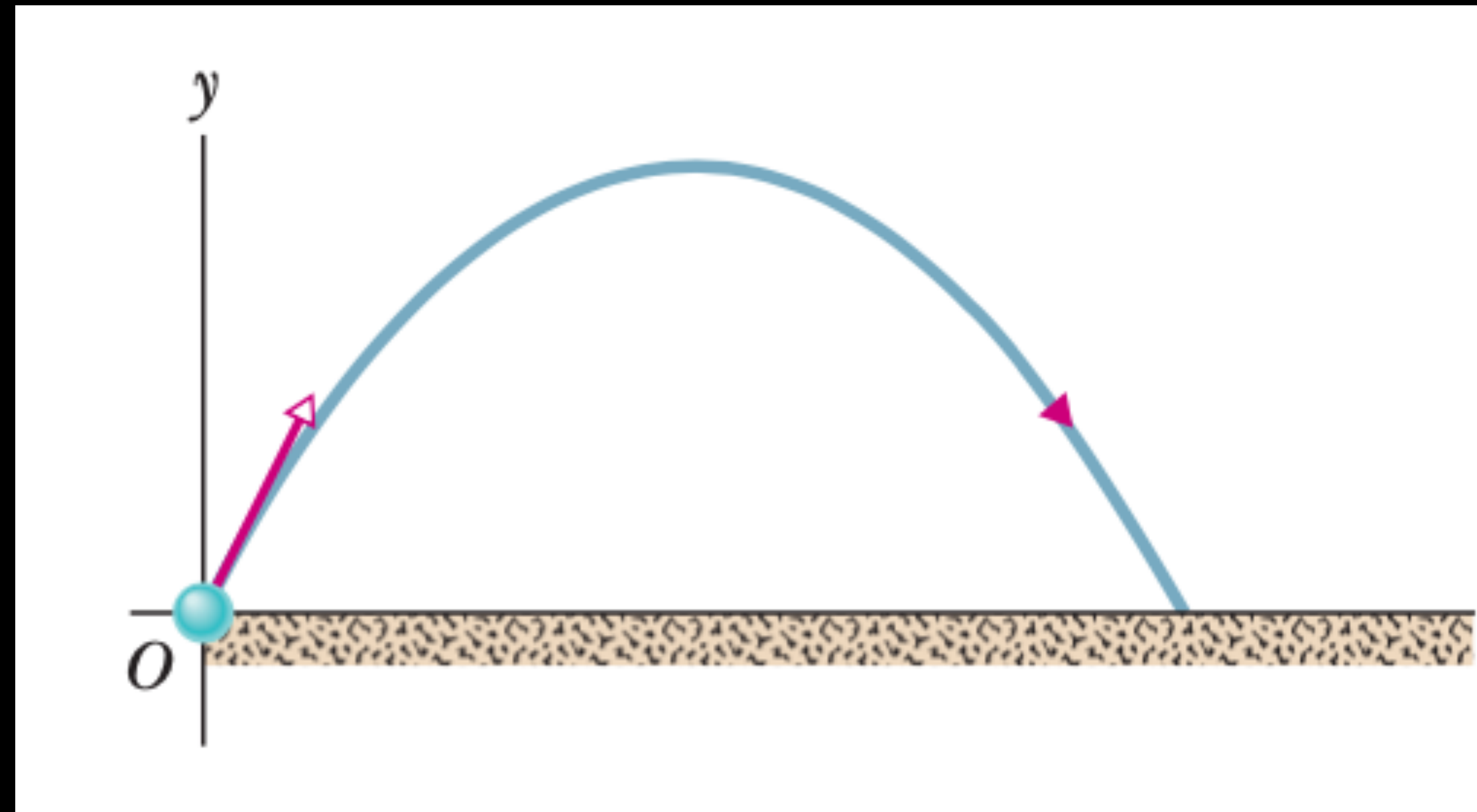
X axis:

$$X = V_{0x}t = V_0 \cos \theta_0 t$$

Y axis:

$$Y = V_{0y}t - \frac{gt^2}{2} = V_0 \sin \theta_0 t - \frac{gt^2}{2}$$

$$V_y = V_{0y} + gt, \quad V_y^2 = V_{0y}^2 + 2g(y - y_0)$$



# Projectile motion

X axis:

$$X = V_{0x}t = V_0 \cos \theta_0 t \quad t = \frac{x}{V_0 \cos \theta_0}$$

Y axis:

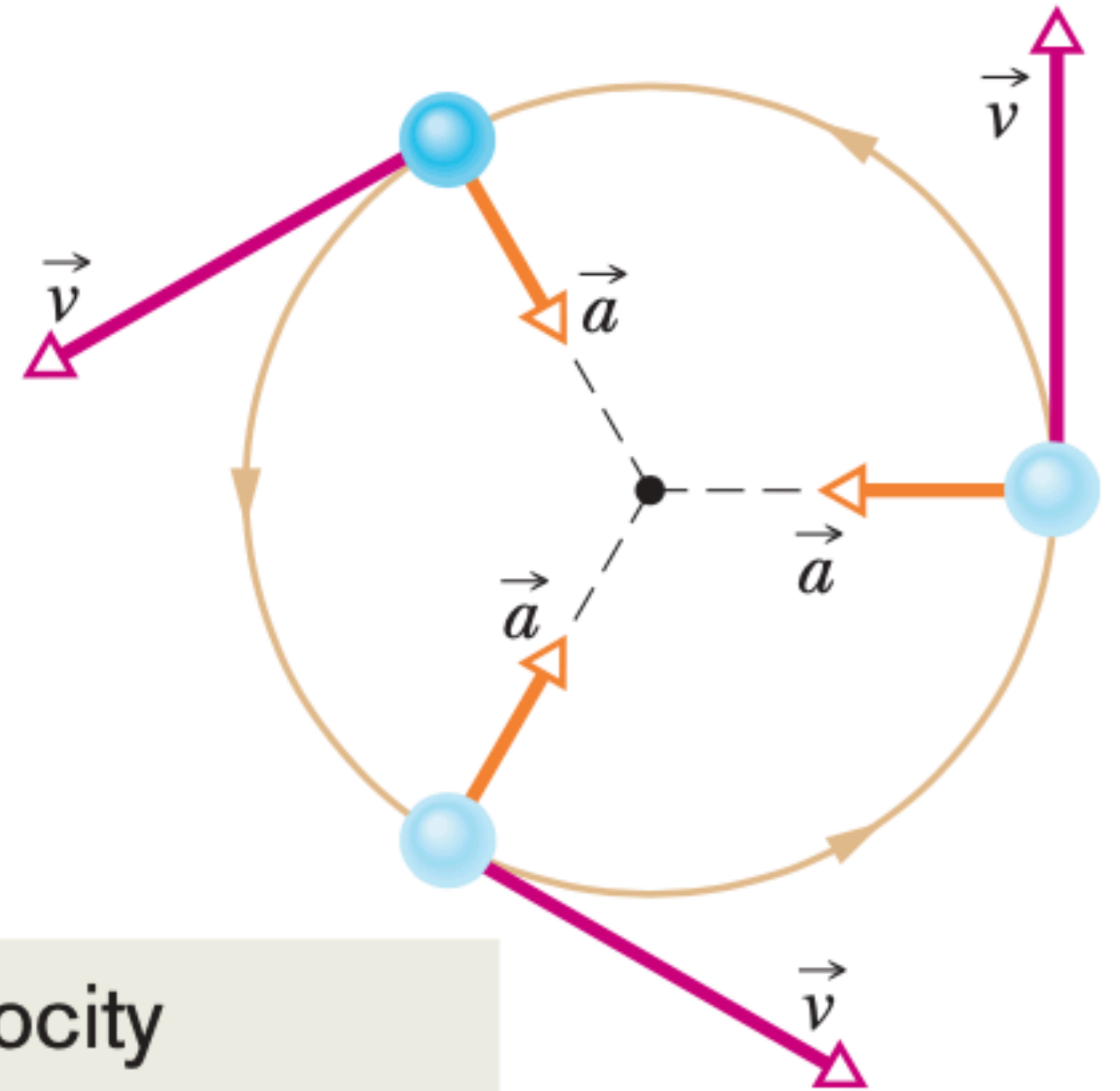
$$Y = V_{0y}t - \frac{gt^2}{2} = V_0 \sin \theta_0 t - \frac{gt^2}{2}$$

$$= \tan \theta x - \frac{gx^2}{2V_0^2 \cos^2 \theta}$$

**Trajectory!!**

# Circular motion

The acceleration vector always points toward the center.



The velocity vector is always tangent to the path.